Electrostatic Energy

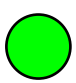
Capacitors and Dielectrics

Energy of a Charge Distribution

How much energy (≡ work) is required to assemble a charge distribution ?

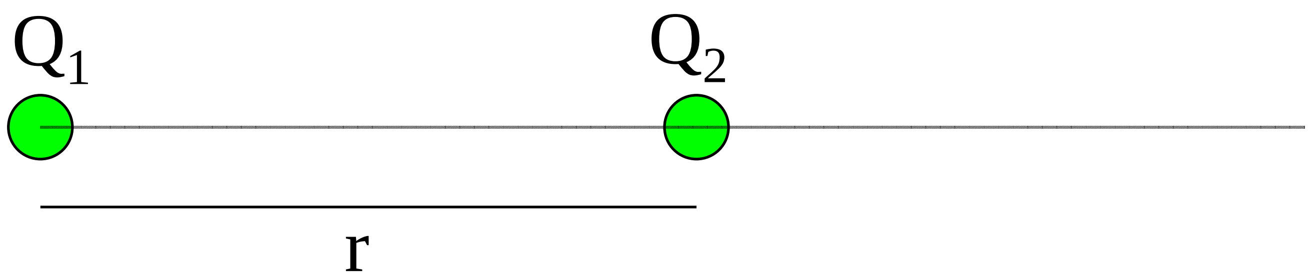
CASE I: Two Charges

Bringing the first charge does not require energy (≡ work)



Energy of a Charge Distribution

CASE I: Two Charges



Bringing the second charge requires to perform work against the field of the first charge.

$$W = Q_2 V_1 \quad \text{with} \quad V_1 = (1/4\pi\varepsilon_0)(Q_1/r)$$

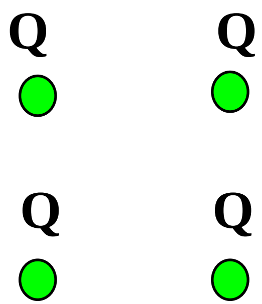
$$\Rightarrow W = (1/4\pi\varepsilon_0)(Q_1 Q_2/r) = U$$

$$U = (1/4\pi\varepsilon_0)(Q_1 Q_2/r)$$

U = potential energy of two point charges

Energy of a Charge Distribution

CASE II: Several Charges



How much energy is stored in this square charge distribution?, or …
What is the electrostatic potential energy of the distribution?, or …
How much work is needed to assemble this charge distribution?

The three statements represent the same question. To answer it is necessary to add up the potential energy of each pair of charges

$$\Rightarrow \; U = \sum U_{ij}$$ where:

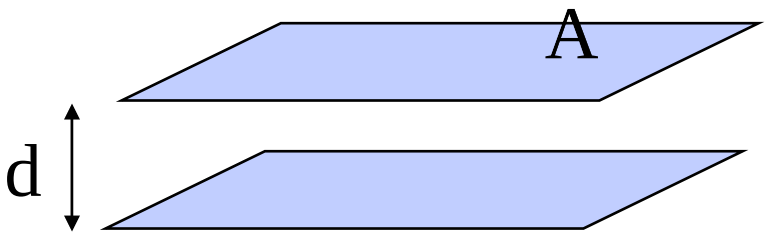
$$U_{ij} = (1/4\pi\varepsilon_0)\,(Q_i\, Q_j\,/r)$$

U_{ij} = potential energy of a pair of point charges

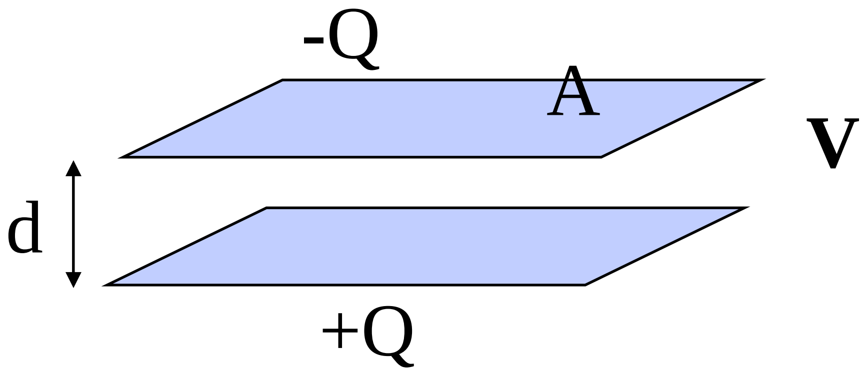
Capacitor

Two conductors, separated by a finite distance constitute a capacitor

One particular form of capacitor is the parallel plate capacitor shown in the figure: to parallel conducting plates, each of area A, separated by a distance d



Capacitance



If a potential difference V is applied between the plates, charges + Q and – Q appear on the plates.

