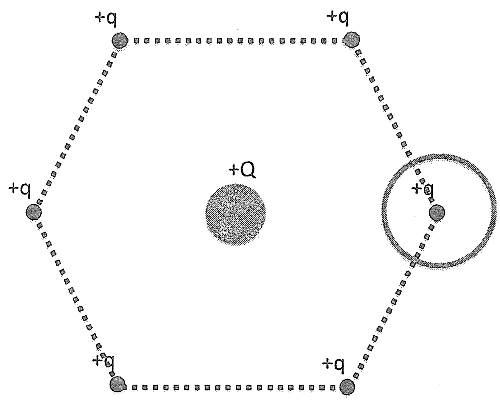
Final Exam

Name:

Problem 1



6 charges (+q) are held in a hexagonal arrangement by some plastic. Each side of the hexagon has length a.

(a) Calculate the net force (magnitude and direction) on the charge +Q at the center of hexagon

(b) Calculate the force (magnitude and direction) on the charge +Q if the charge indicated by a red circle is removed.

a) 0

b) $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\hat{x}$

Problem 2

Consider a uniformly charged sphere of radius R. The charge density is given by ρ/m^3.

Calculate electric field (magnitude and direction) for
(a) r>R
(b) r<R

Calculate voltage (assuming V=0 at r=infinity) for
(a) r>R
(b) r<R

a) $$\frac{1}{\cancel{4\pi}\varepsilon_0}\,\frac{\cancel{4\pi}R^3}{3\,\cancel{4\pi}r^2}\,\rho = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

b) $$\frac{4\cancel{\pi}r^3}{3}\,\rho\,\frac{1}{\varepsilon_0} = 4\cancel{\pi}r^2 E$$

$$E = \frac{\rho r}{3\varepsilon_0}$$

c) $$V(r) = \frac{\rho R^3}{3\varepsilon_0 r}$$

c) $$V(r) = \frac{\rho(R^2 - r^2)}{6\varepsilon_0} + \frac{\rho R^2}{3\varepsilon_0}$$

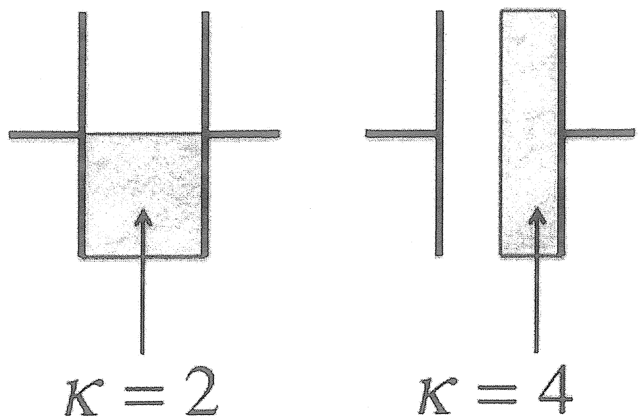
Problem 3

Calculate the capacitance of a spherical capacitor, which is composed of two spheres (one inside another), with inner radius a and outer radius b.

$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

Problem 4

Parallel capacitors are half filled by dielectric materials as shown below. (a) calculate the capacitances in terms of ε_0, A, d. (b) Which capacitor has higher capacitance?

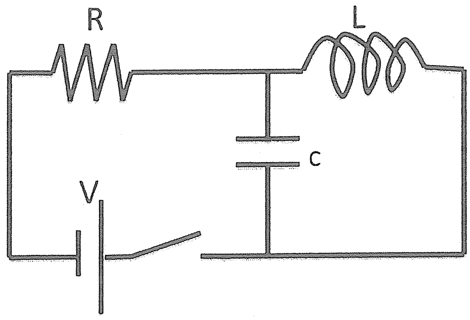


$C = 1.5 \frac{\varepsilon_0 A}{d}$ $C = 1.6 \frac{\varepsilon_0 A}{d}$

a)

b) RIGHT ONE

Problem 5



At t=0, the switch is closed.
(a) Calculate the current sourced by the battery at t=0
(b) Calculate the current sourced by the battery at t=infinity
(c) what is the voltage across the capacitor at t=infinity
(d) After a long time, the switch is released. Calculate the current through the capacitor as a function of time after the switch is released.

a) $I = \frac{V}{R}$

b) $I = \frac{V}{R}$

c) 0 VOLTS

d) $-L\frac{dI}{dt} + \frac{Q}{C} = 0$

$\frac{dI}{dt} = \frac{Q}{LC}$

$\frac{d^2I}{dt^2} = \frac{I}{LC}$

$I(t) = I_0 \cos(\omega t)$

$\boxed{I_0 = \frac{V}{R}}$

$\boxed{\omega = \frac{1}{\sqrt{LC}}}$

OSCILLATES

Problem 6
Consider a cylindrical wire with radius R with current I flowing through it. Calculate magnetic field for
(a) r>R
(b) r<R
assuming that current is uniformly distributed

a) $\int B \cdot ds = \mu_0 I$

$2\pi r B = \mu_0 I$

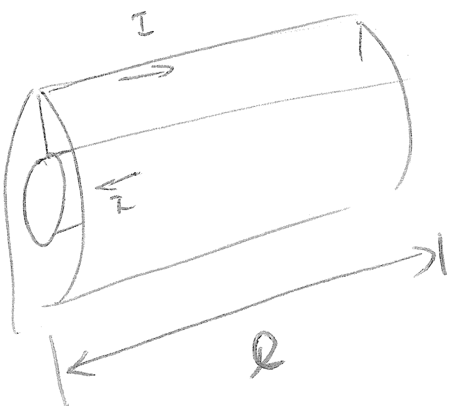
$$B = \frac{\mu_0 I}{2\pi r}$$

b) $2\pi r B = \mu_0 I \frac{r^2}{R^2}$

$$B = \frac{\mu_0 r}{2\pi R^2}$$

Problem 7

Consider a coaxial cable as depicted below. Calculate the inductance per unit length for the cable if the inside wire has the diameter of a and the outside wire has a diameter of b.



$a < r < b$

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$dA = dr\, l$$

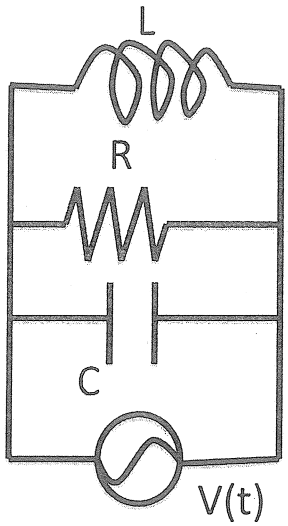
$$\Phi_B = \int_a^b \frac{\mu_0 I l}{2\pi r}\, dr$$

$$\Phi_B = \frac{\mu_0 I l}{2\pi} \ln(b/a)$$

$$\frac{\Phi_B}{l} = \frac{\mu_0 I}{2\pi} \ln(b/a) = \frac{LI}{l}$$

$$\boxed{\frac{L}{l} = \frac{\mu_0 I}{2\pi} \ln(b/a)}$$

Problem 8



Find the complex impedance for the parallel RLC circuit as shown. Assume that we have a voltage source for which we can adjust the angular frequency ω. $V(t)=V_{max}\cos(\omega t)$.

(a) Find the total complex impedance of the circuit
(b) Calculate $I(\omega)$.
(c) Find an expression for the phase ϕ.

(d) Given R=500 ohms, L= 1 henry, C=1.0μF and ω=100 rad/sec. Will the current lag or lead voltage and by how much?

a) PARALLEL CIRCUIT

$$\frac{1}{Z_{TOTAL}} = \frac{1}{R} + i\omega C + \frac{1}{i\omega L}$$

$$Z_{TOTAL} = \frac{1}{\frac{1}{R} + i\omega C + \frac{1}{i\omega L}}$$

b)

$$I(\omega) = \frac{V}{Z_{TOTAL}} = V\left(\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)\right)$$

$$|I(\omega)| = V\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}$$

c)

$$\phi = \tan^{-1}\left[\frac{\omega C - \frac{1}{\omega L}}{1/R}\right]$$

LEAD VOLTAGE.

$$\phi = \tan^{-1}\frac{100\times10^{-6} - \frac{1}{100}}{1/500} = \tan^{-1} 500\cdot\left(-\frac{1}{100}\right)$$

$$= -1.37 \text{ RAD} = -78^\circ$$