

1. SQUID: SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE

(Adapted from the Mr. Squid Manual (Star Cryotronics) by Jerry Gollub, with updates by Suzanne Amador Kane 5/2005)

We are used to thinking of quantum mechanics as a way of understanding the properties of small things: atoms, nuclei, etc. By contrast, the macroscopic world often follows the classical laws of Newtonian mechanics. The reason for this classical behavior on the macroscopic scale is that the quantum phases of the complex wavefunctions of different atoms are unrelated and can vary in complex ways from moment to moment. We say that there is no **quantum phase coherence** for macroscopic objects. For example, electrons that travel by different paths in a piece of aluminum cannot interfere with each other. However, something amazing happens if the aluminum is cooled to below 1K: it changes from a normal metal to become a **superconductor**, a phase in which a large number of electrons condense into a single quantum state. It then *does* have phase coherence on a macroscopic scale, and hence can show interference between electrons that travel by quite different paths. The superconducting state exhibits some remarkable effects by virtue of its quantum mechanical coherence: it can conduct electrical current with zero resistance, it can completely exclude magnetic flux within the superconductor (the **Meissner effect**), and it enables the construction of novel electronic devices. We will study “macroscopic quantum interference” in this experiment, using “high temperature superconductors” discovered in the 1980’s that become superconducting above the temperature of liquid nitrogen (77 K). The difference in behavior of electrons in a metal below and above the superconducting transition temperature is analogous to that between coherent laser light and ordinary incoherent sunlight or light from a lightbulb. In both cases, a well-defined phase that is coherent over large distances allows interference effects to be seen on a macroscopic scale.

Superconductivity was first discovered in 1911 in a sample of mercury metal that lost its resistance just four degrees above absolute zero. The phenomenon of superconductivity has been a major focus of research and applications even since, although until recently efforts were limited by the low temperatures required. The first experiments only revealed the zero resistance property of superconductors, and more than twenty years passed before discovery of the Meissner effect. Magnetic flux quantization – the key to SQUID operation – was predicted theoretically only in 1950 and was finally observed in 1961. The Josephson effects were predicted and experimentally verified a few years after that by Brian Josephson, who won a Nobel Prize for his work while still in his 20’s.

SQUIDs (superconducting quantum interference devices) were first studied in the mid-1960s, soon after the first Josephson junctions were made. These sensitive magnetic field detectors are also important tools in quantum computing experiments and tests of the fundamental properties of quantum mechanics. Practical superconducting wire for use in moving machines and magnets also became available in the 1960’s. For the next twenty years, the field of superconductivity slowly progressed toward practical applications and to more profound understanding of the underlying phenomena. A great revolution in superconductivity came in 1986 when the era of high-temperature

superconductivity began. The existence of superconductivity at liquid nitrogen temperatures has opened the door to applications that are simpler and more convenient than were ever possible before. Nevertheless, the product you have in your hands today was made possible by many aspects of the 80 years of discovery that preceded it. You can read about current applications of high temperature superconductors in the article “High Temperature Cuprate Superconductors get to work”, *Physics Today*, pp.41-47, April 2005, Vol 58 (4), available in the lab binder.

Introduction to Superconductivity and SQUIDS

There are certain materials – actually, many thousands of them by now – that exhibit a remarkable transition in their ability to pass electrical currents: when they are cooled down to a sufficiently low temperature, their electrical resistance vanishes completely. How this behavior comes about was a mystery that occupied the minds of theoretical physicists for nearly half a century after it was first observed. The answer turned out to be tied to the quantum-mechanical nature of solids, in particular, to the tendency of electrons to form organized collective quantum mechanical states. One instance of this is **Cooper pairs** of electrons that behave cooperatively in certain materials and form a single quantum-mechanical state. While it is hard to see why two particles of like charge would form a pair, the reason why is actually easy to understand intuitively. **Phonons** (the quantum mechanical version of sound waves in solids) can promote the pairing of electrons by creating a local concentration of positive electrical charge when two nuclei move closer together as a sound wave move past them; a nearby electron can both be attracted to this local charge concentration and further concentrate it. The resulting even greater positive charge imbalance can attract and bind the second Cooper pair electron. This process has been described as similar to how a depression in a bed will encourage a person to roll into the dent, thus creating an even deeper depression and encouraging a second person in the bed to roll toward the first. This explanation also helps one understand why electrical currents can flow without resistance in a superconductor. This is because in normal conductors resistance to electrical current is caused by electrons scattering off of vibrating atomic nuclei (in other words, off of phonons); this scattering changes the electron’s direction and transfers energy from the electrical current to sound waves. In superconductors, electrons also interact with phonons, but instead of scattering off of them and diminishing the flow of electrical current, the phonons actually promote the formation and movement of Cooper pairs.

In the following discussions, we can only explain these concepts briefly and without theoretical rigor. Fortunately, many such books exist and we refer you to some in the References in Section 10 (of the full documentation). What this guide will try to do is give you some idea of the underlying physical principles behind Mr. SQUID®.

The Superconducting State

A fundamental aspect of physical systems is that they naturally seek a state of lowest free energy. An example of this is that a ball will roll to the lowest spot on an uneven surface (the lowest potential energy). An external source of energy (such as kicking the ball) is required to raise it to a higher spot (energy level). Similarly, systems of particles, such as the electrons in a metal, will occupy a lowest-energy state known as the *ground state*, unless they are excited

by some external source of energy. In certain materials, it is possible for electrons to achieve a ground state with lower energy than otherwise available by entering the superconducting state. In this ground state, *all* the electrons are described by the same wavefunction. What does this mean?