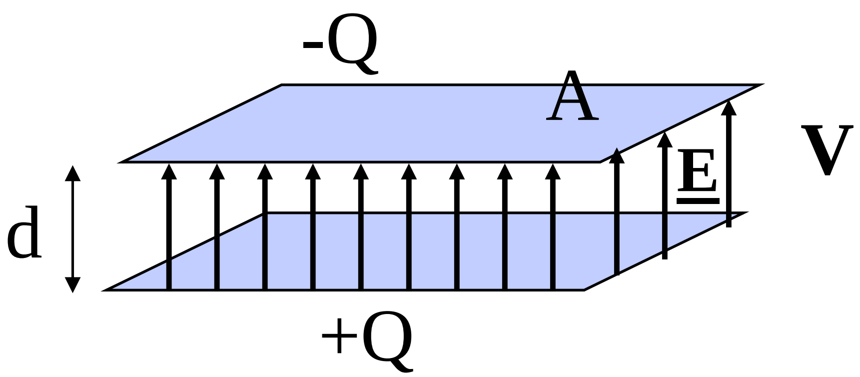
The charge Q is proportional to the applied voltage V

The ratio **C = Q / V** is called the **capacitance**

C = Q / V

[Units: Coulomb /Volt = Farad]

Parallel Plate Capacitor



The electric field between the plates is **$E = Q / A \varepsilon_0$**

The potential difference between the plates is **$V = E d = Q d / A \varepsilon_o$**

$\Rightarrow$ The relation between Q and V is

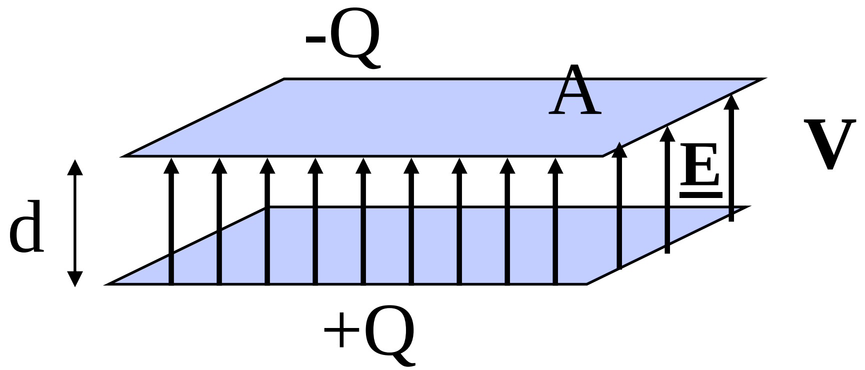
$V = Q d / A \varepsilon_0$ or $Q = V A \varepsilon_0 /d$

and the ratio $C = Q / V = A \varepsilon_0 / d$ is the capacitance of the parallel plate capacitor

$$C = \varepsilon_0 A / d$$

Capacitance

C = Q / V



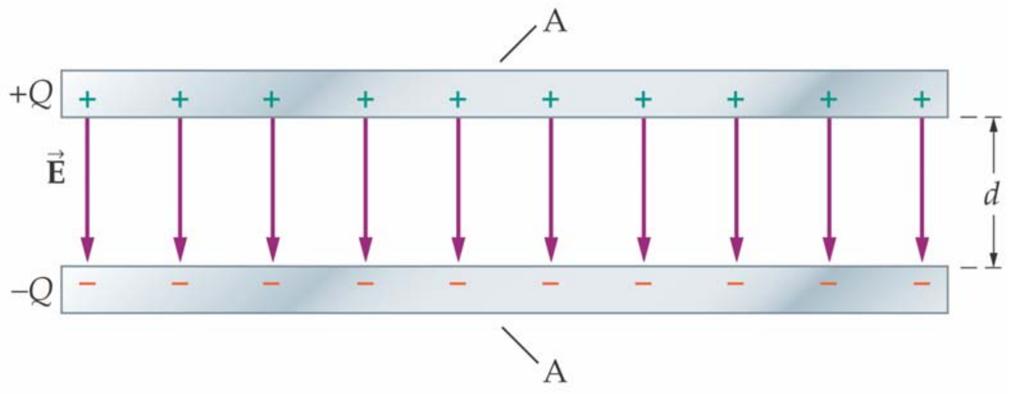
The relationship C = Q / V is valid for any charge configuration (Indeed this is the definition of capacitance or electric capacity)

