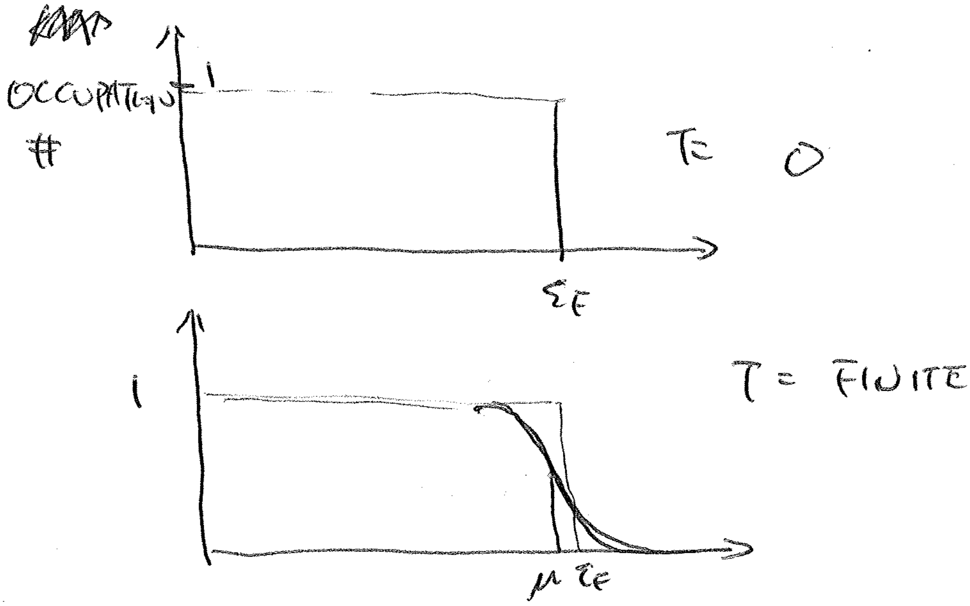
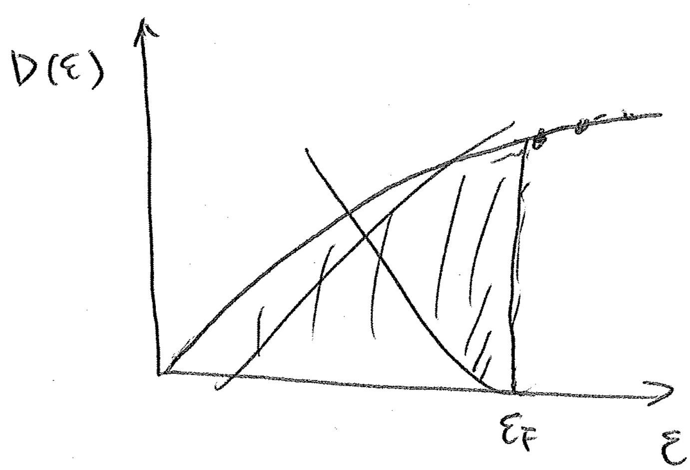
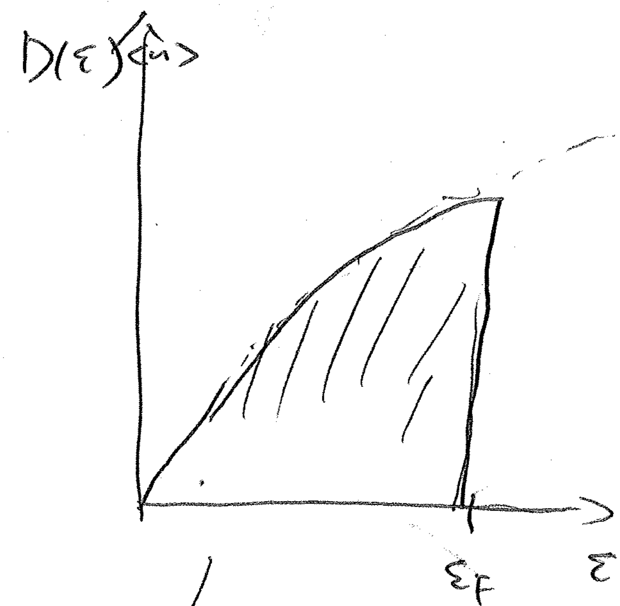
T=0

