# Unit 4: Electric Potential & Capacitors

I. Potential Energy in a field[1]

By building on your intuitive and formal understanding of potential ("stored") energy in a gravitational field, you can understand *electric potential energy*, the "stored" energy of a charge in an electric field.

A. Suppose this diagram represents part of the gravitational field produced by a non-spherical asteroid. The dotted line represents the "ground" of potential energy, *i.e.,* points at which we set the gravitational potential energy equal to zero.
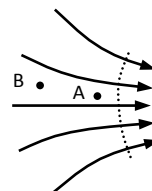
1. Briefly describe the motion of a rock released from rest from point B.

2. At which point, A or B, would the rock have more gravitational potential energy? Explain.

3. An astronaut must do 30 joules of work to lift the rock of mass 0.50 kg from the dotted line to point A. From the given information, can you figure out the rock's gravitational potential energy at point A? If so, how much does it have? If not, what additional information do you need?

4. Although it's common to think of "work" in terms of an equation, the underlying concept is this: **The work done on a object equals the energy given to that object as a result of getting pushed through a distance**. Did you use this definition, at least implicitly, in question 3? If not, re-evaluate your reasoning and answer to make sure they agree with this definition.

B. Now suppose the above diagram represents part of the electric field produced by a negatively charged blob. The dotted line again represents the "ground" of potential energy, i.e., points at which we set the electric potential energy equal to zero.

1. Briefly describe the motion of a positively-charged particle released from rest from point B.

2. At which point, A or B, would the positive particle have more electric potential energy? Explain.

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[1]Section I adapted from *Open Source Tutorials in Physics Sensemaking, Suite 2* © University of Maryland Physics Education Research Group, 2010

3. By measuring the work needed to "lift" the positive particle from the dotted line to point A, a student determines its potential energy at A to be 30 millijoules. Does it make sense to say that "point A has a potential energy of 30 millijoules?" Explain.

✯ *Consult an instructor before you proceed.*

C. This diagram shows a new, uniform electric field.

1. At which of the three labeled points would a +1 coulomb charge have the most electric potential energy? The second most electric potential energy? The least? (*Hint:* Think about whether a particle must "rise against" the field or "fall" with the field to get from one point to another. Does "falling" correspond to gaining or losing potential energy?)
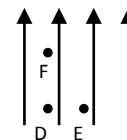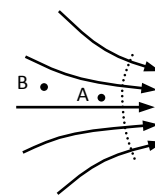
2. Now we'll help you check your answer. Imagine the +1 coulomb charge moving from D to F. Does the charge naturally "fall" through that distance, or must you "lift" it from D to F?

3. So, does the charge gain or lose electric potential energy going from D to F? Does this agree with your reasoning in question 1 above?

4. Now imagine the +1 coulomb charge moving from D to E. Does it "fall," does it get "lifted," or something else?

5. So, going from D to E, does the charge gain potential energy, lose potential energy, or neither? Does this agree with your reasoning in question 1 above?

✯ *Consult an instructor before you proceed.*

II. From potential energy to potential[2]

Again consider the electric field from the previous page, redrawn here. A student determines the potential energy of various charged beads at point A by measuring how much work it takes to "lift" each bead from the dotted line to point A.

A. The work done on a 2.0 coulomb bead, and hence its electric potential energy at point A, was 20

joules. A new bead carries twice as much charge, 4.0 coulombs. Intuitively, how much electric potential energy do you expect the new bead to have at point A? Why?

B.   Same question, different bead: How much work does it take, and therefore how much potential energy is given to a 5.0 coulomb bead moved from the dotted line to point A?

C.   Suppose you want to use a *single number* to best express potential-energy information about point A, a number that applies to the 2.0 coulomb bead, the 4.0 coulomb bead, the 5.0 coulomb bead, and any other bead at point A — even though those different beads all have different potential energies. To be useful, the number should make it easy to calculate the electric potential energy of *any* charge at point A.