In the particular case of a parallel plate capacitor

$$\mathbf{C = Q / V = \varepsilon_0 A / d}$$

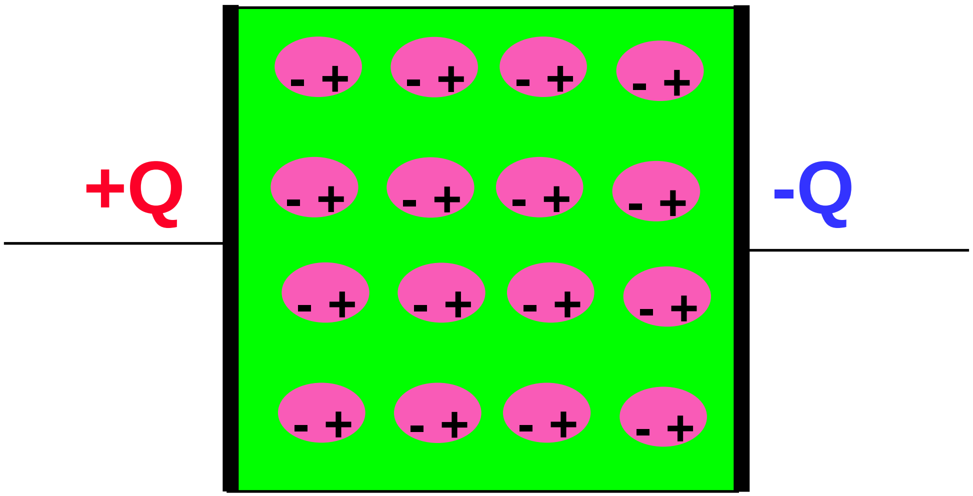
The capacitance is directly proportional to the area of the plates and inversely proportional to the separation between the plates

Given that: $A = 0.0280\ m^2$, $d = 0.550\ mm$, and $V = 20.1\ V$, find the magnitude of the charge Q on each plate.



Dielectrics in Capacitors

- Suppose we fill the space between the plates of a capacitor with an insulating material (a “dielectric”):

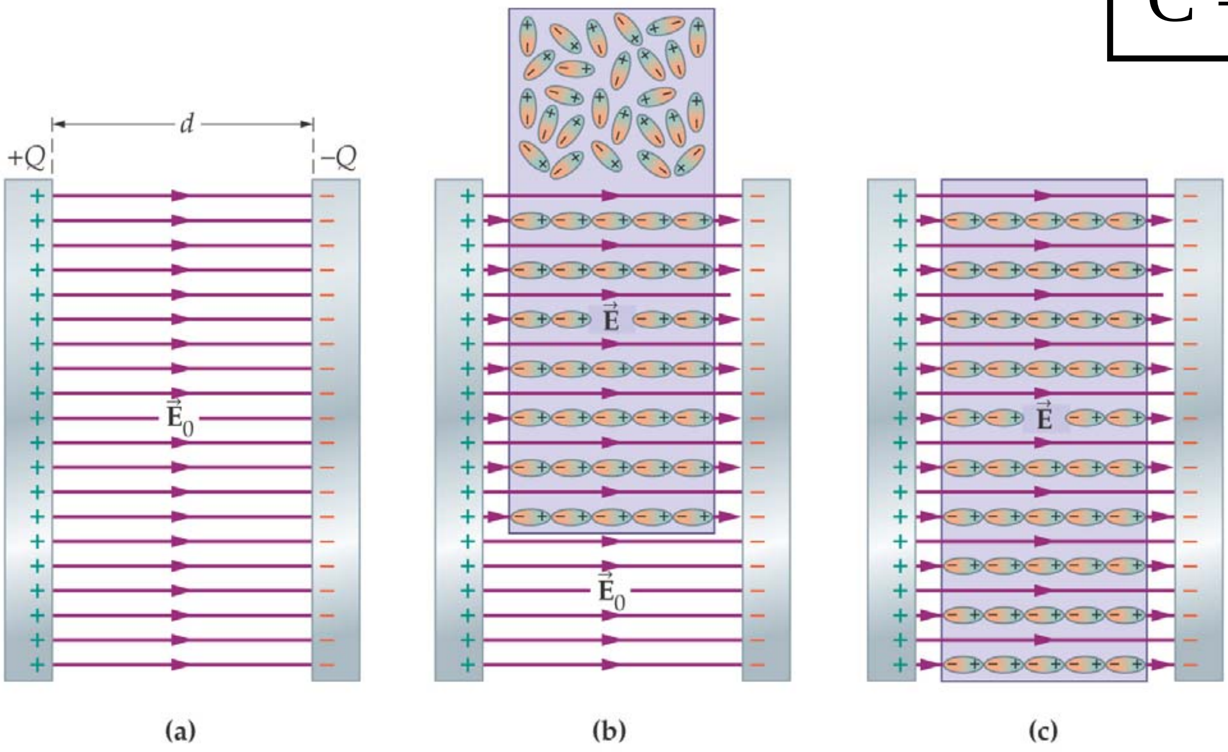


- The material will be “polarized” - electrons are pulled away from atom cores

- Consequently the E field within the capacitor will be reduced

Effect of a dielectric on the electric field of a capacitor

C = Q/V

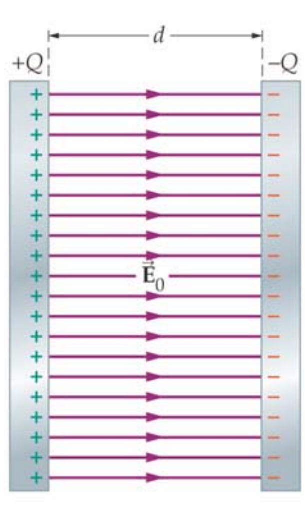


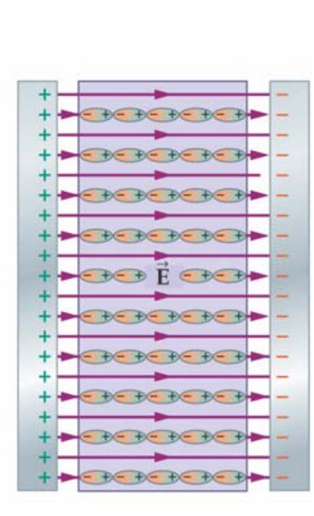
The dielectric decreases the electric field between the plates,
as well as the voltage between the plates,
and consequently increases the capacitance of the capacitor

