

$$\frac{Q}{C} - L \frac{dI}{dt} = 0$$

$$I = -\frac{dQ}{dt}$$

$$\frac{Q}{C} + L \frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{Q}{LC}$$

$$Q = Q_{\text{MAX}} \cos(\omega t + \phi)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

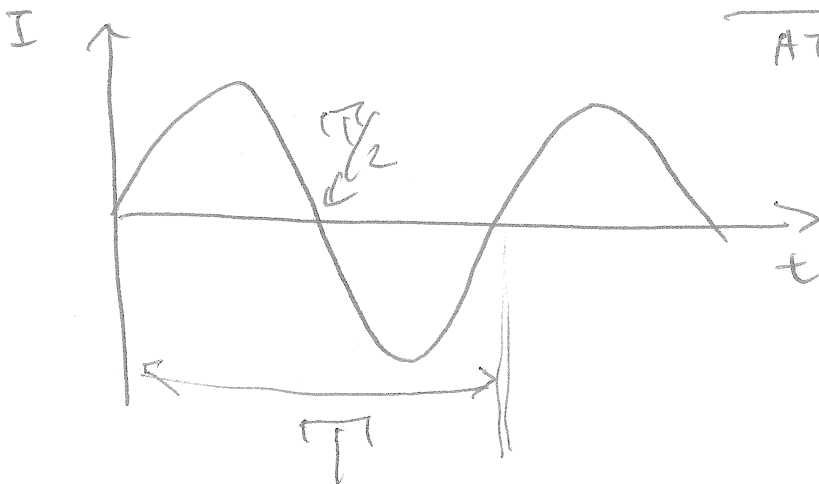
INITIALIZATION

$$I = -\frac{dQ}{dt} = -(-\omega Q_{\text{MAX}} \sin(\omega t + \phi))$$

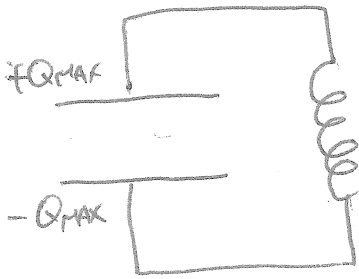
$$= \omega Q_{\text{MAX}} \sin(\omega t + \phi)$$

AT $t=0$ $I=0$

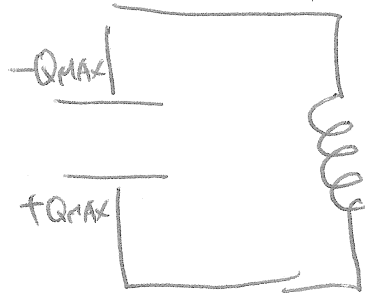
$$\phi = 0$$



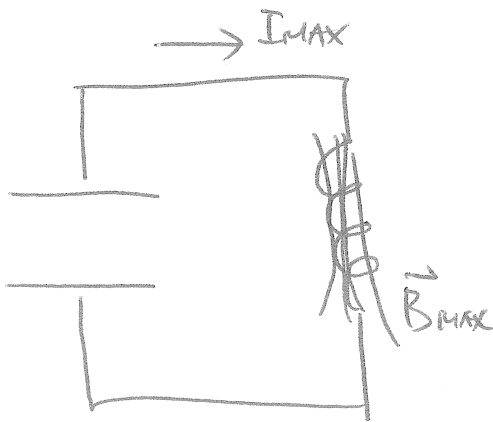
$$At \epsilon = 0$$



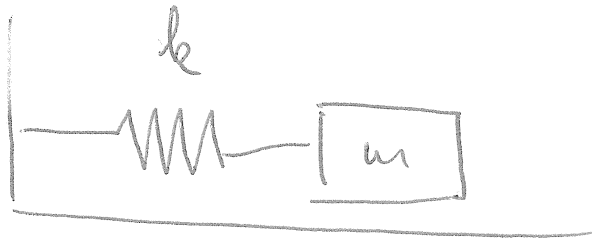
$$At \ t = \pi/2$$



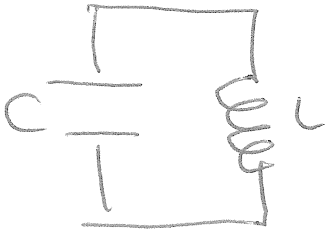
$$At \ t = \pi/4 \quad I(\pi/4) = I_{max} = \omega Q_{max}$$