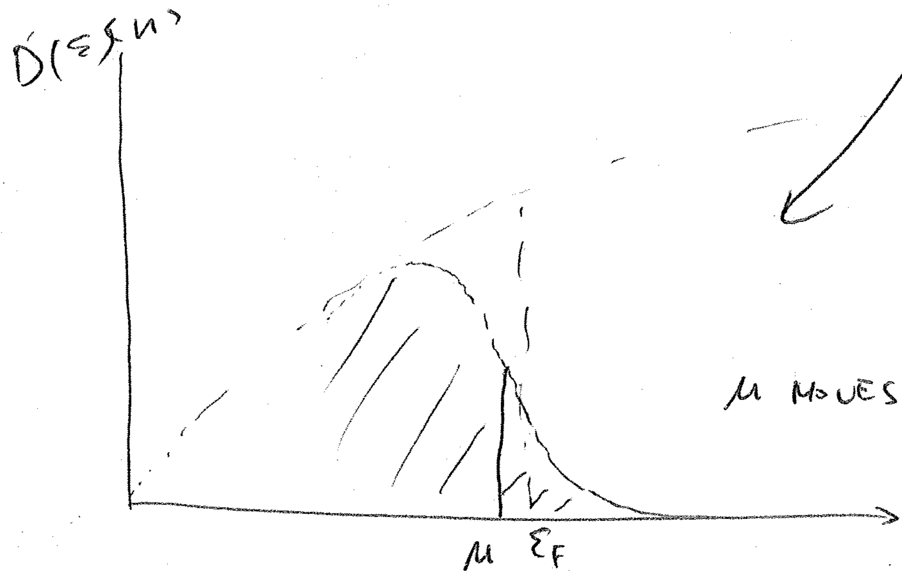


TR 1:30 ~ 4:20 pm
6 hrs

WHAT HAPPENS AT T = FINITE







μ MOVES SLIGHTLY TO LEFT TO KEEP N_{TOTAL} CONST

---

i.e.

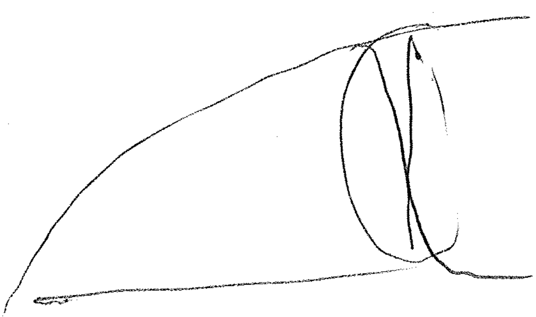
$$N_{TOTAL} = \int_0^\infty g(\varepsilon)\, \cancel{\langle n\rangle}\, f(\varepsilon)\, d\varepsilon$$

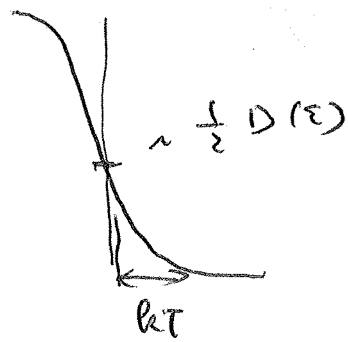
$$= \int_0^\infty g(\varepsilon) \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}\, d\varepsilon$$

$$U = \int_0^\infty \varepsilon D(\varepsilon)\, f(\varepsilon)\, d\varepsilon = \int_0^\varepsilon \varepsilon D(\varepsilon) \frac{1}{e^{(\varepsilon-\mu)/kT}+1}\, d\varepsilon$$

REMEMBE

$$\frac{\partial U}{\partial T} = \underline{\underline{C_V}}$$





$$\# = \frac{1}{2} D(\varepsilon)\, kT$$

$$\Delta E \cong kT$$

$$\# \,\Delta E \cong (kT)^2$$

$$\frac{\Delta E}{\partial T} \cong \underline{T}$$

BUT TO DO IT PRECISELY YOU NEED TO EMPLOY SUMMERFELD EXPANSION

$$D(\varepsilon) = \frac{\pi V}{2}\left(\frac{8m}{h^2}\right)^{3/2} \varepsilon^{1/2} = D_0\, \varepsilon^{1/2}$$

$$N = \int_0^\infty D(\varepsilon) f(\varepsilon)\, d\varepsilon \qquad f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$

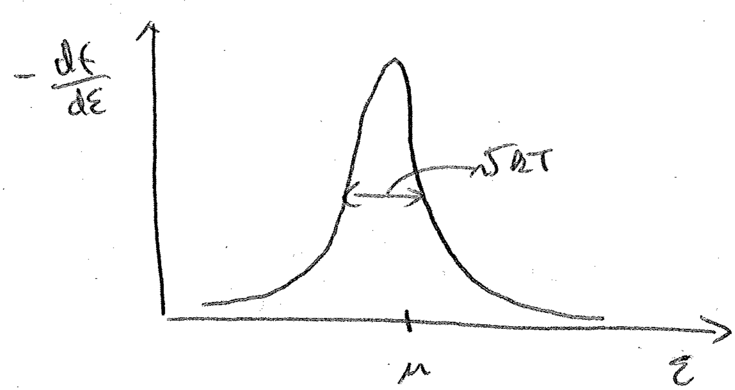
$$= \int_0^\infty D_0\, \varepsilon^{1/2} f(\varepsilon)\, d\varepsilon$$

