2049c Exam 1

SOLUTION

Name:

PID:

Useful formulas

$\vec{E} = -\vec{\nabla} V$

$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$

In capacitors, Q = CV, where C is the capacitance

1

2

3

4

5

Total

Problem 1: Consider a **conducting** sphere of radius a, charged to +Q.

Calculate electric field (magnitude and direction) for
(a) r>a

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\frac{4\pi\rho a^3}{4\pi\varepsilon_0 r^3} = \frac{\rho a^3}{3\varepsilon_0}$$

$$\frac{\rho a^3}{3\varepsilon_0 r^2}$$

OK

(b) r<a

$$\vec{E} = 0$$

Calculate voltage (take V=0 at r=infinity) for
(a) r>a

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$\frac{\rho a^3}{3\varepsilon_0 r}$$ ok

(b) r<a (Hint: make certain that V(a) matches from inside sphere and outside sphere. i.e. voltage here has to be continuous)

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$$

(c) Why is the voltage continuous at r=a?

BECAUSE E IS FINITE AT r=a

Problem 2 Consider a charged sphere with charge distribution $\rho(r)=Dr$ (C/m^3) [where D is some constant] and radius a. (Hint: $dV=4\pi r^2 dr$)

Calculate electric field (magnitude and direction) for

(a) r>a

$$Q_{ENC} = \int_0^a Dr\, 4\pi r^2 dr = \pi D r^4 \Big]_0^a = \pi D a^4$$

$$E \cdot A = E \cdot 4\pi r^2 = \frac{\pi D a^4}{\varepsilon_0} \qquad E = \frac{Da^4}{4r^2\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{Da^4}{4\varepsilon_0 r^2}\hat{r}}$$

(b) r<a

$$Q_{ENC} = \pi D r^4$$

$$E \cdot 4\pi r^2 = \frac{\pi D r^4}{\varepsilon_0}$$

$$E = \frac{Dr^2}{4\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{Dr^2}{4\varepsilon_0}\hat{r}}$$

Calculate voltage (take V=0 at r=infinity) for

(a) r>a

$$V(r) = \int -\frac{Da^4}{4\varepsilon_0 r^2} dr = \frac{Da^4}{4\varepsilon_0 r} + C \qquad \underline{C=0}$$

$$\boxed{V(r) = \frac{Da^4}{4\varepsilon_0 r}}$$

(b) r<a (make certain that V(a) matches from inside sphere and outside sphere. i.e. voltage here has to be continuous)

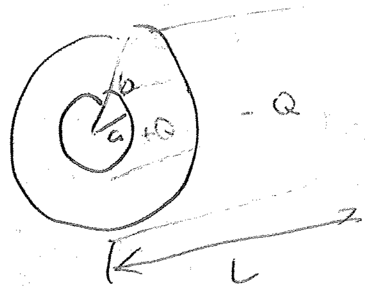
$$V(r) = -\int \frac{Dr^2}{4\varepsilon_0} dr = -\frac{Dr^3}{12\varepsilon_0} + C$$

V(a) HAS TO BE $\frac{Da^3}{4\varepsilon_0}$

$$\text{So} \quad C = \frac{Da^3}{4\varepsilon_0} + \frac{Da^3}{12\varepsilon_0} = \frac{1}{3}\frac{Da^3}{\varepsilon_0}$$

$$\boxed{V(r) = \frac{Da^3}{3\varepsilon_0} - \frac{Dr^3}{12\varepsilon_0}}$$

Problem 3: Calculate the capacitance per unit length (C/L) of a cylindrical capacitor, which is composed of two concentric infinite cylinders (one inside another), with inner radius a and outer radius b.



$$E \Rightarrow \quad E \cdot 2\pi r L = \frac{Q}{\varepsilon_0}$$

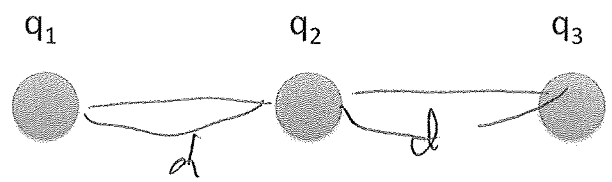
$$E = \frac{Q}{2\pi\varepsilon_0 r L}$$

$$|\Delta V| = \int_a^b \frac{Q}{2\pi\varepsilon_0 r L}\, dr = \frac{Q}{2\pi\varepsilon_0 L} \ln b/a$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln b/a}$$

$$\boxed{C/L = \frac{2\pi\varepsilon_0}{\ln b/a}}$$

Problem 4:



As shown q_1, q_2, and q_3 are equally spaced with distance d between them. q_3 experiences no forces. Find q_1 in terms of q_2.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{4d^2} + \frac{q_2 q_3}{d^2}\right)\hat{x}$$

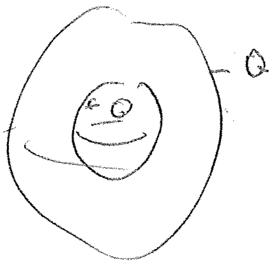
$$\Downarrow$$

$$\frac{q_1 q_3}{4d^2} = -\frac{q_2 q_3}{d^2}$$

$$\boxed{q_1 = -4q_2}$$

Problem 5: Consider a spherical capacitor, which is composed of two spheres (one inside another), with inner sphere having radius a and outer sphere having radius b. Inside sphere is charged to +Q and outer sphere is charged to -Q.

(a) Calculate the voltage of the inside sphere with respect to the outer sphere.



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad \text{INSIDE } (a<r<b)$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a} - \frac{Q}{b}\right)$$

$$\boxed{\Delta V = \frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}}$$

(b) How much work must be done to take an electron with charge -1.6×10^{-19} C from outer sphere to inner sphere? [note: in this problem, sign error will be scored as a wrong answer]

$$\boxed{\text{WORK} = -1.6\times10^{-19}\left[\frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}\right] \text{ J}}$$