In quantum mechanics, physical entities such as electrons are described mathematically by *wavefunctions*. Like ordinary waves in water or electromagnetic waves such as light waves, quantum mechanical wavefunctions are described by an amplitude (the height of the wave) and a phase (whether it is at a crest or a trough or somewhere in between). When you are describing waves of any kind, these two parameters are all that is necessary to specify what part of the wave you are discussing and how large it is. Moving waves oscillate both in time and in space. If we sit at one point in space, the wavefront will oscillate in time. If we look at one moment in time, the wavefront undulates in space. The quantum mechanical wavefunction is a mathematical entity that describes the behavior of physical systems such as electrons and light waves.

So far, you have probably used a wavefunction required to describe each particle in a physical system. In the usual or “normal” state, the wavefunctions describing the electrons in a material are unrelated to one another. In a superconductor, on the other hand, a *single* wavefunction describes the *entire population* of superconducting electron pairs. That wave function may differ in phase from one place to another within the superconductor, but knowing the function in one place determines it in another. Physicists call such a wavefunction a “many-body wavefunction.” Strictly speaking, electrons are indistinguishable particles; there is no way to keep track of an individual electron in the population and it in fact has no meaning to speak of one. The pairs of electrons that inhabit the superconducting state are constantly forming, breaking, and reforming such that the wavefunction that describes the superconducting state remains the same. As we will see, it is the existence of this coherent wavefunction that accounts for the phenomena associated with superconductivity.

The Quantum of Flux

In quantum mechanics, it is common to find that, for example, the energy of a bound particle or the energy of a photon of light can only occur in discrete amounts called **quanta** rather than being continuously divisible. We also see quantization occurring in nature with respect to electrical charge, for example. To our knowledge, all electrical charge (excluding quarks and some exotic solid state systems for the moment) occurs in units of $e = 1.6 \times 10^{-19}$ C, the charge on a single electron. Therefore, nothing in nature has 1.5 times this charge, for example. Electromagnetic energy is quantized in units called photons, whose magnitude is set by the product of the frequency of the radiation times Planck's constant. In general, quantization is only readily apparent when we are dealing with microscopic objects. Macroscopic objects are composed of enormous numbers of elementary particles whose energies, complicated interactions and chaotic thermal motion completely masks the discrete nature of the microscopic world.

Superconductivity offers a unique opportunity to observe the quantization of a physical quantity in a macroscopic, readily observable system. The key quantum property to be studied in this experiment is the *quantization of the*

magnetic flux (field times area), Φ , penetrating a hole in a superconducting ring (or penetrating any superconducting closed circuit). Recall that flux is defined by the integral:

$$\Phi = \oiint \vec{B} \cdot d\vec{a} \quad \text{Eq.1}$$

over some loop enclosing an area a . The vector $d\vec{a}$ points everywhere normal to the loop's surface. The unit of quantization (or **flux quantum**) is $h/2e = 2.07 \times 10^{-15}$ Wb or weber, where h is Planck's constant. (Note that the SI unit of flux, is 1 weber = 1 Wb = 1 Tesla-m².) If the flux through the hole changes (e.g. as a result of a change in a small magnetic field in the hole), circulating currents will arise in the ring that will precisely cancel the change. These circulating currents can be detected using a phenomenon called the **Josephson effect**, and hence, the small magnetic field can be measured; this is the basis of SQUID operation. The sensitivity to changes in flux is about 0.001 of one flux quantum, good enough to measure magnetic fields generated outside your skull by the process of thinking (e.g. thinking about this write-up). For example, the magnetic field of the earth passing through the area of a typical SQUID sensor corresponds to over 100 flux quanta.

Superconducting Rings

We will now explore why superconducting rings exhibit this flux quantization. Consider the following experiment. We cool a ring of superconductor in a small magnetic field that corresponds to one flux quantum threading the ring. We now have a superconducting ring threaded by a single flux quantum. Suppose we now turn off the applied field. According to Faraday's Law of Induction, the moment that we change the field lines that thread the ring, a current flows in the ring. The current induced tries to oppose the change in magnetic field by generating a field to replace the field we removed. In an ordinary material, that current would rapidly decay away. In the superconductor, something entirely different happens. If the induced current decreased just a little bit in the ring, then the flux threading the ring would be a little less than a flux quantum. This is not allowed. The next allowable value of flux would be zero flux. Therefore, the current would have to abruptly cease rather than decay away. Because the superconducting state is composed of an enormous number of electrons that are paired up and occupying the same quantum state, a current reduction of the sort needed would require all the electrons to jump into another state simultaneously. This is an extraordinarily unlikely event. Practically speaking, it will never happen. As a result, the current induced in a superconducting ring will flow indefinitely, and it prevents the magnetic flux through the loop from changing from its initial value. People have actually tried this experiment for years on end. As long as the ring is kept cold, the current flows without resistance.

But why is the flux quantized rather than simply constant at its initial value? The answer lies in the long-range coherence of the superconducting wavefunction. As we said before, the value of the wavefunction in one place in a superconductor is related to the value at any other place by a simple phase change. The case of a superconducting ring places special restrictions on the superconducting wavefunction. The wavefunction at the point marked by the black dot on the ring in Fig. 1 must have the same value as the wavefunction obtained by traveling around the ring

one full circuit – it is the same spot. The phase change for this trip must be 2π in order for the wavefunction to have a single value at a given point in space. In ordinary wave language, if the wave was at a crest at the starting point, it must be at a crest 360° (2π) around the circle.