INTEGRATE BY PARTS

$$N = \frac{2}{3} D_0\, \underbrace{\varepsilon^{3/2} f(\varepsilon)\Big]_0^\infty}_{\text{GOES TO ZERO}} + \frac{2}{3} D_0 \int_0^\infty \varepsilon^{3/2}\left(-\frac{df}{d\varepsilon}\right) d\varepsilon$$

$$-\frac{df}{d\varepsilon} = -\frac{d}{d\varepsilon}\left(e^{(\varepsilon-\mu)/kT}+1\right)^{-1} = \frac{1}{kT}\,\frac{e^x}{(e^x+1)^2}$$

$$x = \frac{\varepsilon-\mu}{kT}$$



$$N = \frac{2}{3} D_0 \int_0^\infty \varepsilon^{3/2}\left(-\frac{df}{d\varepsilon}\right) d\varepsilon = \frac{2}{3} D_0 \int_{-\mu/kT}^\infty \frac{e^x}{(e^x+1)^2}\, \varepsilon^{3/2}\, dx$$

INTEGRAND GOES TO ZERO IF $|\varepsilon-\mu| >> kT$

$$\varepsilon^{3/2} = \mu^{3/2} + (\varepsilon-\mu)\frac{d}{d\varepsilon}\varepsilon^{3/2}\Big|_{\varepsilon=\mu} + \frac{1}{2}(\varepsilon-\mu)^2\frac{d^2}{d\varepsilon^2}\varepsilon^{3/2}\Big|_{\varepsilon=\mu}$$

$$\varepsilon^{3/2} = \mu^{3/2} + \frac{3}{2}(\varepsilon-\mu)\mu^{1/2} + \frac{3}{8}(\varepsilon-\mu)^2\mu^{-1/2} + \dots$$

$$N = \frac{2}{3}D_0\int_{-\infty}^{\infty}\frac{e^x}{(e^x+1)^2}\left[\mu^{3/2} + \frac{3}{2}xkT\mu^{1/2} + \frac{3}{8}(xkT)^2\mu^{-1/2} + \dots\right]$$

1st $$\int_{-\infty}^{\infty}\frac{e^x}{(e^x+1)^2}dx = \int_{-\infty}^{\infty}-\frac{df}{d\varepsilon}d\varepsilon = f(-\infty) - f(\infty)$$
$$= 1 - 0 = 1$$

2nd $$\int_{-\infty}^{\infty}\frac{x e^x}{(e^x+1)^2} = \int_{-\infty}^{\infty}\frac{x}{(e^x+1)(1+e^{-x})}dx = 0$$ ODD fn

3rd $$\int_{-\infty}^{\infty}\frac{x^2 e^x}{(e^x+1)^2}dx = \frac{\pi^2}{3}$$

$$N = \frac{2}{3}D_0\mu^{3/2} + \frac{1}{4}D_0(kT)^2\mu^{-1/2}\frac{\pi^2}{3} + \dots$$

$$= N\left(\frac{\mu}{\varepsilon_F}\right)^{3/2} + N\frac{\pi^2}{8}\frac{(kT)^2}{\varepsilon_F^{3/2}\mu^{1/2}}$$

$$\frac{\mu}{\varepsilon_F} \approx \left[1 - \frac{\pi^2}{8}\left(\frac{kT}{\varepsilon_F}\right)^2\right]^{2/3} \qquad [1 - x]^{2/3} \approx 1 - \frac{2}{3}x$$

$$\approx 1 - \frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2$$

$$U = \frac{3}{5} N \frac{\mu^{5/2}}{\varepsilon_F^{3/2}} + \frac{3\pi^2}{8} N \frac{(kT)^2}{\varepsilon_F}$$

$\Downarrow$ ☆ MORE ALGEBRA?

$$U = \frac{3}{5} N \varepsilon_F + \frac{\pi^2}{4} N \frac{(kT)^2}{\varepsilon_F}$$

$$\frac{dU}{dT} = \frac{\pi^2}{\cancel{4}2} N \frac{k^2 T}{\varepsilon_F} = C_V$$

BLACK BODY RADIATON