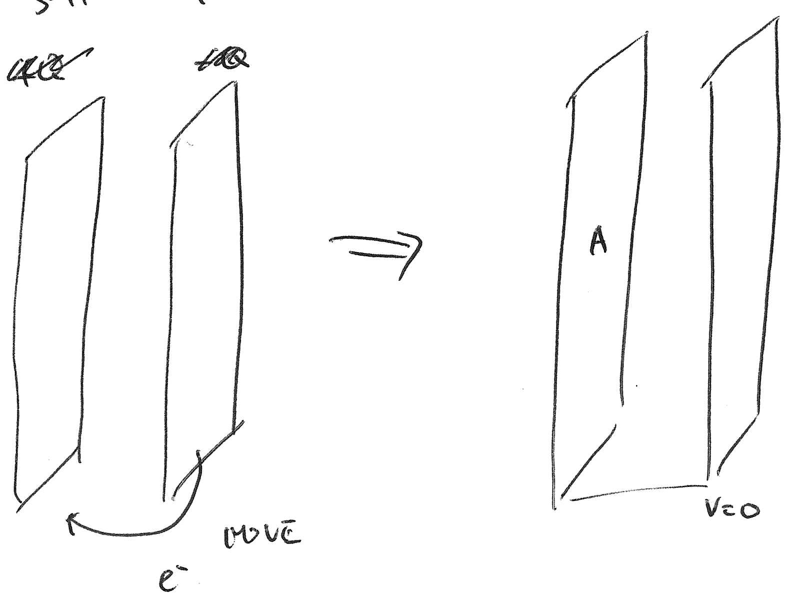
WEEK 4

DEMO: MON EM 357 LARGE PARALLEL PLATES
WED EM 352, 357 SCREW DRIVER DISCHARGE
FRI EM 351 LEYDEN JAR

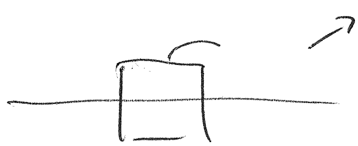
CAPACITANCE

$$C = \frac{Q}{|\Delta V|} \quad \text{(FARAD)} \quad = 1\ C/V$$

SUPPOSE I MOVE CHARGE



$$\sigma = \frac{Q}{A}$$

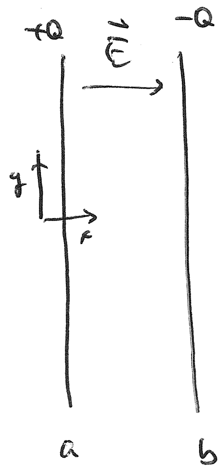


GAUSS'S LAW

$$|\vec{E}| = \frac{\sigma}{\varepsilon_0}$$

$$|E| = \frac{Q}{\varepsilon_0 A}$$

$$V_+ =$$



$$V(b) = 0$$

$$W_{ab} = \int_a^b q_T E \, dx = q_T E (b - a)$$

$$= U(a) - U(b) = q_T (V(a) - V(b))$$

$$\cancel{q_T} E (b-a) = \cancel{q_T} (V(a) - 0)$$

$$Ed = V(a)$$

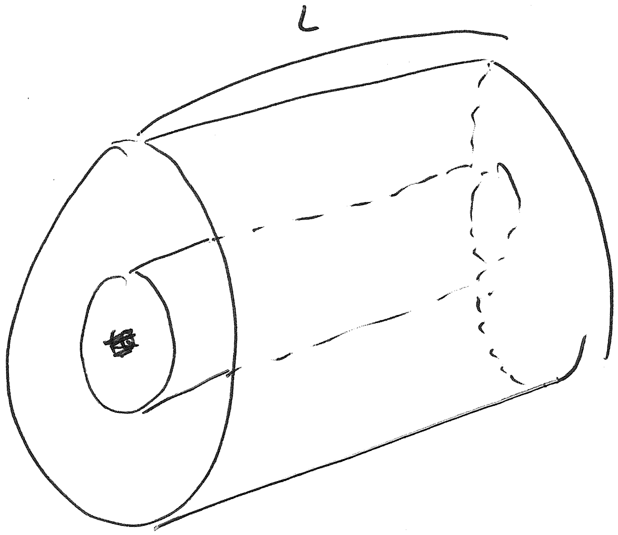
$$V(b) = 0$$

$$C = \frac{Q}{V}$$

$$V = \frac{Qd}{\varepsilon_0 A} \qquad \therefore \quad C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}d}{\varepsilon_0 A}}$$