Effect of a dielectric on a capacitor

$V_0 = E_0\, d$

$C_0 = Q / V_0$





$E = E_0 / \kappa$

κ: dielectric constant

When the dielectric is inserted: $$V = Ed = \left(\frac{E_0}{\kappa}\right)d = \frac{E_0 d}{\kappa} = \frac{V_0}{\kappa}$$

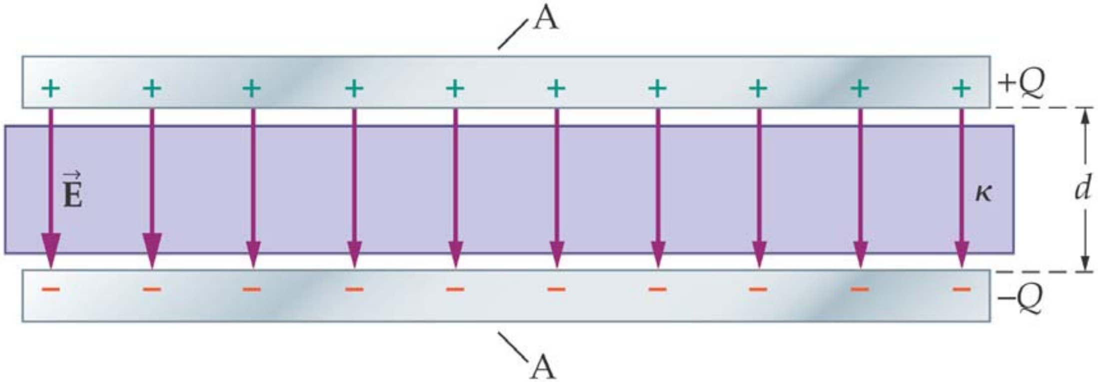
and for the capacitance: $$C = \frac{Q}{V} = \frac{Q}{\left(V_0/\kappa\right)} = \kappa\frac{Q}{V_0} = \kappa C_0$$

$C = \kappa\, C_0$: The capacitance increases when the dielectric is present

Effect on Capacitance

- A dielectric reduces the electric field by a factor κ [$E = E_0/\kappa$]
- A dielectric reduces the voltage by a factor of κ [$V = V_0/\kappa$]
- and C = Q/V is increased by κ [$C = C_0 \kappa$]
- Adding a dielectric ***increases*** the capacitance.

Parallel plate capacitor filled with dielectric

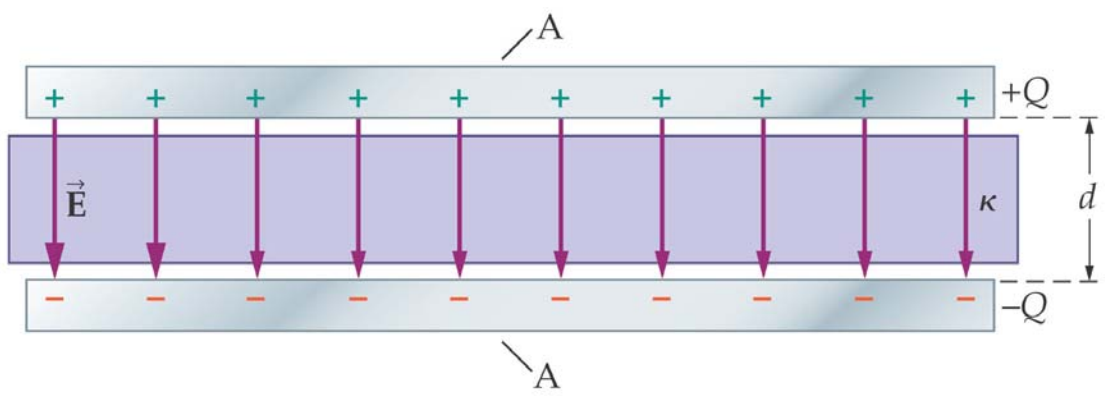


$$C = \frac{\kappa \varepsilon_0 A}{d}$$

Dielectric constant κ of some common substances

Water	80.4
Neoprene	6.7
Pyrex	5.6
Mica	5.4
Paper	3.7
Mylar	3.1
Teflon	2.1
Air	1.00059
Vacuum	1

Given A = 0.0280 m^2, d = 0.550 mm,
V = 12 V, and Q = 3.62×10^{-8} C:
Find κ



What is the value of the capacitance when there is no dielectric ?

What Does a Capacitor Do?

- Stores electrical charge.
- Stores electrical energy.

