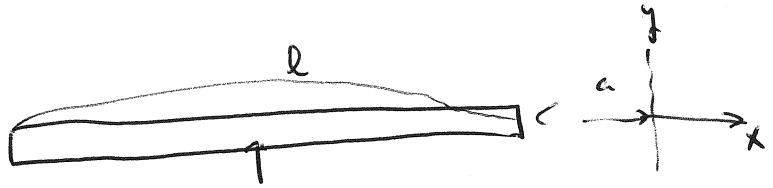
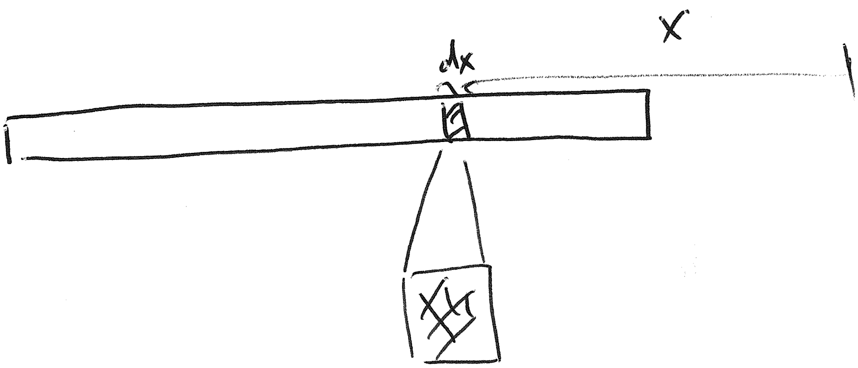
MORE COMPLEX EXAMPLE



Q UNIFORM CHARGE



CHARGE ON SEGMENT = $\frac{dx}{l} Q$

$$\vec{dE} = \frac{1}{4\pi\varepsilon_0} \frac{\frac{dx}{l} Q}{x^2} \hat{x}$$

$$\vec{dE} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{l} \frac{dx}{x^2} \hat{x}$$

$$\vec{E} = \int_{-(l+a)}^{-a} \frac{1}{4\pi\varepsilon_0} \frac{Q}{l} \frac{dx}{x^2} \hat{x}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{l} \cdot \left(-\frac{1}{x}\right) \Big]_{l+a}^{-a} \hat{x}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{l} \left(\frac{1}{a} - \frac{1}{l+a} \right) \hat{x}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{l} \left(\frac{l}{a(l+a)} \right) \hat{x}$$

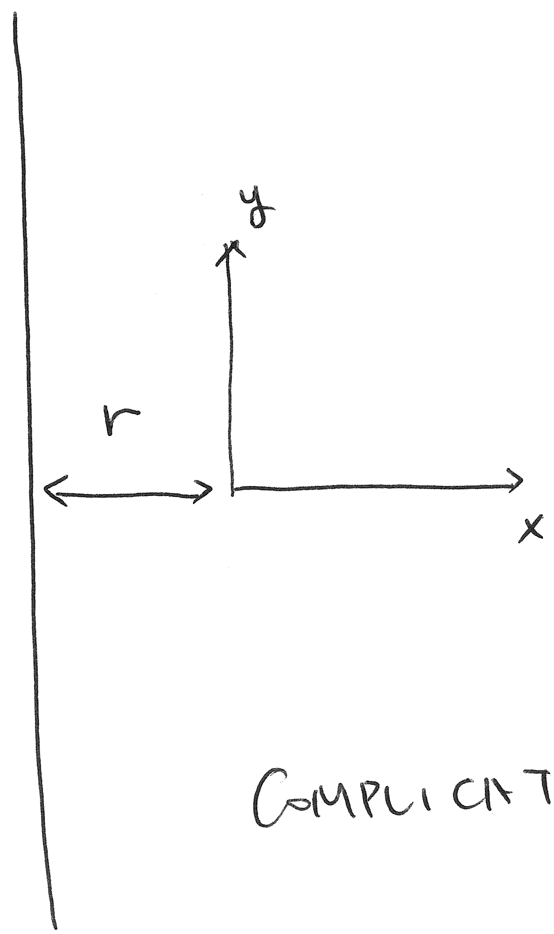
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{a(l+a)} \hat{x}$$

$a \to \infty$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{a^2} \checkmark$$