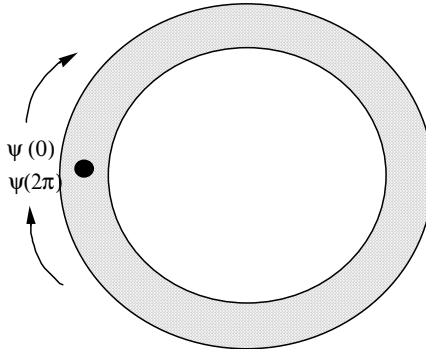


Figure 1: A superconducting ring showing the single-valued condition on the wavefunction's phase.

This condition of “single-valuedness of the wavefunction” constrains the flux to quantized values or **fluxons**. According to electromagnetic theory, applying a magnetic field to a superconductor induces a change in the phase of the wavefunction. In quantum mechanics, you learned that a free electron wavefunction can be written as $\Psi(x) = A e^{i p x / \hbar}$, where p is the linear momentum along the x -direction. Now, advanced electricity and magnetism and quantum mechanics explain that you really should replace the linear momentum, p , in this relationship with what is called the **canonical momentum**, $p + qA$, where p is the mechanical momentum, q is charge ($-2e$ for Cooper pairs), and A is the **magnetic vector potential**. (The magnetic vector potential can be used to generate the magnetic field using the equation $\mathbf{B} = \nabla \times \mathbf{A}$.) This equation reveals why a given amount of magnetic field creates a specific phase change in the wavefunction, since now the phase depends upon the magnetic field through A :

$$\Psi(x) = e^{i k x} = e^{i p x / \hbar} = e^{i p_{\text{canonical}} x / \hbar} \quad \text{Eq. 2}$$

This means that there is a term in the phase equal to not just (px/\hbar) but $(px/\hbar - 2eAx/\hbar)$. Now we again apply the single-valuedness condition for the phase to this new expression. When the particle goes around a loop, the second phase term becomes a line integral of $2eA/\hbar$ around the loop, but that can be transformed into an area integral of the curl of A (same as the magnetic field) over the area enclosed by the loop, a quantity equal to the magnetic flux. This means that the change in phase, $\Delta\phi$, as a particle goes around a loop as in Fig. 1 is :

$$\begin{aligned}
\Delta\phi &= \frac{-2e}{\hbar} \oint \vec{A} \cdot d\vec{s} \\
&= \frac{-2e}{\hbar} \oiint \nabla \times \vec{A} \cdot d\vec{a} \\
&= \frac{-2e}{\hbar} \oiint \vec{B} \cdot d\vec{a} \\
&= \frac{-2e}{\hbar} \Phi
\end{aligned}
\tag{Eq. 3}$$

So, if we consider a loop within a superconductor, or any other quantum mechanical system containing magnetic flux, the phase includes a term proportional to the magnetic flux. (This is called the **Aharonov-Bohm effect** and has been measured in many different experiments.) Since the phase change going completely around the ring must be some multiple of 2π in order to maintain the single-valuedness of the wavefunction, the amount of flux contained within in the ring can only assume certain discrete values: $\Delta\phi = 2e\Phi/\hbar = 2\pi$. From this relationship, we derive the flux quantum as: $\Phi_0 = \hbar 2\pi/2e = \hbar/2e$. This quantum mechanical property is the origin of flux quantization.

The critical current I_c and critical magnetic field, B_c

Superconductors can only remain superconducting if their currents remain below a critical current, and if they are surrounded by magnetic fields less than a critical magnetic field magnitude. The origin of both effects is related to the Meissner effect—that is, the exclusion of magnetic fields from inside the superconductor. Now, recall from electricity and magnetism that the energy due to the presence of a magnetic field of magnitude B is:

$$U = V B^2/8\pi \tag{Eq. 4}$$

where V = volume of the material. Now, to *exclude* the magnetic field a region of space *costs* a corresponding amount of energy. The magnetic field is excluded from the material when it is converted from a normal metal to a superconductor. This energy *cost* is offset by an overall *lowering* of the free energy of the system, which changes from the normal state free energy G_n to that of the superconducting state with free energy G_s . (Why free energy rather than just plain energy? There is an entropic change in going from one phase to another as well as an energy difference. The free energy takes into account both changes.) In the absence of a magnetic field and at sufficiently low temperatures, the material *lowers* its internal energy by going superconducting: in other words $G_s < G_n$. The balance between the energy cost of excluding magnetic field ($V B^2/8\pi > 0$) and energy lowering by going from normal to superconducting ($G_s - G_n < 0$) is expressed as:

$$\Delta U = (G_s - G_n) + V B^2/8\pi \tag{Eq. 5}$$

Now, for $B = 0$, clearly the superconducting state is advantageous energetically. As either T (temperature) or B increases, the energy difference, ΔU reaches zero, then becomes positive and the superconducting state is no longer

favorable. The material becomes a normal conductor again. The threshold value of B at which this occurs is the critical field, B_c , and it is defined by:

$$\Delta U = (G_s - G_n) + V B_c^2 / 8\pi = 0 \quad \text{Eq. 6}$$

So, B_c is the highest magnetic field at which the superconducting state is energetically favorable. Higher field values cause the superconductor to “go normal”.

Now we can understand why there is also a critical current, I_c . Associated with a current in any wire is a magnetic field. For a simple cylindrical wire of radius r , the highest current is at the surface of the wire, and its value is:

$$B = 2 I / (c r) \quad \text{Eq. 7}$$

Here c is the speed of light. However, this magnetic field cannot exceed B_c , so we also get a limit on the maximum current that the superconductor can support before it goes normal:

$$I_c = B_c c r / 2 \quad \text{Eq. 8}$$

However, if only the surface layer goes normal, then the current is just pinched down to smaller radius within the wire, giving an even higher magnetic field. By this reasoning, you can see that the entire wire must go normal, not just the region at the surface. The critical current limits current throughout the superconductor as a result.