$$C = \frac{\varepsilon_0 A}{d}$$

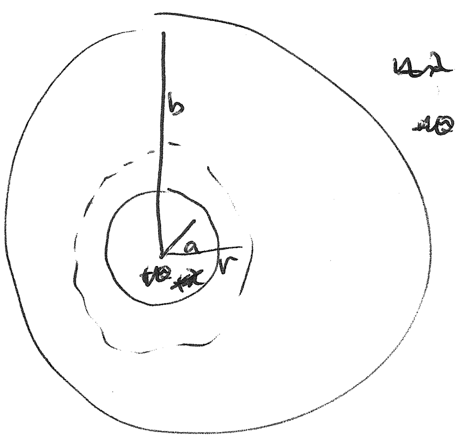
HOW ABOUT FOR TWO METALLIC CYLINDERS?



L IS LONG

CONTRIBUTION FROM FRINGE FIELD IS LOW

HOW DO WE CALCULATE CAPACITANCE?



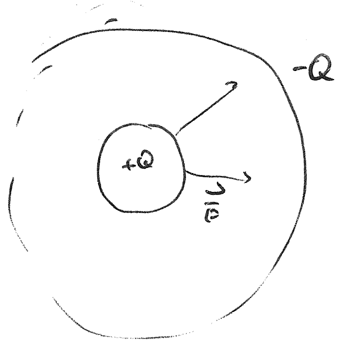
GAUSSIAN SURFACE IS A CYLINDER

$$\Phi = \frac{Q_{ENC}}{\varepsilon_0}$$

$$\Phi = E(r) \cdot 2\pi r \cancel{L} = E(r) \cdot 2\pi r L$$

$$\frac{Q_{ENC}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \cancel{Q_{ENC}} = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{Q}{2\pi r L \varepsilon_0}$$



TAKE $V(a) = 0$

$$W_{ab} = \int q_T \vec{E} \cdot dr\, \hat{r}$$

$$= \int_a^b q_T \frac{Q}{2\pi r L \varepsilon_0} \cdot dr$$

$$= \frac{q_T Q}{2\pi L \varepsilon_0} \ln r \Big]_a^b$$

$$= \frac{q_T Q}{2\pi L \varepsilon_0} \ln b/a$$

$$= q_T \left(V(a) - V(b)\right)$$

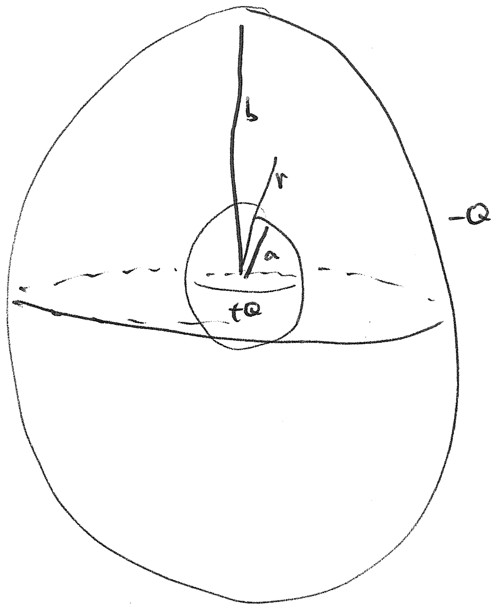
$$V(a) - V(b) = \frac{\cancel{q_T} Q}{2\pi L \varepsilon_0} \ln b/a$$

$$V(b) = \frac{-Q}{2\pi L \varepsilon_0} \ln b/a$$

$$|\Delta V| = \frac{Q}{2\pi L \varepsilon_0} \ln b/a$$

$$C = \frac{Q}{|\Delta V|} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi L \varepsilon_0} \ln b/a} = \frac{2\pi L \varepsilon_0}{\ln(b/a)}$$

SPHERICAL CAPACITOR



$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$$

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr$$

$$= \frac{1}{4\pi\varepsilon_0} \left. -\frac{Q}{r} \right]_a^b$$

$$= \frac{1}{4\pi\varepsilon_0} \left[-\frac{Q}{b} + \frac{Q}{a} \right]$$

$$V(a) - V(b) = \frac{1}{4\pi\varepsilon_0} \left[\frac{Q}{a} \cancel{+} - \frac{Q}{b} \right]$$

$$\Delta V = \frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

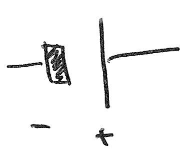
$$C = \frac{4\pi\varepsilon_0}{\frac{b-a}{ab}}$$

$$\boxed{C = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a}}$$

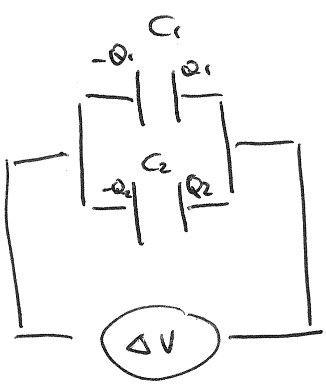
END FOR?