Capacitors are basic elements of electrical circuits both macroscopic (as discrete elements) and microscopic (as parts of integrated circuits).

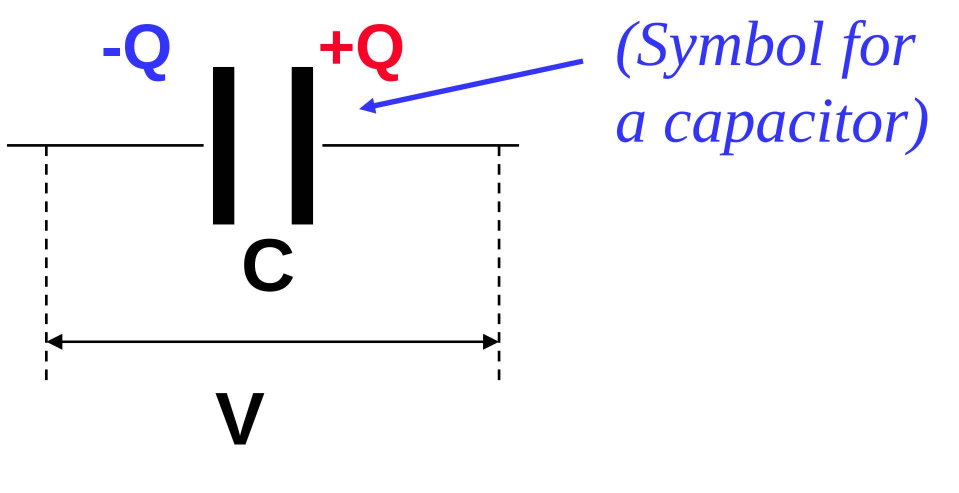
Capacitors are used when a sudden release of energy is needed (such as in a photographic flash).

Electrodes with capacitor-like configurations are used to control charged particle beams (ions, electrons).

What Does a Capacitor Do?

- Stores electrical charge.
- Stores electrical energy.

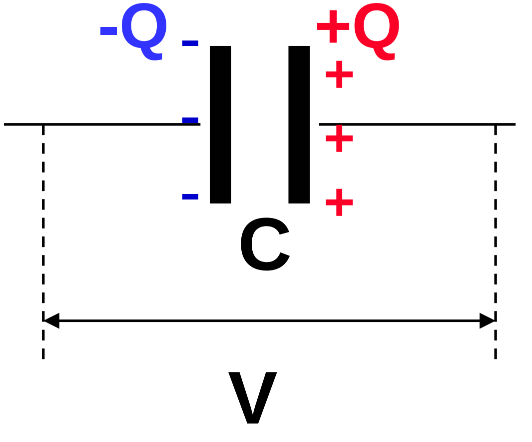
The charge is easy to see. If a certain potential, V, is applied to a capacitor C, it must store a charge **$Q = C V$**



What Does a Capacitor Do?

- Stores electrical charge.
- Stores electrical energy.

It takes a certain amount of energy to charge the capacitor.
This energy resides in the capacitor until it is discharged.



Energy Stored in a Capacitor

Suppose we have a capacitor with charge q (+ and -)

Then we transfer the charge Δq from the – to the + plate

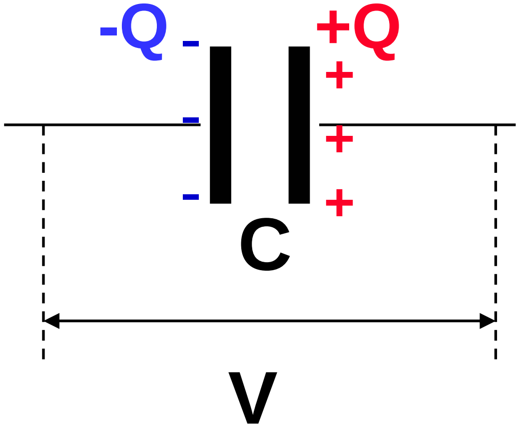
We must do work $\Delta W = V\,\Delta q$ to increase the charge

The potential energy of the capacitor increases as it gets charged

Since the voltage increases linearly with charge,

the total energy U stored in the capacitor charged with charge Q can be written as:

$$U = Q\,V_{AVE} = \tfrac{1}{2}\,Q\,V$$



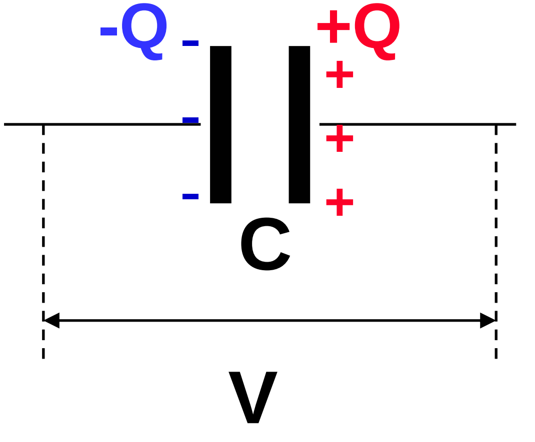
Energy Stored in a Capacitor

The total energy U stored in a charged capacitor with charge Q and potential difference V is:

