The Electric Field

The test charge \( q_0 \) experiences a force:

\[
F = k \frac{qq_0}{r^2}
\]

We say that the charge \( q \) creates an electric field \( E \) given by:

\[
E = k \frac{q}{r^2}
\]

The electric field \( E = F/q_0 \) is the force per unit charge experienced by the test charge \( q_0 \).

If we place a charge \( Q \), in a region of space where there is an electric field \( E \), the charge experiences a force \( F = E Q \).
The Electric Field

Direction of the electric field

Superposition of electric field

Strength of electric field
Electric Field Lines

Electric field lines (lines of force) are continuous lines whose direction is everywhere that of the electric field. 

Electric field lines:
1) Point in the direction of the electric field $\mathbf{E}$
2) Start at positive charges or at infinity
3) End at negative charges or at infinity
4) Are more dense where the field has greater magnitude
Electric Field Lines
Electric field lines (lines of force) are continuous lines whose direction is everywhere that of the electric field.
Force Due to an Electric Field

Just turn the definition of $\mathbf{E}$ around. If $\mathbf{E}(\mathbf{r})$ is known, the force $\mathbf{F}$ on a charge $q$, at point $\mathbf{r}$ is:

$$\mathbf{F} = q \mathbf{E}(\mathbf{r})$$

The electric field at $\mathbf{r}$ points in the direction that a positive charge placed at $\mathbf{r}$ would be pushed.

Electric field lines are bunched closer where the field is stronger.
An electric dipole consists of two equal and opposite charges \((q\) and \(-q\)) separated a distance \(d\).
The Electric Dipole

We define the **Dipole Moment** \( \mathbf{p} \)

- magnitude = \( qd \)
- direction = from -q to +q
The Electric Dipole

Suppose the dipole is placed in a uniform electric field (i.e., $\mathbf{E}$ is the same everywhere in space). Will the dipole move??
What is the total force acting on the dipole?
What is the total force acting on the dipole?
The Electric Dipole

What is the total force acting on the dipole?

Zero, because the force on the two charges cancel: both have magnitude $qE$. The center of mass does not accelerate.
What is the total force acting on the dipole?

Zero, because the force on the two charges cancel: both have magnitude \( qE \). The center of mass does not accelerate.

But the charges start to move. Why?
What is the total force acting on the dipole?
Zero, because the force on the two charges cancel: both have magnitude $qE$.

The center of mass does not accelerate.

**But the charges start to move (rotate). Why?**

There’s a torque because the forces aren’t colinear.
The torque is:
\[ \tau = (\text{magnitude of force}) (\text{moment arm}) \]

\[ \tau = (qE)(d \sin \theta) \]

and the direction of \( \tau \) is (in this case) into the page, or the dipole tends to rotate clockwise.
Field Due to an Electric Dipole at a point $x$ straight out from its midpoint

Electric dipole moment

$$p = qd$$

Calculate $\mathbf{E}$ as a function of $p$, $x$, and $d$
The electric field of a spherical charge distribution is that of a point charge (with the same total charge) located at its center (for points outside the charge distribution).

The electrical field near a large charged thin plate, is uniform in direction and magnitude (perpendicular to the plate and with magnitude independent of the distance to the plate).
Parallel Plate Capacitor

Two parallel conducting plates, charged with equal opposite charges, and separated by a small distance $d$, constitute a parallel plate capacitor.

The electric field is uniform between the plates, and zero outside the plates (away from the edges of the structure).
Properties of Conductors

In a conductor there are large number of electrons free to move. This fact has several interesting consequences

Excess charge placed on a conductor moves to the exterior surface of the conductor

The electric field inside a conductor is zero when charges are at rest

A conductor shields a cavity within it from external electric fields

Electric field lines contact conductor surfaces at right angles

A conductor can be charged by contact or induction

Connecting a conductor to ground is referred to as grounding

The ground can accept or give up an unlimited number of electrons
Electric Field Flux

The electric flux $\Phi$ is defined as:

$$\Phi = E \cdot A \cos (\theta)$$

$E$: uniform electric field
$A$: area
$\theta$: angle between $E$ and $A$

For a closed surface:

The flux is positive for field lines that leave the enclosed volume.
The flux is negative for field lines that enter the enclosed volume.
Gauss’s Law

If a charge $q$ is enclosed by an arbitrary surface, the electric flux through the surface is given by:

$$\Phi = \frac{q_{enclosed}}{\varepsilon_0}$$

SI unit of $\Phi$: N m$^2$/C

$$k = (4\pi\varepsilon_0)^{-1} = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$\varepsilon_0 = \text{permitivity of free space} = 8.86 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

Gauss’s Law is used to calculate the electric field in situations of high symmetry
Applying Gauss’s’s Law

Gauss’s law is useful only when the electric field is constant on a given surface

1. Select Gauss surface
   In this case a cylindrical pillbox

2. Calculate the flux of the electric field through the Gauss surface
   \[ \Phi = 2 \mathbf{E} \mathbf{A} \]

3. Equate \( \Phi = \frac{q_{\text{encl}}}{\varepsilon_0} \)
   \[ 2EA = \frac{q_{\text{encl}}}{\varepsilon_0} \]

4. Solve for \( E \)
   \[ E = \frac{q_{\text{encl}}}{2 \mathbf{A} \varepsilon_0} = \frac{\sigma}{2 \varepsilon_0} \]
   with \( \sigma = \frac{q_{\text{encl}}}{\mathbf{A}} \)
<table>
<thead>
<tr>
<th>CHARGE DENSITY</th>
<th>SPHERICAL (point or sphere)</th>
<th>CYLINDRICAL (line or cylinder)</th>
<th>PLANAR (plane or sheet)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Depends only on radial distance from central point</td>
<td>Depends only on perpendicular distance from line</td>
<td>Depends only on perpendicular distance from plane</td>
</tr>
<tr>
<td>GAUSSIAN SURFACE</td>
<td>Sphere centered at point of symmetry</td>
<td>Cylinder centered at axis of symmetry</td>
<td>Pillbox or cylinder with axis perpendicular to plane</td>
</tr>
<tr>
<td>ELECTRIC FIELD $E$</td>
<td>$E$ constant at surface $E \parallel A - \cos \theta = 1$</td>
<td>$E$ constant at curved surface and $E \parallel A$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E \perp A$ at end surface $\cos \theta = 0$</td>
<td>$E \perp A$ at curved surface $\cos \theta = 0$</td>
<td></td>
</tr>
<tr>
<td>FLUX $\Phi$</td>
<td></td>
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</tbody>
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