1.   What number would you choose?

2.   What are the units of that number?

The number you just came up with is called the electric *potential*, to distinguish it from electric *potential energy*. Electric potential is denoted *V.*

D.   Let *EPE* denote electric potential energy of a bead and *q* denote the charge of that bead. Generalizing the reasoning you just used, come up with an equation relating *EPE*, *V*, and *q*.

E.   We saw above that it makes no sense to say point A has an electric potential energy, because different beads at that point have different potential energies. Does it make sense to say point A has an electric potential? Explain.

F.   At which point, A or B, is the electric potential higher? Explain. (You already answered a similar question about the electric potential energy, but not about electric potential.)

III. Equipotential maps[3]

So far in this tutorial, our diagrams have shown electric fields. To introduce diagrams of electric potential, we'll start with an analogy…

A.   Suppose this diagram is a contour map of a hill, showing lines of equal height. For instance, the curvy line that includes points P and R consists of points 20 meters above sea


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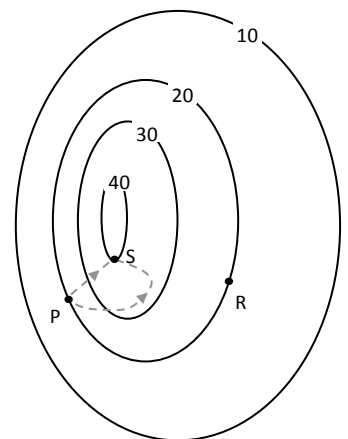
[3]Section III adapted from Open Source Tutorials in Physics Sensemaking, Suite 2 © University of Maryland Physics Education Research Group, 2010

level. The dashed lines show two paths by which a hiker can climb from P to a scenic view at S.

1. A ball is placed on the ground at point P and released from rest. It starts rolling. On the map, draw an arrow showing its initial direction of motion. Do the same for a ball at point R. Explain.

2. At which point, P or R, is the hill steeper? Articulate your reasoning, because you'll build on it below.

3. An **equipotential line** is a line (or curve) along which you could walk without gaining or losing potential energy. Are the lines of equal height on the contour map above also equipotential lines? Give an intuitive explanation; don't just cite an equation.

4. A backpack gets carried from P to S. Along which dashed path, if either, would it gain more gravitational potential energy: the direct path or the curvy path? Explain.

5. Along which path, if either, is more work done on the backpack? Hint: Think about the relation between work and change in energy.

6. The work is the same along either path, since all the work done on the backpack goes into increasing its gravitational potential energy, and the gain in gravitational potential energy is the same along either path. This path-independence of the work initially seems counterintuitive to most people. Can you make intuitive sense of this result?   Hint: Think about which path is longer vs. which path is steeper (on average).

★ *Consult an instructor before you proceed.*

B. Now suppose the diagram shows electric equipotential lines corresponding to an unpictured, irregularly-shaped charged blob — *not* a point charge! So, when a charged particle "walks" along one of those curves, it neither gains nor loses electric potential energy. For instance, point P and R both have potential V = 20 volts (joules per coulomb)

1. A positively charged dust particle is released from rest from point P. Draw an arrow showing its initial direction of motion. Do the same for a positive particle released from point R.
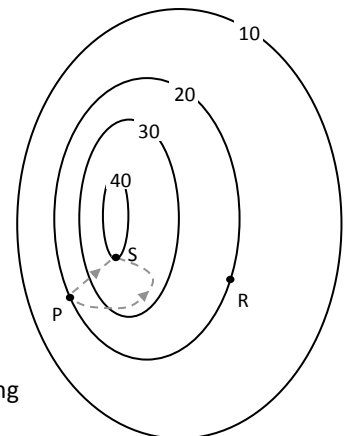
Explain.

2. Do the arrows you just drew also show the direction of the electric field at those points? Explain why or why not.

3. At which point, P or R, is the electric field stronger? How do you know? Hint: Think of a rock falling in a strong gravitational field vs. a weak gravitational field. In which field must the rock fall farther to lose a given amount of potential energy? What conclusions can you reach about the relation between field strength and the spacing of equipotential lines?