ANALOGOUS TO



$$\frac{1}{2} kx^2 + \frac{1}{2} m v^2 = \text{CONSTANT}$$

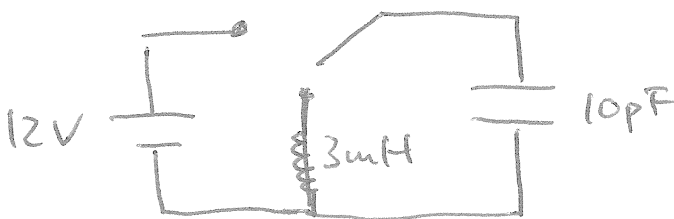


$$\frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \text{CONSTANT}$$

$$Q = CV$$

$$\frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} L I^2 = \text{CONSTANT}$$

EXAMPLE



a) FIND OSCILLATION FREQUENCY

$$f = \frac{1}{T} \quad \text{where } T = \frac{2\pi}{\omega}$$

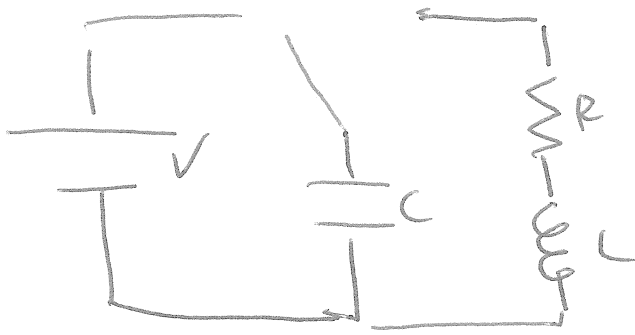
$$2\pi f = \omega$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi \sqrt{LC}}$$

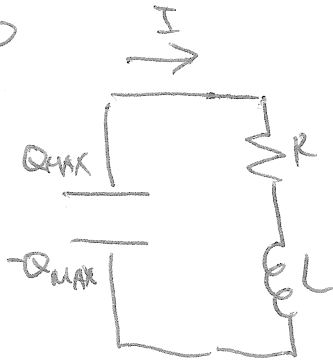
b) MAXIMUM CURRENT AND CHARGE.

$$Q_{\text{MAX}} = CV = 12V \times 10pF = 120pC$$

$$I_{\text{MAX}} = \omega Q_{\text{MAX}} \Rightarrow \text{CAN BE CALCULATED}$$



$t=0$



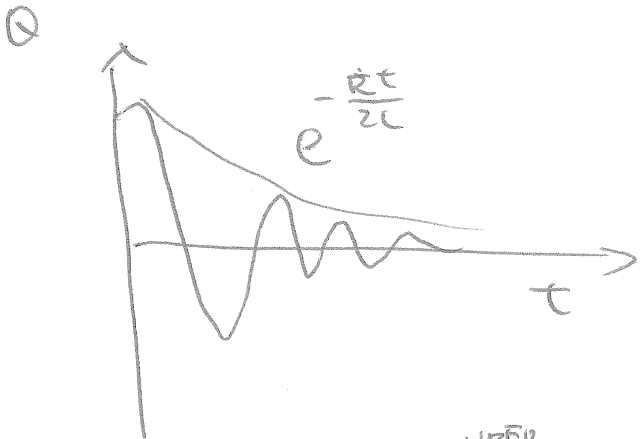
$$\frac{Q}{C} - IR - L \frac{dI}{dt} = 0$$

$$I = - \frac{dQ}{dt}$$

$$\boxed{\frac{Q}{C} + R \frac{dQ}{dt} + L \frac{d^2Q}{dt^2} = 0}$$

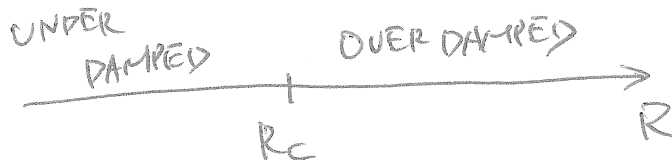
$$Q(t) = Q_{MAX} e^{-\frac{Rt}{2L}} \cos(\omega t)$$

$$\omega = \left[\frac{1}{LC} - \left(\frac{R}{2L} \right)^2 \right]^{1/2}$$

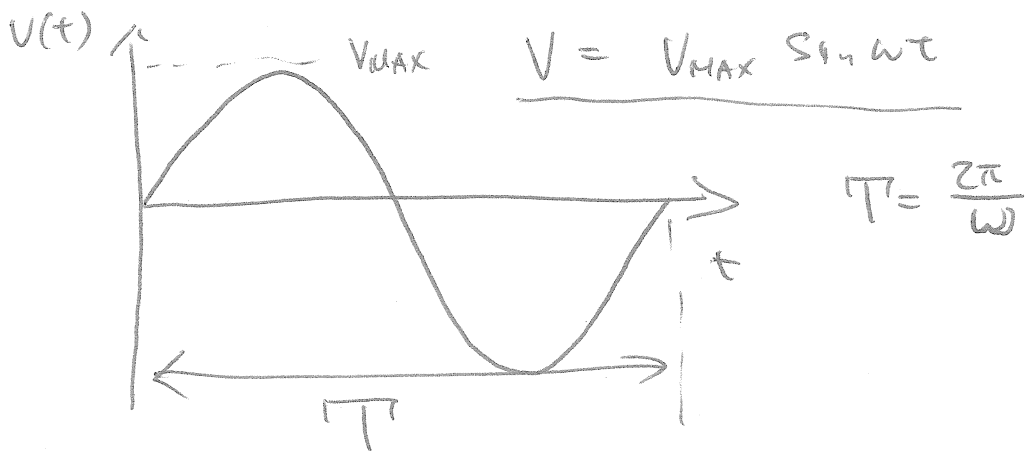


$$R_c = \sqrt{\frac{4L}{C}}$$

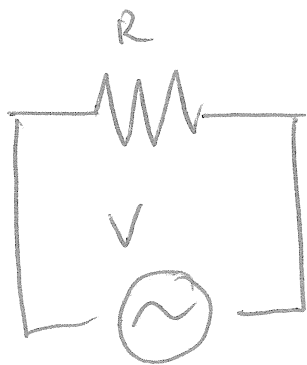
NO OSCILLATION OCCURS



AC CIRCUITS



EXAMPLE



GIVEN $V(t) = V_{MAX} \sin \omega t$
WHAT IS $I(t)$?

$$V - IR = 0$$

$$V = IR$$

$$I = \frac{V}{R} = \frac{V_{MAX} \sin \omega t}{R}$$

ROOT MEAN SQUARE (RMS)

$$I_{RMS} = \sqrt{(I^2(t))_{AVERAGE}}$$

$$I(t) = I_{MAX} \sin \omega t$$

$$I^2(t) = I_{MAX}^2 \sin^2 \omega t$$

$$I_{MAX} = \frac{V_{MAX}}{R} \text{ IN THIS CASE}$$

$$\left(\sin^2 \omega t \right)_{AVERAGE} = \frac{1}{T} \int_0^T \sin^2(\omega t) dt = \frac{1}{T} \left[\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega} \right]_0^T = \frac{1}{2}$$

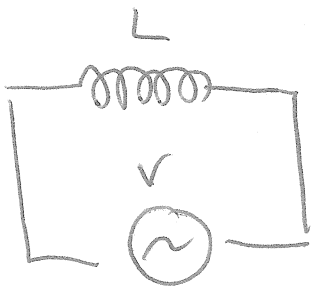
$$I_{RMS} = \frac{I_{MAX}}{\sqrt{2}}$$

EXAMPLE

$$V(t) = 200 \sin(\omega t)$$

WHAT IS THE RMS VOLTAGE?

$$\Delta V_{RMS} = \frac{200}{\sqrt{2}} = 141V$$



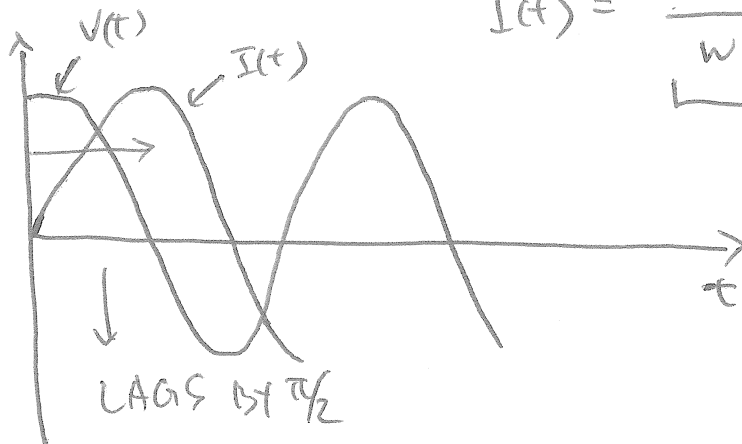
$$V(t) = V_{MAX} \cos(\omega t)$$

$$V(t) - L \frac{dI}{dt} = 0$$

$$V(t) = L \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{V(t)}{L} = \frac{V_{MAX} \cos(\omega t)}{L}$$

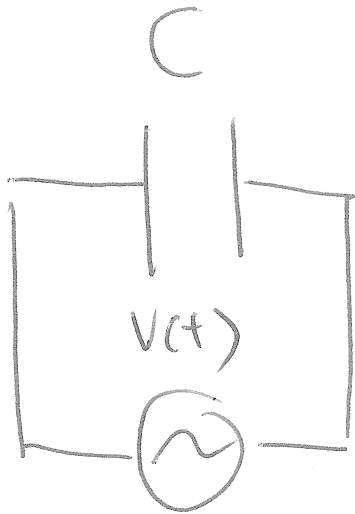
$$I(t) = \frac{V_{MAX}}{\omega L} \sin(\omega t)$$



$$I_{MAX} = \frac{V_{MAX}}{\omega L}$$

X_L : REACTANCE OF INDUCTOR

$$\boxed{X_L = \omega L}$$



~~$$V(t) = \frac{Q}{C}$$~~

$$V(t) - \frac{Q}{C} = 0$$

$$V(t) = V_{MAX} \cos(\omega t)$$

$$Q(t) = CV(t)$$

$$= CV_{MAX} \cos \omega t$$

$$I = \frac{dQ}{dt} = -\omega C V_{MAX} \sin(\omega t)$$

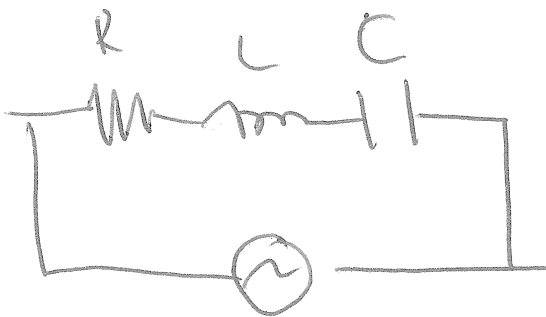
$$\sin(\omega t) = -\cos(\omega t + \pi/2)$$

$$= +\omega C V_{MAX} \cos(\omega t + \pi/2)$$

CURRENT LEADS VOLTAGE

$$I_{MAX} = \omega C V_{MAX}$$

$$X_C = \frac{1}{\omega C}$$



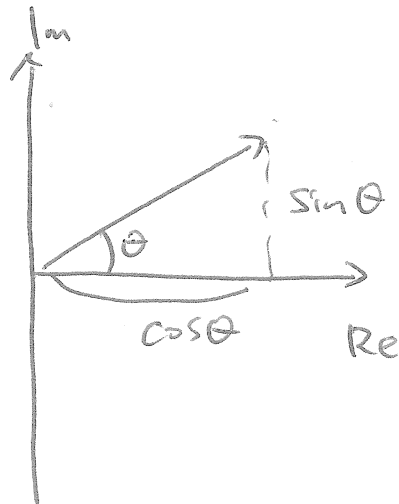
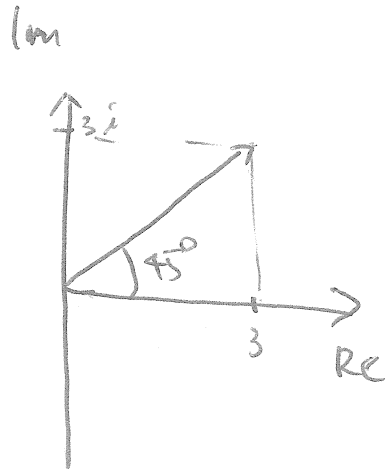
~~$$V(t) = IR$$~~

$$V(t) - IR - L \frac{dI}{dt} - \frac{Q}{C} = 0$$

$$i = \sqrt{-1}$$

COMPLEX NUMBER

$$3 + 3i$$



$$\cos \theta + i \sin \theta$$

EULER RELATIONSHIP

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\cos \omega t = \text{Re} \left[e^{i\omega t} \right]$$

$$= \underline{\underline{\text{Re} \left[\cos \omega t + i \sin \omega t \right]}}$$



$$V(t) = V_m \cos \omega t = V_m e^{i\omega t}$$

$$I(t) = I_m e^{i(\omega t - \phi)}$$

$$\frac{V(t)}{I(t)} = \frac{V_m e^{i\omega t}}{I_m e^{i(\omega t - \phi)}} = \frac{V_m}{I_m} e^{i\phi}$$

$$\underline{\underline{\phi = 0}}$$