Josephson Junctions

The Josephson effect is yet another manifestation of what we call the long-range quantum coherence of superconductors. It also takes advantage of the critical current idea introduced above. The simplest picture of this effect is shown in Fig. 2.

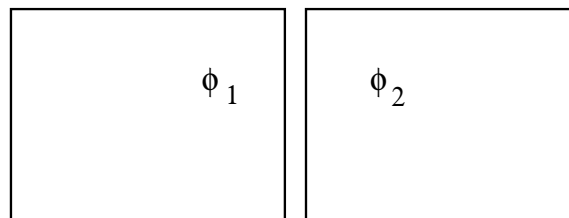


Figure 2 Schematic diagram of two superconducting regions separated by a thin gap, in which the phase, ϕ , differs between the two regions.

Two regions of superconductor are placed very close to one another, but separated by a thin region of a normal conductor. The quantum mechanical phase on the left is ϕ_1 and the phase on the right is ϕ_2 . In an ordinary material, the phases at two different spots are unrelated. In a single piece of superconductor, the phases at two different places have a specific relationship to one another. This arrangement assures the superconducting lower energy ground state. In the picture above, what will the phases do? The surprising answer is that if the two regions of

superconductor are close enough together, their phases will also be related, even though the intervening material is not superconducting. In other words, they will act like a single superconductor. (The origin of this effect isn't really so mysterious. The superconducting wavefunction always extends a very short distance into a surrounding normal metal, in a fashion similar to single particle wavefunctions undergoing exponential decay in a classically forbidden region. The *Josephson junction* (or weak link) is just formed by placing two such regions so close together that the wavefunctions haven't decayed to zero on either side before they meet in the center of the junction.) Functionally speaking, electrical currents can flow between the two regions with zero resistance. Such currents are called *Josephson currents* and physical systems composed of two regions of superconductor that exhibit this property are called Josephson junctions.

How are such structures formed? You might not know it, but most metals are actually crystals. That is, they are composed of atoms arranged in a regular lattice, as shown Fig. 3(a). Most metal wires are polycrystalline—they are made of multiple, randomly oriented tiny crystals called crystallites or domains. Within each domain, the atoms are ordered regularly. Domains share walls between each region of local crystalline order. One way these walls can arrange themselves is so as to form part of each domain on either side. The resulting wall is called a **grain boundary**, as shown in Fig. 3(b). Electrical resistance can be thought of as due to scattering of electrons from vibrating atomic nuclei in the crystalline lattice. The extra disorder induced by a grain boundary introduces more scattering and hence greater electrical resistance. In addition, the disorder caused by grain boundaries can destabilize superconductivity at these points.

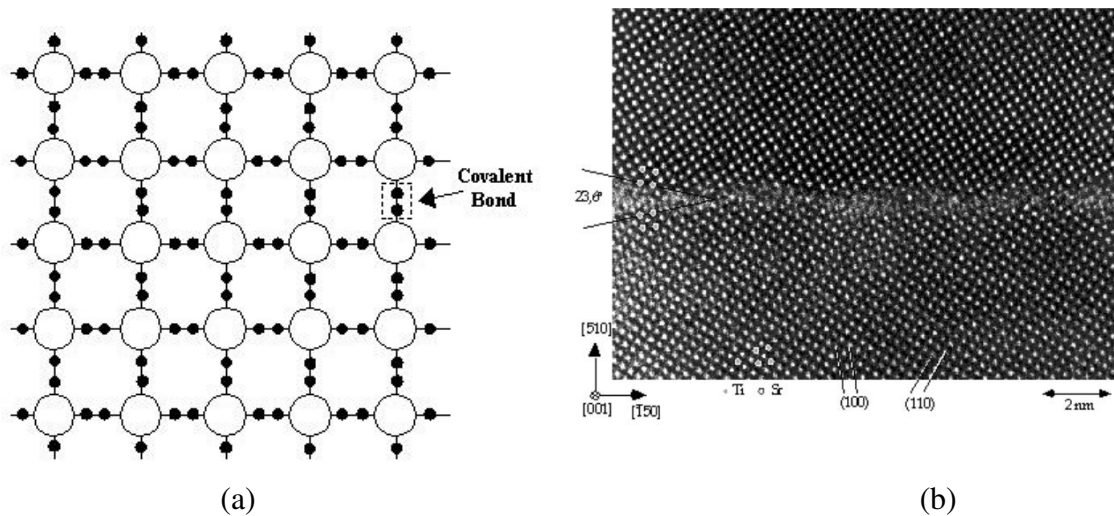


Figure 3: (a) Schematic diagram of a crystal lattice. (b) Electron micrograph of strontium-titanate showing the two mismatched crystal lattices (top and bottom) meeting at a grain boundary (center line) Image sources: (a) <http://www.localhistory.scit.wlv.ac.uk/Museum/Engineering/Electronics/history/Transistordetails.htm> (b) <http://pruffle.mit.edu/~ccarter/NANOAM/images/>

Any weak electrical coupling between two regions of superconductor – such as tiny constrictions, microscopic point contacts, weakly conducting layers, or certain crystallographic grain boundaries – exhibit the Josephson effect. This can be exploited to make Josephson junctions. The Josephson junctions in Mr. SQUID[®] are grain boundary weak-

link junctions, as shown in Fig. 4. As a result, they have a lower critical current than the rest of the superconducting ring. Josephson junctions are the essential active devices of superconductive electronics, much as the transistor is the essential active device of semiconductor electronics. Junctions can be used in a variety of electronic circuits as switching devices, as sensors, as variable inductors, as oscillators (because of the ac Josephson effect), and for other applications. People have built Josephson electronic circuits that contain up to tens of thousands of junctions. At the opposite extreme, one of the most useful circuits made from Josephson junctions is the dc SQUID, which contains only two junctions.