4. A tiny positively-charged bead gets carried from P to S. Along which path, if either, would it gain more electric potential energy: the direct path or the curvy path? Explain.

5. Along which path, if either, is more work done on the bead? Can you make intuitive sense of this result?

IV. Capacitors[4]

You may be wondering what sort of object can create a uniform electric field like that seen in section I., part C. (shown again here).
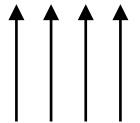
One such object is a parallel plate capacitor. Any two conductors seperated by an insulator can be electrically charged so that one conductor has a net excess positive charge and the other conductor has an equal amount of excess negative charge; such an arrangement is called a capacitor. Capacitors can come in many shapes and sizes, but it is the parallel plate capacitor that creates the above uniform electric field when attached to a voltage source, as pictured here. When connected to a votage source, electrons will flow off one conductor (leaving excess positive charge) and on to the other until the potential differnce between the two conductors is the same as that of the voltage source. The potenital difference between the two plates can be expressed in terms of the work done on a positive test charge, q, as it moves from the positive to negative plate aong the electric field, i.e., $\Delta V = E * d$. The capacitance, C, of a given capacitor is defined as the ratio of the amount of access charge, $|q|$, on either one of the conductors to the size of the potential difference, $|\Delta V|$, across the two conductors, so that C = $|q|$ / $|\Delta V|$.
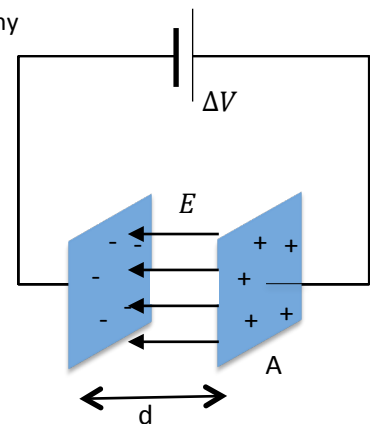
[4]Section IV adapted from Laws, P.W. *The Physics Suite: Module 4, Electricity and Magnetism.* Wiley, 2004. & Georgia State University. Hyperphsyics. Capacitance of Parallel Plates. <http://hyperphysics.phy-astr.gsu.edu/hbase/electric/pplate.html>, 2014.

V. More Problems

1.[5] Consider the following diagram:
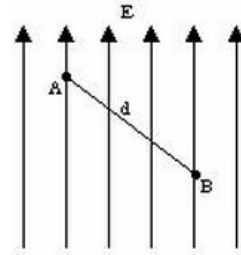
(i) A charge $+q$ travels a distance d from point A to point B in a uniform electric field of magnitude E. The path lies at a 45° angle to the field lines. What is the work done by the field on the charge? Explain your reasoning.

(ii) A charge $-q$ travels a distance d from point A to point B in a uniform electric field of magnitude E. The path lies at a 45° angle to the field lines. What is the work done by the field on the charge? Explain your reasoning.

(iii) Is the potential difference between point A and point B positive or negative? Explain your reasoning.

2.[6] In this diagram, the thin lines are equipotential lines, labeled in volts. The thick lines represent hollow plastic tubes through which a charged bead could slide frictionlessly from point L to point N or vice versa. Point L is located at a 7 volt potential and point N is located at a 19 volt potential.
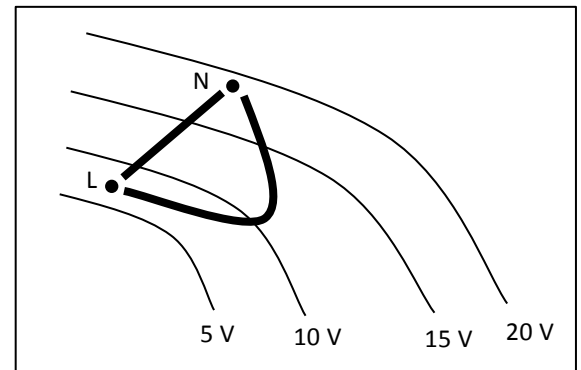
i. A positive bead of charge 2.0 C and mass 1 g is put into the straight tube at L. Does it naturally move to point N, or must it be pushed through the tube to reach N? Explain. Calculate how much energy the bead gains or how much work the pusher must do in this situation.

ii. Two of the same beads from part i are released from rest into the straight and curvy tube, respectively, at point N. Which bead (if either) reaches point L with more speed? Explain. Calculate at what speeds do the beads reach point L.