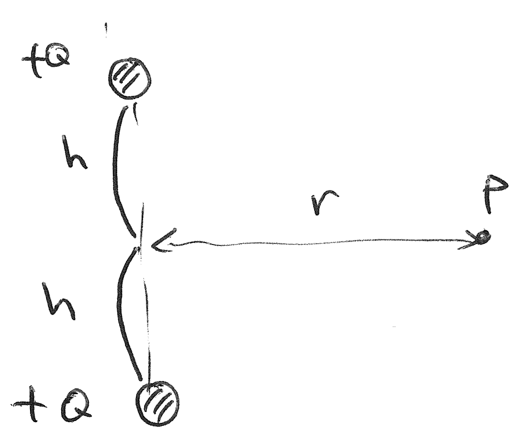
EXAMPLE

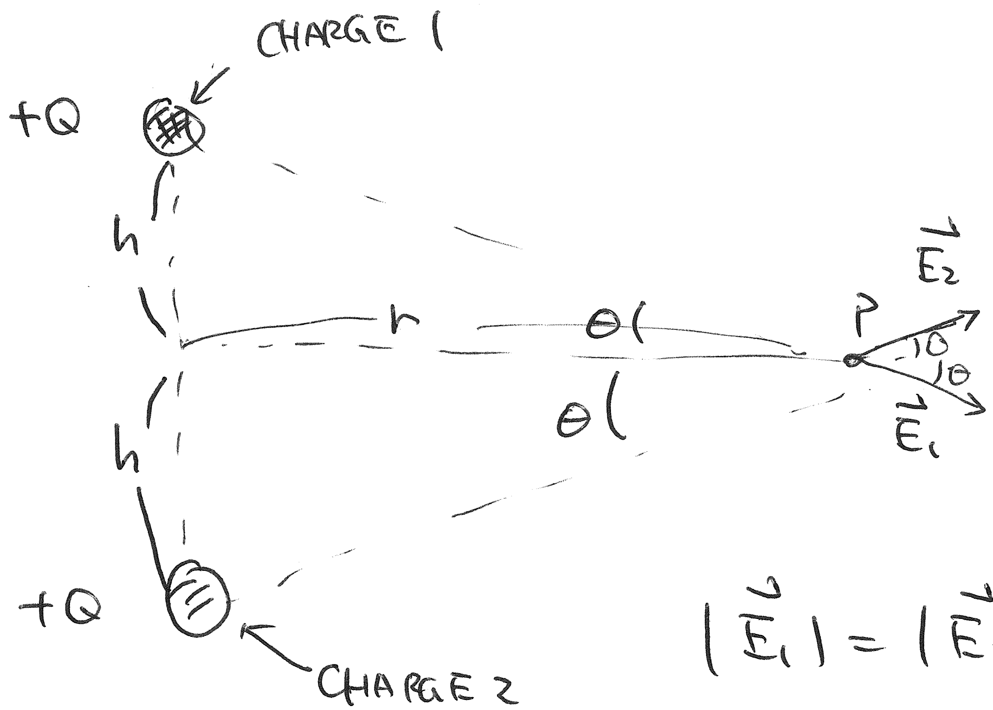
UNIFORMLY CHARGED INFINITE ROD



COMPLICATED

CONSIDER

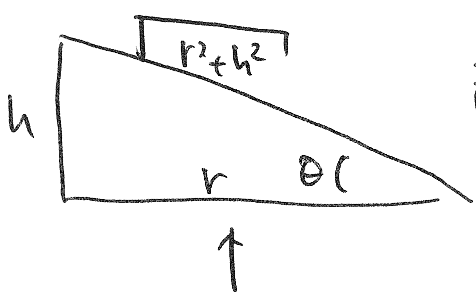




$$|\vec{E}_1| = |\vec{E}_2|$$

SO y COMPONENTS CANCEL

SO WE HAVE



$$\vec{E}_{TOTAL} = |\vec{E}_1| + |\vec{E}_2|\ \hat{x}$$

$$= 2E_{1x}\ \hat{x}$$ ← BECAUSE $E_{1x} = E_{2x}$

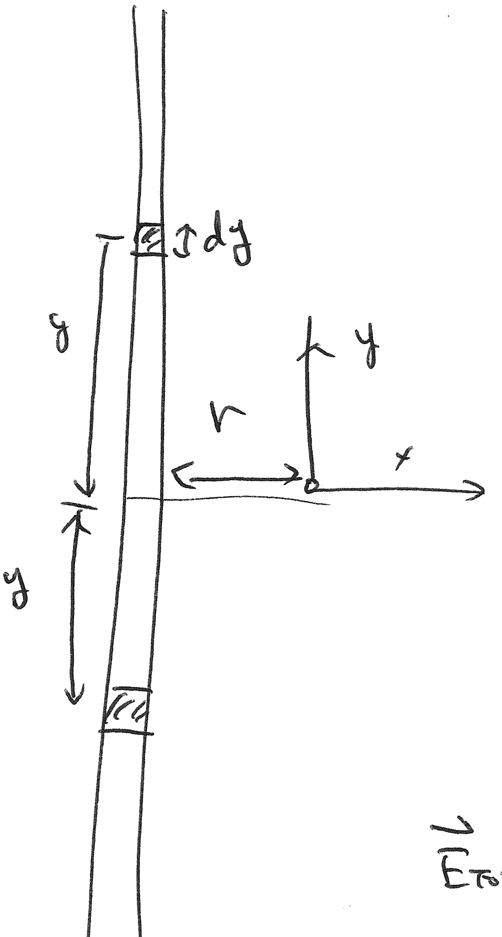
$$E_{1x} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(\sqrt{r^2+h^2})^2} \times \cos\theta$$

$$\cos\theta = \frac{r}{\sqrt{r^2+h^2}}$$

$$\therefore\ \vec{E}_{TOTAL} = 2\times\frac{1}{4\pi\varepsilon_0}\frac{Q}{(r^2+h^2)}\frac{r}{\sqrt{r^2+h^2}}\ \hat{x}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{Qr}{(r^2+h^2)^{3/2}}\ \hat{x}$$

USE THAT TO SOLVE



$$dq = \lambda\, dy$$

SUBSTITUTE

$Q \to dq$

$h \to y$

$$d\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{dq\, r}{(r^2+y^2)^{3/2}}\, \hat{x}$$

$$d\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{r\lambda\, dy}{(r^2+y^2)^{3/2}}\, \hat{x}$$

$$\vec{E}_{TOT} = \int d\vec{E} = \int_0^{\infty} \frac{1}{2\pi\varepsilon_0} \frac{r\lambda\, dy}{(r^2+y^2)^{3/2}}\, \hat{x}$$

INTEGRATION TABLE

$$= \frac{r\lambda}{2\pi\varepsilon_0} \left[\frac{1}{r^2} \frac{y}{(r^2+y^2)^{1/2}} \right]_0^{\infty} \hat{x}$$

$$\vec{E}_{TOTAL} = \hat{x}\, \frac{r\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r^2}$$

$$\boxed{\vec{E}_{TOTAL} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}\, \hat{x}}$$