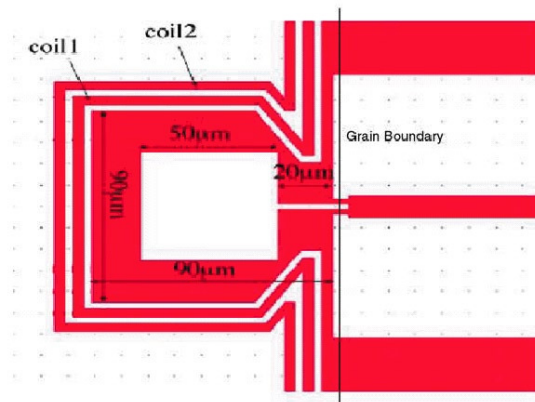


Figure 4: Schematic diagram of Mr. SQUID™ showing the two grain boundaries (black lines) that constitute the Josephson junctions. The superconducting loop is the center circuit, while two outer coils (coil 1 and coil 2) are used for applying magnetic fields to the device. (Star Cryotronics)

The dc SQUID

Now we will describe how SQUIDs work. A dc SQUID is a superconducting loop with two Josephson junctions in it. (Fig. 4) Suppose we pass a constant current, known as a bias current, through the SQUID. If the SQUID is symmetrical and the junctions are identical, the bias current will split equally, half on each side. A dc SQUID is generally represented schematically as shown in Fig. 5.

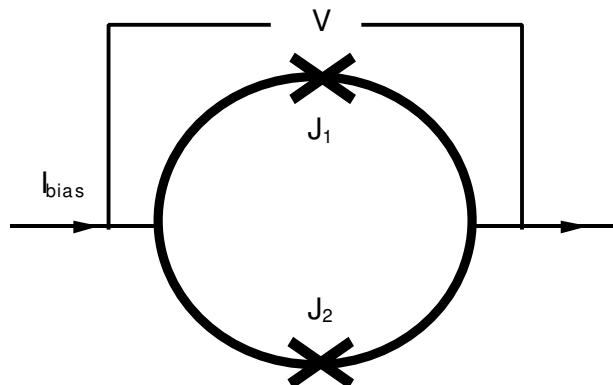


Figure 5: A schematic representation of the dc SQUID. (Star Cryotronics)

A supercurrent (superconducting current) will flow through the SQUID, as long as the total current flowing through it does not exceed the critical current of the Josephson junctions. Since the critical current is the maximum current the SQUID can carry and remain superconducting, it is also the current at which a voltage across the SQUID develops. You could measure the critical current of a SQUID by increasing the bias current up slowly from zero until a voltage appears, then reading the current with an ammeter. The value of current determined in this way is the critical current of the SQUID.

When the two junctions in the SQUID are identical, the loop is symmetrical, and the applied field is zero, both junctions will develop a voltage at the same time. So the critical current of the SQUID is simply twice the critical current of one of its junctions. If the critical current of each junction is 5 microamps, for example, then the critical current of the SQUID is 10 microamps. The voltage-current characteristic, or V - I curve, of a SQUID, looks very much like the V - I curve of a bulk superconductor, except the value of the critical current is smaller. A typical V - I characteristic for Mr. SQUID[®] is shown in Fig. 6, where bias current is along the x-axis and voltage is along the y-axis.

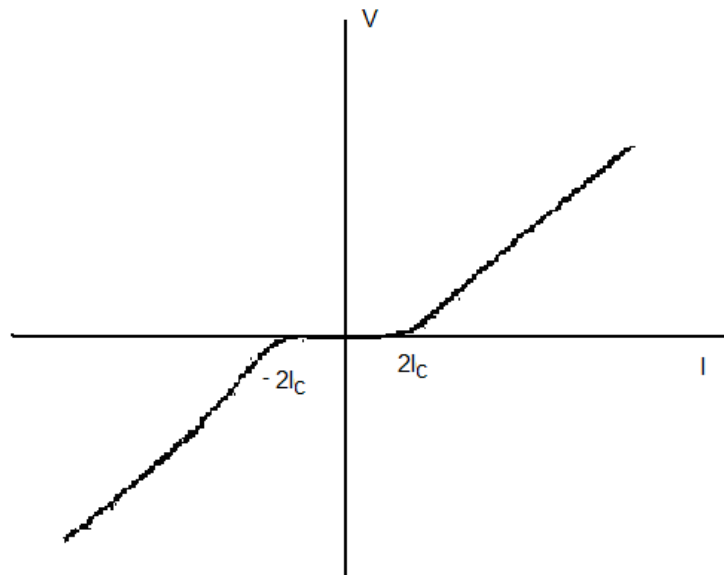


Figure 6: A typical Mr. SQUID[®] V - I characteristic. (Star Cryotronics)

Now imagine what happens if a magnetic field is applied to the SQUID. First, let's bias the SQUID with a current well below its critical current. Then, if we apply a tiny magnetic field to the SQUID, the magnetic field wants to change the superconducting wave function. But the superconducting wavefunction doesn't want to change — as discussed earlier, it must maintain an integral number of wavefunction cycles around the loop. So the superconducting loop does what you would expect: it opposes the applied magnetic field by generating a screening

current I_s , that flows around the loop (Fig. 7(a)). The screening current creates a magnetic field equal but opposite to the applied field, effectively canceling out the net flux in the ring.