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[5]Probelm 1 from Dr. Beth Thacker PHYS 1404 Unit 05 Lab: Electric Potential Difference
[6]Problem 2 adapted from Open Source Tutorials in Physics Sensemaking, Suite 2 © University of Maryland Physics Education Research Group, 2010

3. [7] The figure shows two capacitors, each with a 3 mm separation. A proton is released from rest in the center of each capacitor.
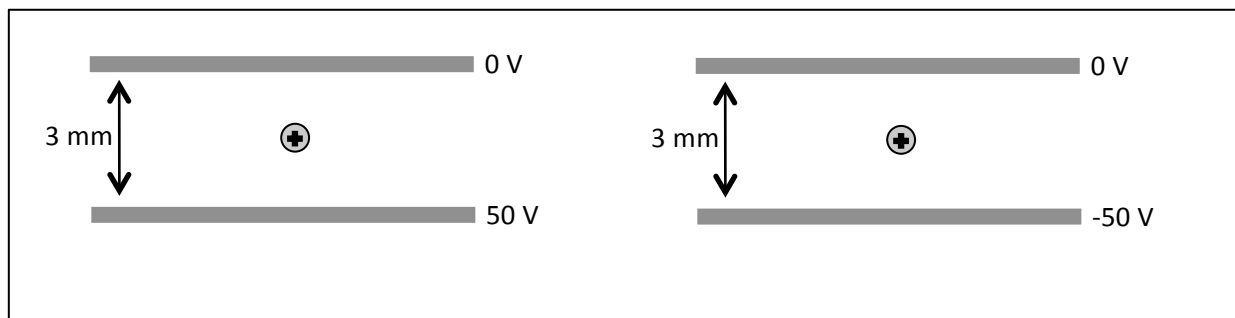
a. Draw an arrow on each proton to show the direction it moves. How do you know that is moves in this way?

b. Describe the electric field in between the two plates. How does it vary? For each capacitor, what is the magnitude of the electric field at the starting point of the proton, i.e., at the center of the capacitor?

c. Which proton reaches a capacitor plate first? Or are they simultaneous? Explain.

4. [8] You have been able to get a part-time job in a University laboratory. The group is planning a set of experiments to study the forces between nuclei in order to understand the energy output of the Sun. To do this experiment, you shoot alpha particles from a Van de Graaf accelerator at a sheet of lead. The alpha particle is the nucleus of a helium atom and is made of 2 protons and 2 neutrons. The lead nucleus is made of 82 protons and 125 neutrons. The mass of the neutron is almost the same as the mass of a proton. To assure that you are actually studying the effects of the nuclear force, an alpha particle should come into contact with a lead nucleus. Assume that both the alpha particle and the lead nucleus have the shape of a sphere. The alpha particle has a radius of 1.0×10^{-13} cm and the lead nucleus has a radius 4 times larger. Your boss wants you to make two calculations:

[7] Problem adapted Knight, R.D., *Physics for Scientists and Engineers Second Edition: A Strategic Approach, Student Workbook.* Pearson, 2008.

[8] Probelm 4 from the University of Minnesota Context Rick Problem Archive.

(a) What is the minimum speed of such an alpha particle if the lead nucleus is fixed at rest? (Think back to Physics I)
(b) What is the potential difference between the two ends of the Van de Graaf accelerator if the alpha particle starts from rest at one end (from a bottle of helium gas)?

KNOW:

WANT:

STRATEGY: