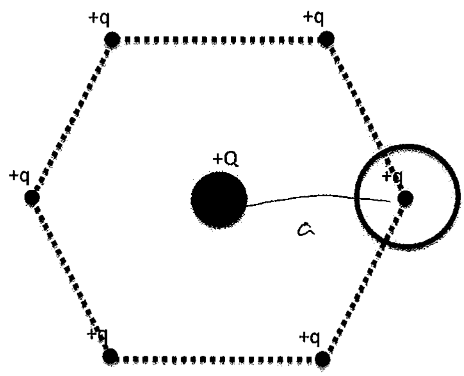
Final Exam

Name:

Problem 1



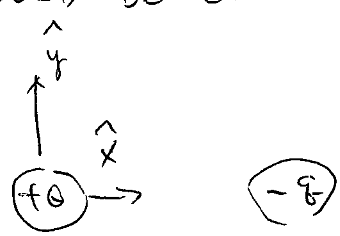
6 charges (+q) are held in a hexagonal arrangement by some plastic. Each side of the hexagon has length a.

(a) Calculate the net force (magnitude and direction) on the charge +Q at the center of hexagon

(b) Calculate the force (magnitude and direction) on the charge +Q if the charge indicated by a red circle is removed.

a) 0 , ALL BALANCED OUT

b) IT WOULD BE EQUIVALENT OF



$$\vec{F} = \hat{x} \frac{1}{4\pi\varepsilon_0} \frac{Qq}{a^2}$$

Problem 2

Consider a uniformly charged sphere of radius R. The charge density is given by ρ/m^3.

Calculate electric field (magnitude and direction) for

(a) r>R

(b) r<R

Calculate voltage (assuming V=0 at r=infinity) for

(a) r>R

(b) r<R

a) $r > R$ $\quad Q_{ENC} = \frac{4\pi R^3}{3}\rho$

$$4\pi r^2 E = \frac{4\pi R^3 \rho}{3\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}}$$

b) $r < R$ $\quad Q_{ENC} = \frac{4\pi r^3}{3}\rho$

$$4\pi r^2 E = \frac{4\pi r^3 \rho}{3\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho r}{3\varepsilon_0}\hat{r}}$$

c) $r > R$

$$\nabla V = -E$$

$$\frac{\partial V}{\partial r} = -E$$

$$V = \frac{\rho R^3}{3\varepsilon_0 r} + C \qquad C = 0$$

$$\boxed{V(r) = \frac{\rho R^3}{3\varepsilon_0 r}}$$

d) $r < R$

$$\nabla V = -E$$

$$\frac{\partial V}{\partial r} = -E$$

$$V = -\int \frac{\rho r}{3\varepsilon_0} dr = -\frac{\rho r^2}{6\varepsilon_0} + C$$

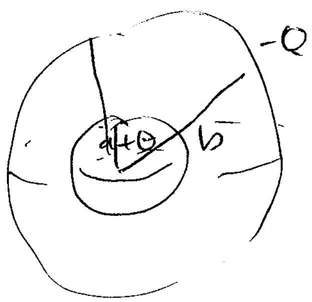
$$\boxed{V(r) = -\frac{\rho r^2}{6\varepsilon_0} + \frac{\rho R^2}{2\varepsilon_0}}$$

CONTINUOUS AT $\boxed{R = r}$

$$-\frac{\rho R^2}{6\varepsilon_0} + C = \frac{\rho R^2}{3\varepsilon_0} \qquad C = \frac{\rho R^2}{3\varepsilon_0}\left[1 + \frac{1}{2}\right] = \frac{\rho R^2}{2\varepsilon_0}$$

Problem 3

Calculate the capacitance of a spherical capacitor, which is composed of two spheres (one inside another), with inner radius a and outer radius b.



$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

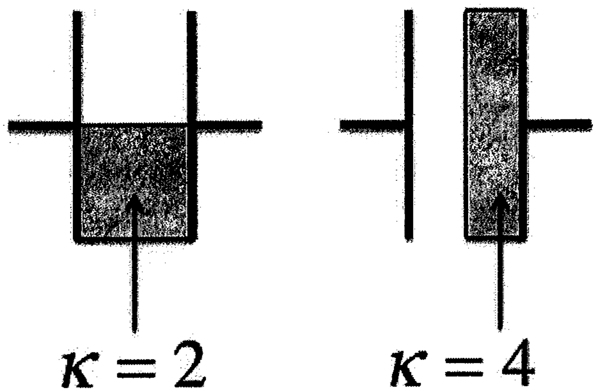
$$|\Delta V| = \frac{1}{4\pi\varepsilon_0} Q \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$|\Delta V| = \frac{1}{4\pi\varepsilon_0} Q \frac{b-a}{ab}$$

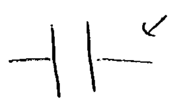
$$Q = q|\Delta V|$$

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a}}$$

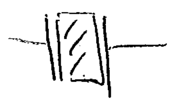
Problem 4
Parallel capacitors are half filled by dielectric materials as shown below. (a) calculate the capacitances in terms of ε_0, A, d. (b) Which capacitor has higher capacitance?



a)



$\frac{\varepsilon_0 A}{2d}$ PARALLEL ADD



$\frac{\varepsilon_0 A}{d}$

$$\frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{d} = \frac{3\varepsilon_0 A}{2d} = C_{TOTAL} = 1.5\frac{\varepsilon_0 A}{d}$$

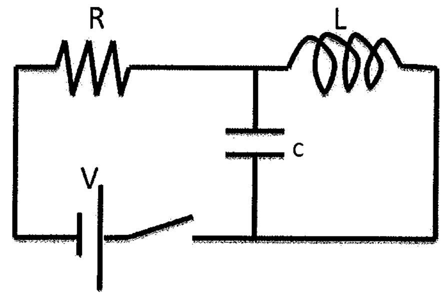
$\frac{2\varepsilon_0 A}{d}$ $\frac{8\varepsilon_0 A}{d}$

$$\frac{1}{C_T} = \frac{d}{2\varepsilon_0 A} + \frac{d}{8\varepsilon_0 A} = \frac{5d}{8\varepsilon_0 A}$$

$$C_T = \frac{8\varepsilon_0 A}{5d} = 1.6\frac{\varepsilon_0 A}{d}$$

b) RIGHT CAPACITOR.

Problem 5



At t=0, the switch is closed.
(a) Calculate the current sourced by the battery at t=0
(b) Calculate the current sourced by the battery at t=infinity
(c) what is the voltage across the capacitor at t=infinity
(d) After a long time, the switch is released. Calculate the current through the capacitor as a function of time after the switch is released.

a) $t=0$ $\frac{V}{R} = I$

b) $t=\infty$ $I = \frac{V}{R}$

c) AT $t=\infty$ $\boxed{V_c = 0}$

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d) $I(t) = \frac{V}{R} \cos \omega t$

$\omega = \frac{1}{\sqrt{LC}}$

Problem 6

Consider a cylindrical wire with radius R with current I flowing through it. Calculate magnetic field for

(a) r>R

(b) r<R

assuming that current is uniformly distributed

a) $2\pi r B = \mu_0 I$

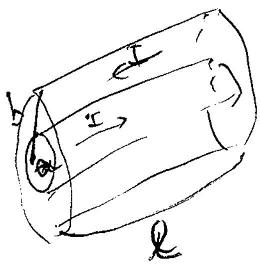
$$B = \frac{\mu_0 I}{2\pi r}$$

b) $2\pi r B = \mu_0 I \frac{r^2}{R^2}$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Problem 7

Consider a coaxial cable as depicted below. Calculate the inductance per unit length for the cable if the inside wire has the diameter of a and the outside wire has a diameter of b.



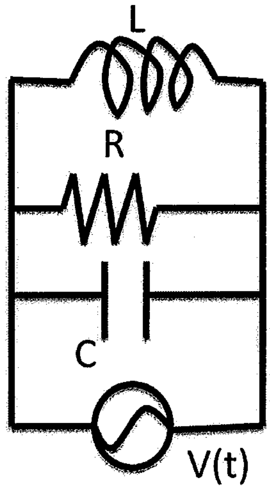
$$\frac{\mu_0 I}{2\pi r} = B$$

$$\Phi = \int_a^b \frac{\mu_0 I \ell}{2\pi r}\, dr = \frac{\mu_0 I \ell}{2\pi} \ln(b/a)$$

$$\frac{d\Phi_B}{dt} = \underbrace{\frac{\mu_0 \ell}{2\pi} \ln(b/a)}_{L} \frac{dI}{dt}$$

$$\boxed{\frac{L}{\ell} = \frac{\mu_0}{2\pi} \ln(b/a)}$$

Problem 8



Find the complex impedance for the parallel RLC circuit as shown. Assume that we have a voltage source for which we can adjust the angular frequency ω. $V(t)=V_{max}\cos(\omega t)$.

(a) Find the total complex impedance of the circuit
(b) Calculate I(ω).
(c) Find an expression for the phase ϕ.

(d) Given R=500 ohms, L= 1 henry, C=1.0μF and ω=100 rad/sec. Will the current lag or lead voltage and by how much?

a)
$$\frac{1}{Z_{TOTAL}} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$$

$$\frac{1}{Z_T} = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)$$

$$Z_T = \frac{1}{\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)} = \frac{\frac{1}{R} - \left(\omega C - \frac{1}{\omega L}\right) i}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

b)
$$|Z_T| = \frac{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

c)
$$\phi = \tan^{-1}\left(\frac{-\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R}}\right) = \tan^{-1}\left(\frac{\frac{1}{\omega L} - \omega C}{\frac{1}{R}}\right)$$

d)
$$\phi = \tan^{-1}\left(\frac{\frac{1}{100} - 1\times 10^{-4}}{\frac{1}{500}}\right) \cong \tan^{-1}(5)$$

LAG VOLTAGE