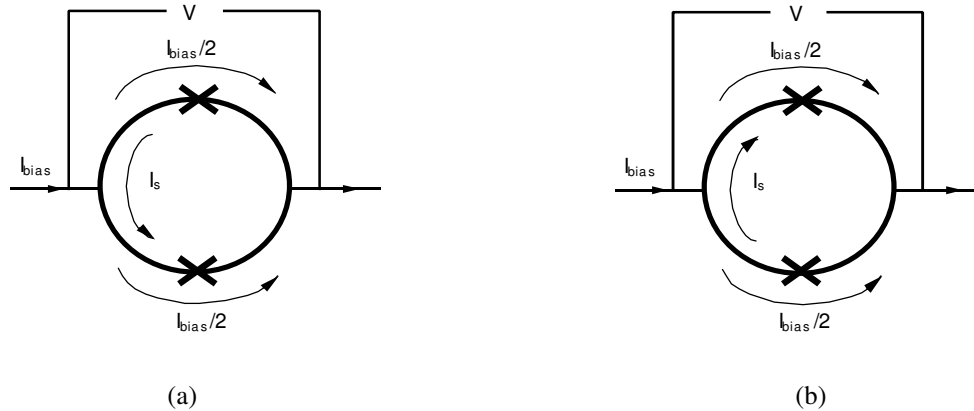


Figure 7: (a) A dc SQUID in the presence of an applied magnetic field. (b) The screening current I_s has reversed its direction after one additional flux quantum enters the loop.

The applied magnetic field has lowered the critical current of the SQUID — in other words, it has reduced the amount of bias current we can pass through the SQUID without generating a voltage. The reason is that the screening current superimposes itself on top of the bias current, but we still must have $I_{bias} + I_{screening} < I_c$ for the junction to stay superconducting. Suppose the critical current of each junction is 5 microamps and the screening current is 1 microamp. Since the bottom junction has to carry 1 microamp of screening current, it can now carry only 4 microamps of bias current before it becomes resistive. When it carries a total of 5 microamps, it becomes normal and resistive. When the junction on the bottom goes normal, all the current goes through the junction on the top, which makes it go normal. That means both paths are now resistive, so a voltmeter will register a voltage across the SQUID.

As we increase the applied magnetic flux, the screening current increases. But when the applied magnetic flux reaches half a flux quantum, something interesting happens. Just the junctions momentarily go normal. The continuity of the superconducting loop is destroyed long enough for one quantum of magnetic flux to enter the loop. Then superconductivity around the loop is restored. Consider what happens to the screening current to see why this is also energetically favorable. Rather than generating enough screening current to keep 0.51 flux quanta out, now all the SQUID has to do is generate enough screening current to keep 0.49 flux quanta in, which is, of course, a little easier (that is, lower in energy.) Of course, the screening current has to change direction, as shown in Fig. 7(b).

Thus, the junctions serve as gates that allow magnetic flux to enter (or leave) the loop. The voltage read with an oscilloscope is the average voltage across the SQUID. Although the experimenter observes a non-zero dc voltage (hence the SQUID appears “resistive” just above I_c as shown in Figure and the left side of) the instantaneous voltage across the SQUID and the circulating current are actually oscillating at high frequencies in the microwave range in response to an applied magnetic field.ⁱ

Now we will see why the critical current (and hence the voltage measured across the SQUID) is periodic in magnetic flux. In the discussion above, we have assumed that an external magnetic field B_{ext} is generating the magnetic flux in this discussion. (This could be caused by currents flowing within coil 1 or coil 2 in your Mr. SQUID™, as shown in Fig. 4.) There is also a magnetic field generated by the total current, I , flowing in the SQUID. The total magnetic field B_{total} inside the superconductor is thus:

$$B_{total} = B_{ext} + 2I/(cr) \quad \text{Eq. 9}$$

However, this means that it is the *total* magnetic field, B_{total} , that has to be below the critical field for superconductivity to exist:

$$B_{total} = B_{ext} + 2I/(cr) \leq B_c \quad \text{Eq. 10}$$

This also means that when the total magnetic field equals the critical magnetic field, the resulting critical current is lower than before. As a matter of fact, it's equal to:

$$I_c = (rc/2)(B_c - B_{ext}) \quad \text{Eq. 11}$$

This is why the V - I curve below exhibits a lower I_c values for greater magnetic fluxes—the critical current decreases as B_{ext} increases. (Fig. 8) The critical current (and hence the V - I curve) are periodic because the entire cycle repeats every time one more flux quantum enters the SQUID ring.

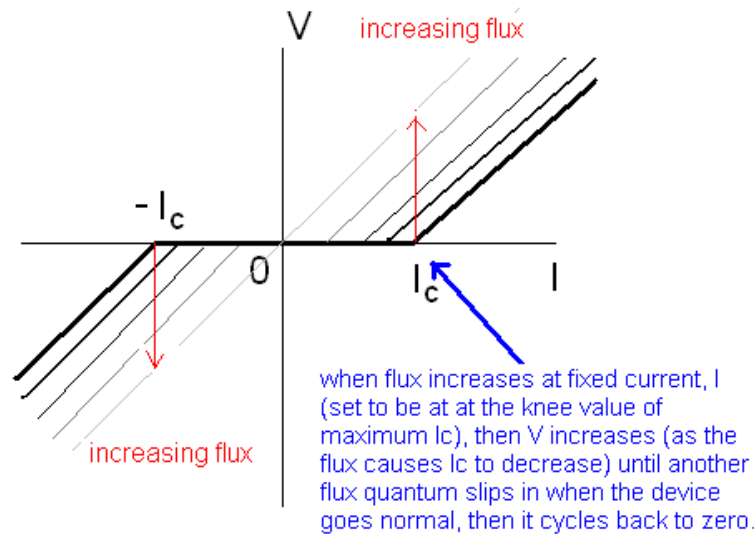


Figure 8: Schematic depiction of how the V - I curve of the SQUID varies as the applied magnetic flux increases. Different V - I curves correspond to different values of flux. The greatest value of critical current occurs when the applied flux equals an integral number of flux quanta and no screening current is required

Fig. 8 also illustrates why the SQUID's voltage is periodic in the applied magnetic flux. The critical current goes through a maximum when the applied magnetic flux is an integer multiple of the flux quantum, because that's when the screening current is zero. It goes through minima when the applied magnetic flux is an integer of the flux

quantum plus one half, because that's when the screening current is largest. For fixed bias current through the device, the corresponding SQUID voltage depends upon flux. Because the critical current of the SQUID is periodic in the way you just saw, the V - I curve of a SQUID oscillates periodically between two extremes as shown in Fig. 9

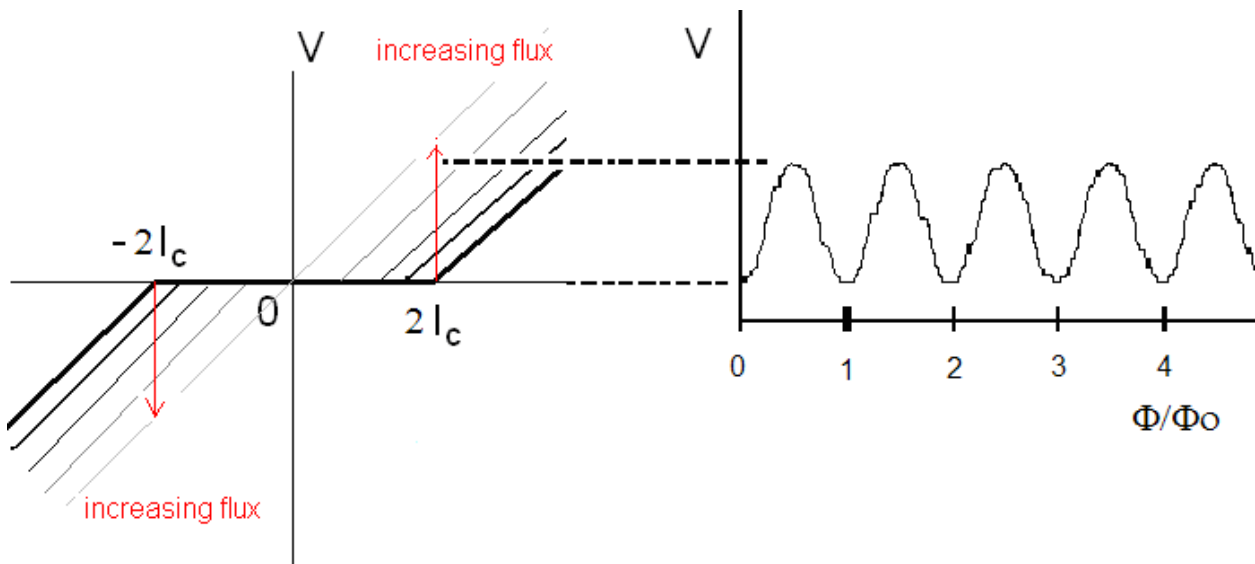


Figure 9: Periodic dependence of the SQUID voltage on applied flux. Note that the slope of each V - I curve is R_N in the normal region. As a result, the maximum value of V (twice the amplitude of the V vs. Φ curve) vs. Φ is equal to $I_c R_N$.

To make a magnetic field detector, we operate the SQUID with a constant bias current slightly greater than the critical current, so the SQUID junctions are always resistive. Under these conditions, there is a periodic relationship between the voltage across the SQUID and the applied magnetic flux, with a period of one flux quantum (Fig. 9). This flux vs. voltage curve will appear on your oscilloscope during your experiments with Mr. SQUID[®], and it serves as the basis for magnetic flux measurements using SQUID's. While it would be possible to obtain quite a sensitive measure of a magnetic signal simply by counting flux quanta, practical SQUID systems involve control electronics that interpolates between whole numbers of flux quanta and greatly enhances their ultimate sensitivity. SQUID sensitivity is finally limited by the intrinsic noise in the device, which in 4.2 K niobium dc SQUIDs, for example, typically approaches a millionth of a flux quantum (corresponding to a few billionths of the earth's field passing through a 100- μm diameter SQUID).

The inherent periodicity of the SQUID implies that it cannot distinguish between zero applied field and any other field that generates an integral number of flux quanta. This allows the dynamic range of the SQUID to be extended almost indefinitely by re-zeroing the SQUID in a controlled way. It also means that in order to measure the absolute value of an applied field, it is necessary to reset the SQUID in a known field, or to rotate the SQUID with respect to the field. Nevertheless, it is very often the case that only changes in field are of interest; in which case no special measures are necessary.

Although the model of SQUIDs we just discussed is not rigorous, it is a good approximation. This model doesn't really connect quantum interference directly to the SQUID's operation, though. A more complete explanation would describe how the curve showing the critical current of the SQUID varies with applied flux is an interference pattern, analogous to an optical interference pattern. If you pass a current through a dc SQUID, you see maxima and minima of critical current as you raise or lower the applied flux, because the macroscopic quantum wave functions at the two junctions interfere with each other.