CAPACITORS IN PARALLEL AND SERIES

CAPACITORS



BATTERY



$$C = \frac{Q}{V} \Rightarrow CV = Q$$

$$Q_1 = C_1 \Delta V$$

$$Q_2 = C_2 \Delta V$$

$$Q_1 + Q_2 = (C_1 + C_2)\Delta V$$

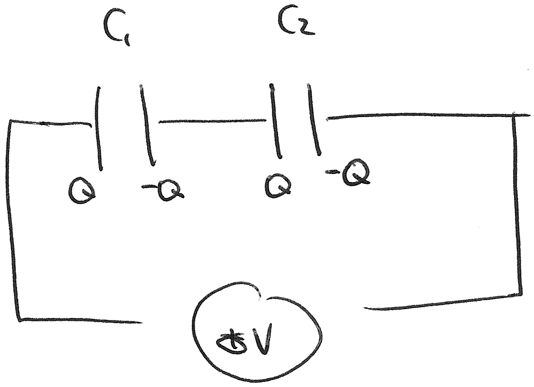
$$Q_T = C_T \Delta V$$

C_T : TOTAL CAPACITANCE

EQUIVALENT

CAPACITANCE

$$\underline{C_T = C_1 + C_2}$$



$$Q = C_1 \Delta V_1$$

$$Q = C_2 \Delta V_2$$

$$\Delta V_1 + \Delta V_2 = \Delta V$$

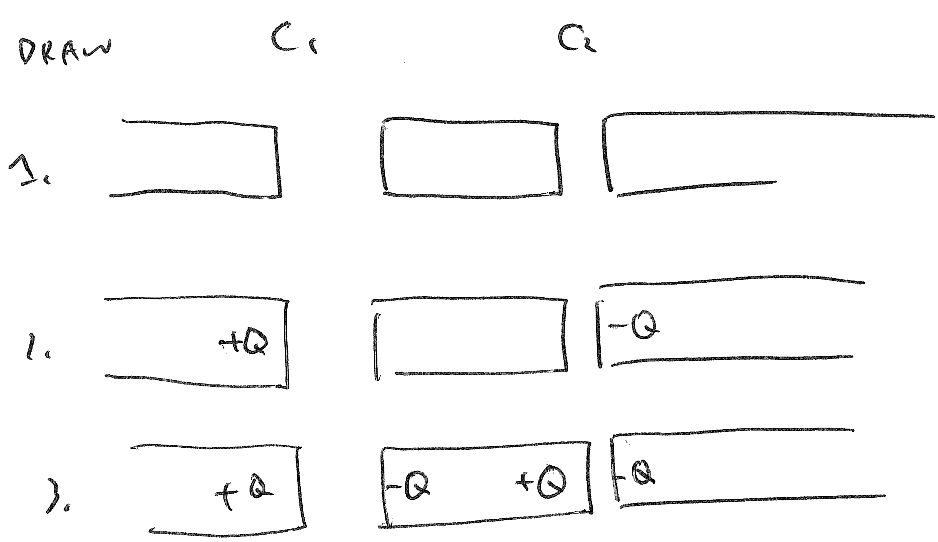
$$\frac{Q}{C_1} + \frac{Q}{C_2} = \Delta V$$

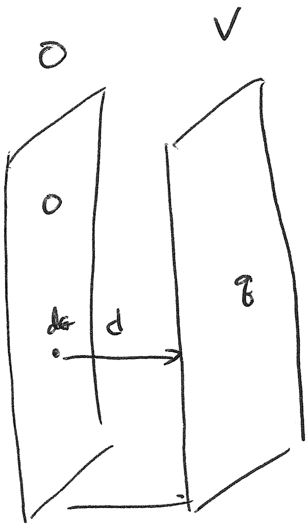
$$Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \Delta V$$

$$Q \cancel{C_{eq}} \frac{1}{C_{eq}} = \Delta V$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