$$U = \frac{1}{2} Q V$$

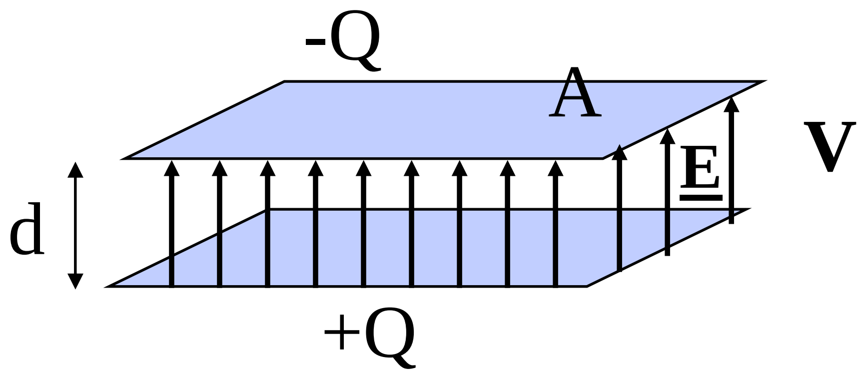
$$U = \frac{1}{2} C V^2$$

$$U = Q^2/2C$$



All these three expressions are equivalent, they give the energy in terms of different variables

Energy Density.



In the case of a parallel plate capacitor $Q = \varepsilon_0 EA$ and $V = Ed$

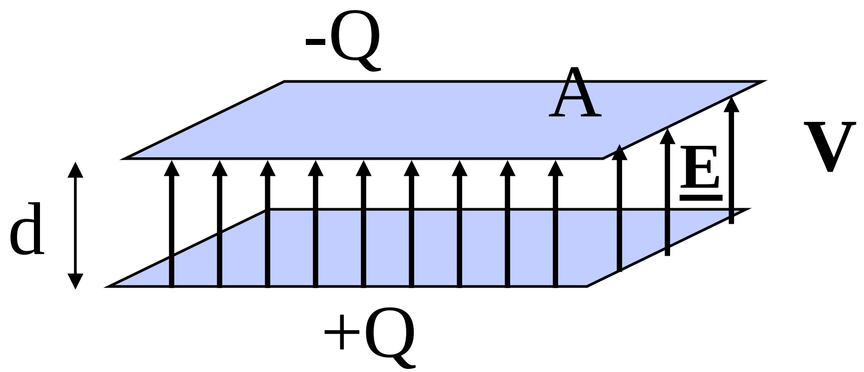
The total energy stored is $U = ½ QV = ½ (\varepsilon_0 EA) (Ed)$

or $U = ½ \varepsilon_0 E^2 (Ad)$, where Ad is the volume between the plates,

and $u_E = ½ \varepsilon_0 E^2$

is the electric energy density (energy per unit volume)

Energy Density.



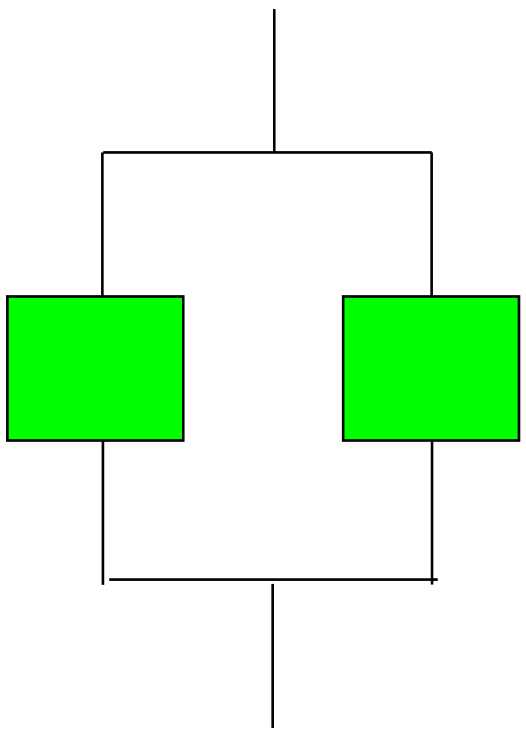
The electric potential energy can be thought of as stored in the electric field existing between the plates of the capacitor.

This result is valid for any electric field (not just that produced by a parallel plate capacitor)

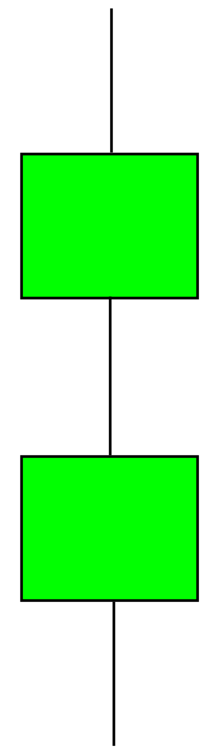
There is an electric energy density $u_E = \frac{1}{2} \varepsilon_0 E^2$ associated with an electric field

The energy is stored in the electric field

Parallel and Series

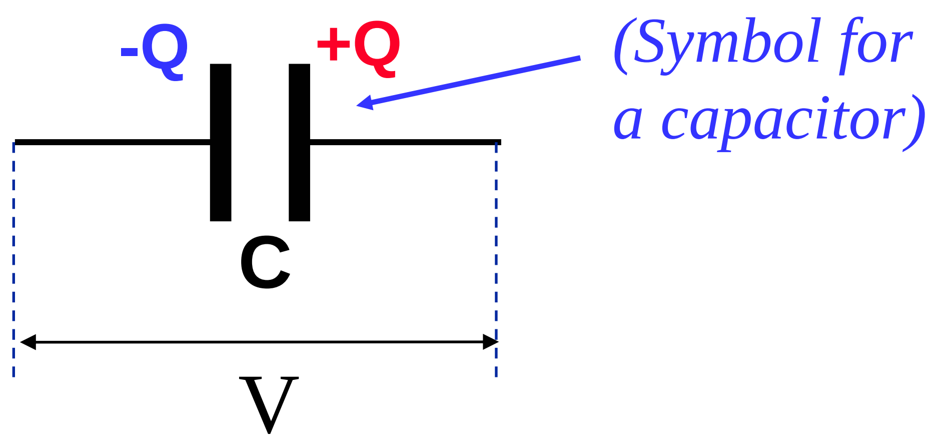


Parallel



Series

Capacitors in Circuits



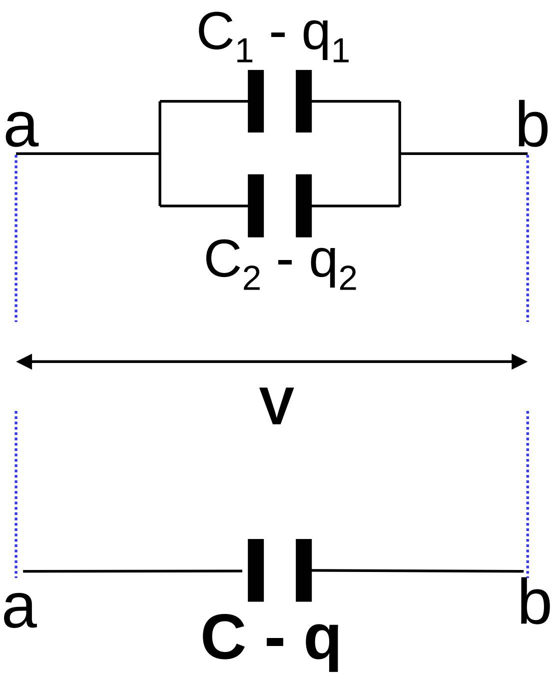
A piece of metal in equilibrium has a constant value of potential.

Thus, the potential of a plate and attached wire is the same.

The potential difference between the ends of the wires is V, the same as the potential difference between the plates.

Capacitors in Parallel

- Suppose there is a potential difference V between a and b.
- Then $q_1\ V = C_1$ & $q_2\ V = C_2$



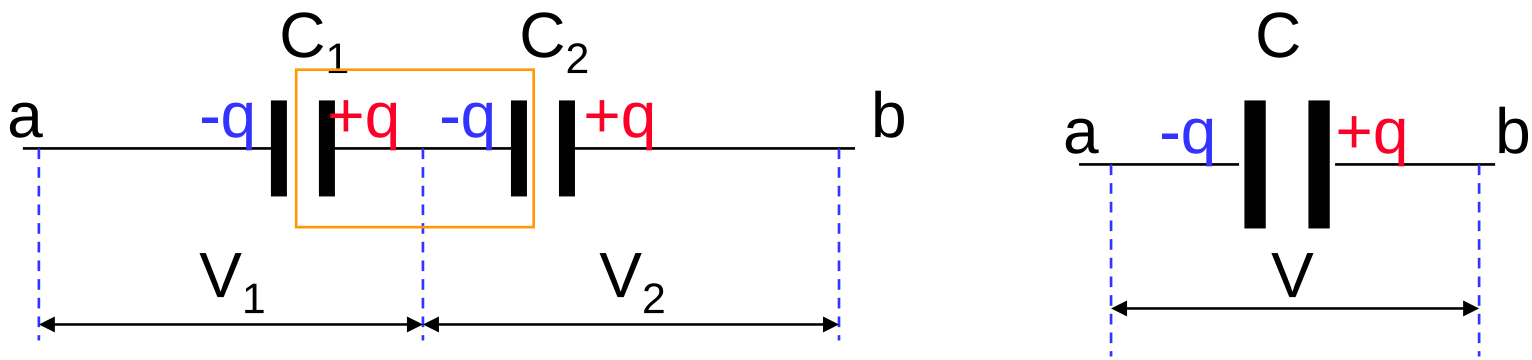
- We want to replace C_1 and C_2 with an equivalent capacitance $C = q\ V$
- The charge on C is $q = q_1 + q_2$

- Then $C = q\ V = (q_1 + q_2)\ V = q_1\ V + q_2\ V = C_1 + C_2$

$$C = C_1 + C_2$$

- This is the equation for capacitors in **parallel**.
- Increasing the number of capacitors increases the capacitance.

Capacitors in Series

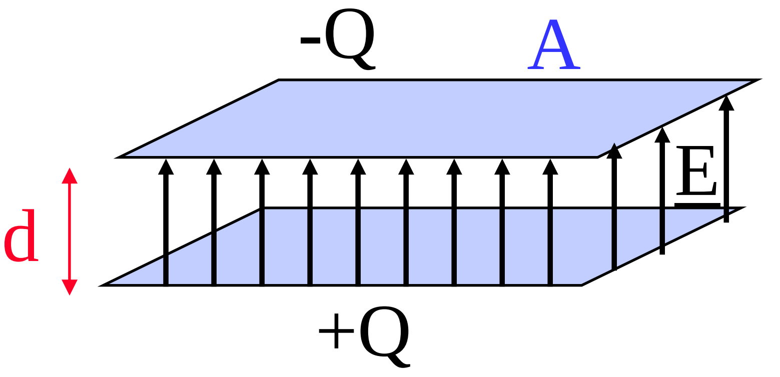


- Here the total potential difference between a and b is $V = V_1 + V_2$
- Also $V_1 = (1/C_1)\, q$ and $V_2 = (1/C_2)\, q$
- The charge on every plate (C_1 and C_2) must be the same (in magnitude)
- Then: $V = V_1 + V_2 = q / C_1 + q / C_2 = [(1/C_1) + (1/C_2)]\, q$
- or, $V = (1/C)\, q \Rightarrow$ $\mathbf{1 / C = 1 / C_1 + 1 / C_2}$

- This is the equation for capacitors in **series**.
- Increasing the number of capacitors decreases the capacitance.

Energy of a Charge Distribution

CASE III: Parallel Plate Capacitor

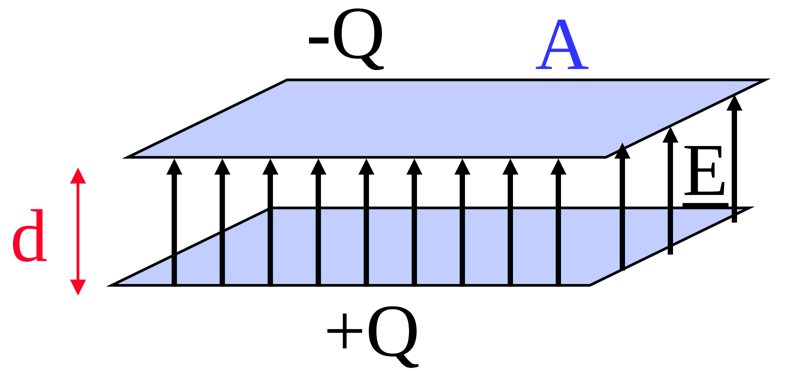


Electric Field $\Rightarrow$ $E = \sigma / \varepsilon_0 = Q / \varepsilon_0 A$ $\quad (\sigma = Q / A)$

Potential Difference $\Rightarrow$ $V = E\, d = Q\, d / \varepsilon_0 A$

Energy of a Charge Distribution

CASE III: Parallel Plate Capacitor



The work done in charging the plates ends up as stored potential energy of the final charge distribution

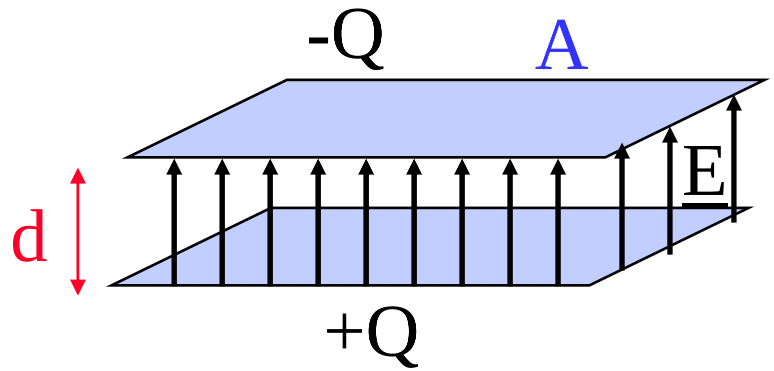
$$W = U = d\,Q^2 / 2\,\varepsilon_0\,A$$

Where is the energy stored ?

The energy is stored in the electric field

Energy of a Charge Distribution

CASE III: Parallel Plate Capacitor



The energy U is stored in the field, in the region between the plates.

$$U = d\, Q^2 / 2\, \varepsilon_0\, A = (1/2)\, \varepsilon_0\, E^2\, A\, d$$

$E = Q / (\varepsilon_0\, A)$

The volume of this region is **Volume = A d**, so we can define the **energy density u_E** as:

$$u_E = U / A\, d = (1/2)\, \varepsilon_0\, E^2$$