DRAW





WORK = $dq\, d E$

$$E = \frac{\sigma}{\epsilon_0} = \frac{q d}{A \epsilon_0}$$

$$d\,\text{WORK} = dq \cdot \frac{q d}{A \epsilon_0}$$

ENERGY STORED IN CAPACITOR

$$\text{ENERGY} = \int_0^Q d\,\text{work} = \int_0^Q dq \, \frac{q d}{A \epsilon_0} = \frac{q^2 d}{2 A \epsilon_0}\Big]_0^Q$$

$$= \frac{Q^2 d}{2 A \epsilon_0} = \frac{Q^2}{2C}$$

$$C = \frac{\epsilon_0 A}{d} \quad \#$$

$Q = CV$

$$\therefore \text{ENERGY} = \frac{Q^2}{2C} = \frac{C^2 V^2}{2C} = \frac{1}{2} C V^2$$

ELECTRIC FIELD STORES ENERGY

$$U = \frac{Q^2}{2C}$$

$$E = \frac{Q}{\varepsilon_0 A} \qquad U = \frac{\varepsilon_0^2 A^2 E^2}{2C} = \frac{\varepsilon_0^2 A^2 E^2}{2 \frac{\varepsilon_0 A}{d}}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{\varepsilon_0^2 A^2 E^2 \, d}{2 \varepsilon_0 A}$$

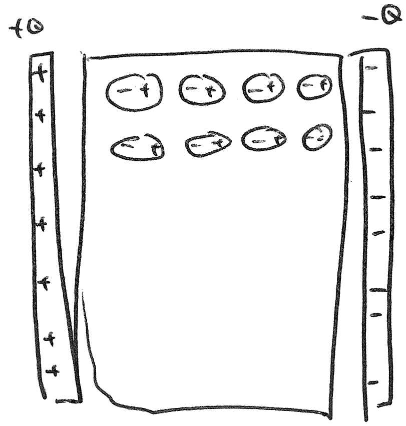
$$= \frac{\varepsilon_0 A d E^2}{2}$$

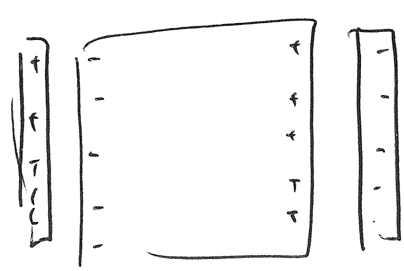
$Ad =$ VOLUME

$$\cancel{U} = \frac{\varepsilon_0 E^2}{2} V$$

$$\frac{U}{V} = \frac{\varepsilon_0 E^2}{2}$$

DIELECTRICS





E INSIDE REDUCED

$$E_{DIEL} = \frac{E}{K}$$

$$V_{DIEL} = \frac{E}{K} \cdot d = \frac{V}{K}$$

$$Q = CV$$

$$Q = C_{DIEL} V_{DIEL}$$

$$Q = C_{DIEL} \cdot \frac{V}{K}$$

$$Q = \frac{C_{DIEL}}{K} \cdot V$$

$$k\,C = \text{DIEL}$$

$K = 1$ VACUUM

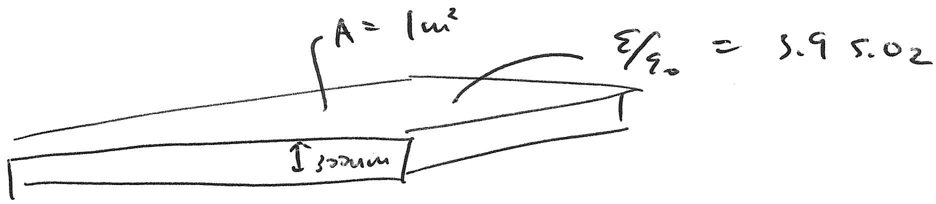
$K = 80$ WATER

$K = 310$ $SrTiO_3$

ε_0 : PERMITTIVITY OF FREESPACE (VACUUM)

$\varepsilon = K\varepsilon_0$ [PERMITTIVITY OF DIELECTRIC]

EXAMPLE CALCULATE CAPACITANCE OF SiO_2



$$C = 3.9 \cdot \frac{\varepsilon_0 A}{d} \quad \text{FARADS}$$

$$= 3.9 \cdot \frac{8.85 \times 10^{-12}}{300 \times 10^{-9}}$$

$$C = 1.15 \times 10^{-4}\ F$$

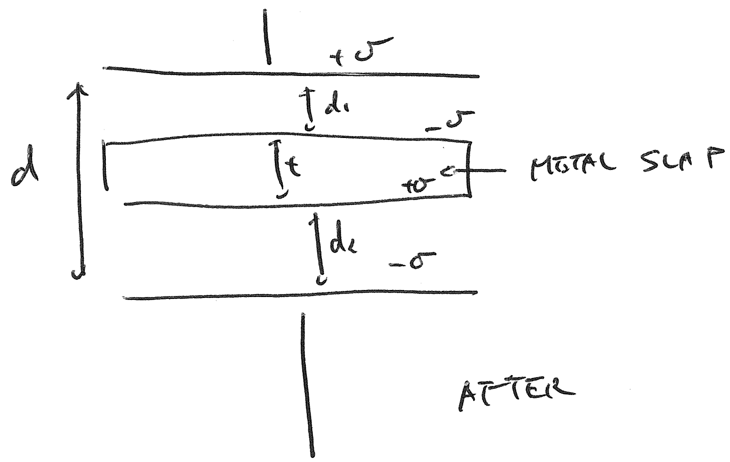
SO CAPACITANCE $1.15 \times 10^{-4}\ F/m^2$

SO PER 1 VOLT

$$1.15 \times 10^{-4}\ F/m^2 \cdot 1V = 1.15 \times 10^{-4}\ C/m^2$$

$$\frac{1.15 \times 10^{-4}}{1.6 \times 10^{-19}} = 7.2 \times 10^{14}\ e^- / m^2\text{-V}$$

EXAMPLE



ORIGINAL CAPACITANCE

$$\frac{\varepsilon_0 A}{d}$$

CAPACITORS IN SERIES

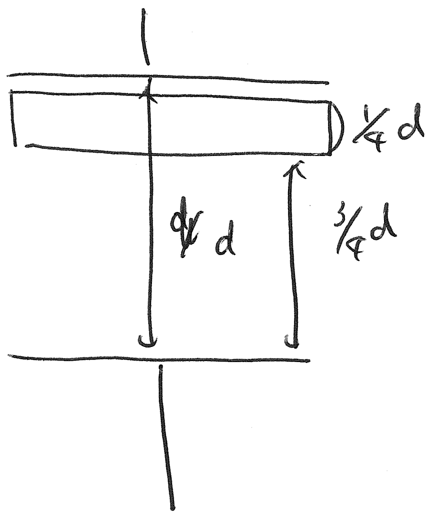
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$d_1 = \frac{d-t}{2} = d_2$$

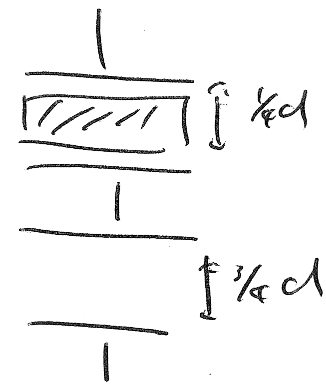
$$C_1 = C_2 = \frac{\varepsilon_0 A}{\frac{d-t}{2}} = \frac{2\varepsilon_0 A}{d-t}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{2}{C_1} = \frac{2}{\frac{2\varepsilon_0 A}{d-t}} = \frac{d-t}{\varepsilon_0 A}$$

$$C_T = \frac{\varepsilon_0 A}{d-t}$$



$\Rightarrow$ EQUIVALENT TO



$$\frac{1}{C_T} = \frac{1}{C_d} + \frac{1}{C_V}$$

$$C_d = \frac{k\varepsilon_0 A}{\frac{1}{4}d} \qquad C_V = \frac{\varepsilon_0 A}{\frac{3}{4}d}$$

$$\frac{1}{C_T} = \frac{1}{\frac{k\varepsilon_0 A}{\frac{1}{4}d}} + \frac{1}{\frac{\varepsilon_0 A}{\frac{3}{4}d}}$$

$$= \frac{\frac{1}{4}d}{k\varepsilon_0 A} + \frac{\frac{3}{4}d}{\varepsilon_0 A}$$

$$\frac{1}{C_T} = \frac{\frac{1}{4}d + \frac{3}{4}kd}{k\varepsilon_0 A}$$

$$C_T = \frac{k\varepsilon_0 A}{\frac{1}{4}d + \frac{3}{4}kd}$$

END WED!

CURRENT AND RESISTANCE FRI

USE NOTES 2009