Experiment A. Magnetic susceptibility vs. Temperature of the YBCO SQUID (the Meissner Effect)

We own superconducting samples in which the high-T superconductor has been formed into a pellet embedded in a plastic casing. The compound used in your sample is called YBCO for yttrium-barium-copper-oxide. Wrapped around the superconductor is a wire coil, or inductor; a thermocouple has also been included in the package to monitor temperature. Now, if the entire inside of the inductor were filled with superconductor, then no magnetic field could pass through that region due to the Meissner effect; in other words, it would have zero inductance. Your wire coil actually has a diameter of 1.36 cm, while the superconductor pellet inside it has diameter 1.26cm, so it only partially fills the inductor coil. That means the coil's inductance, L , is actually given by:

$$L \cong L_o (1 + f \chi) \quad \text{Eq.12}$$

Where f = the fraction of the coil volume occupied by the sample, L_o is the coil inductance in vacuum, and χ is the called the magnetic susceptibility of the superconductor. As a result, if you measure the coil's inductance, you can measure the magnetic properties of the superconductor inside it! A sudden drop in the inductance at the superconducting transition is a direct measure of the Meissner effect. (You can also watch the effect of flux exclusion by levitating a small high field magnet above a block of superconductor. If you have never seen this done, ask your instructor for help in doing this.)

Here is the circuit we will use to measure coil inductance:

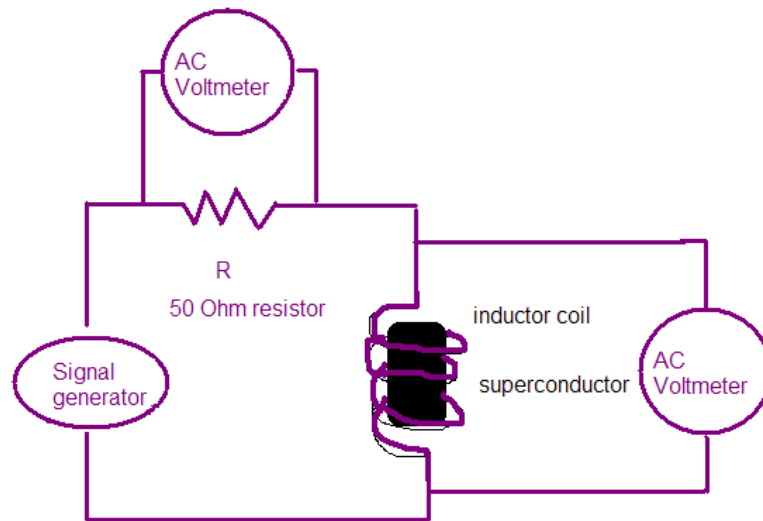


Figure 10: Schematic of the induction measurement.

Conversion from mV to Kelvin

K	0	1	2	3	4	5	6	7	8	9	10	K
60	7.60	7.53	7.46	7.40	7.33	7.26	7.19	7.12	7.05	6.99	6.92	60
70	6.92	6.85	6.78	6.71	6.64	6.56	6.49	6.42	6.37	6.33	6.29	70
80	6.29	6.25	6.21	6.17	6.13	6.09	6.05	6.01	5.97	5.93	5.90	80
90	5.90	5.86	5.83	5.79	5.75	5.72	5.68	5.64	5.60	5.56	5.52	90
100	5.52	5.48	5.44	5.41	5.37	5.34	5.30	5.27	5.23	5.20	5.16	100
110	5.16	5.13	5.09	5.06	5.02	4.99	4.95	4.91	4.88	4.84	4.81	110
120	4.81	4.77	4.74	4.70	4.67	4.63	4.60	4.56	4.53	4.49	4.46	120
130	4.46	4.42	4.39	4.35	4.32	4.28	4.25	4.21	4.18	4.14	4.11	130
140	4.11	4.07	4.04	4.00	3.97	3.93	3.90	3.86	3.83	3.79	3.76	140
150	3.76	3.73	3.69	3.66	3.63	3.60	3.56	3.53	3.50	3.47	3.43	150
160	3.43	3.40	3.37	3.34	3.30	3.27	3.24	3.21	3.18	3.15	3.12	160
170	3.12	3.09	3.06	3.03	3.00	2.97	2.94	2.91	2.88	2.85	2.82	170
180	2.82	2.79	2.76	2.73	2.70	2.67	2.64	2.61	2.58	2.53	2.52	180
190	2.52	2.49	2.46	2.43	2.40	2.37	2.34	2.31	2.29	2.26	2.23	190
200	2.23	2.20	2.17	2.14	2.11	2.08	2.05	2.02	1.99	1.96	1.93	200
210	1.93	1.90	1.87	1.84	1.81	1.78	1.75	1.72	1.69	1.66	1.64	210
220	1.64	1.61	1.59	1.56	1.54	1.51	1.49	1.46	1.44	1.41	1.39	220
230	1.39	1.36	1.34	1.31	1.29	1.26	1.24	1.21	1.19	1.16	1.14	230
240	1.14	1.11	1.09	1.07	1.04	1.02	0.99	0.97	0.94	0.92	0.89	240
250	0.89	0.87	0.84	0.82	0.79	0.77	0.74	0.72	0.69	0.67	0.65	250
260	0.65	0.62	0.60	0.58	0.55	0.53	0.50	0.48	0.45	0.42	0.40	260
270	0.40	0.38	0.36	0.34	0.32	0.30	0.28	0.26	0.24	0.22	0.20	270
280	0.20	0.18	0.16	0.14	0.12	0.10	0.08	0.06	0.04	0.02	0.00	280
290	0.00	-0.02	-0.04	-0.06	-0.08	-0.10	-0.12	-0.14	-0.16	-0.18	-0.20	290
300	-0.20	-0.22	-0.24	-0.26	-0.28	-0.30	-0.32	-0.34	-0.36	-0.38	-0.40	300

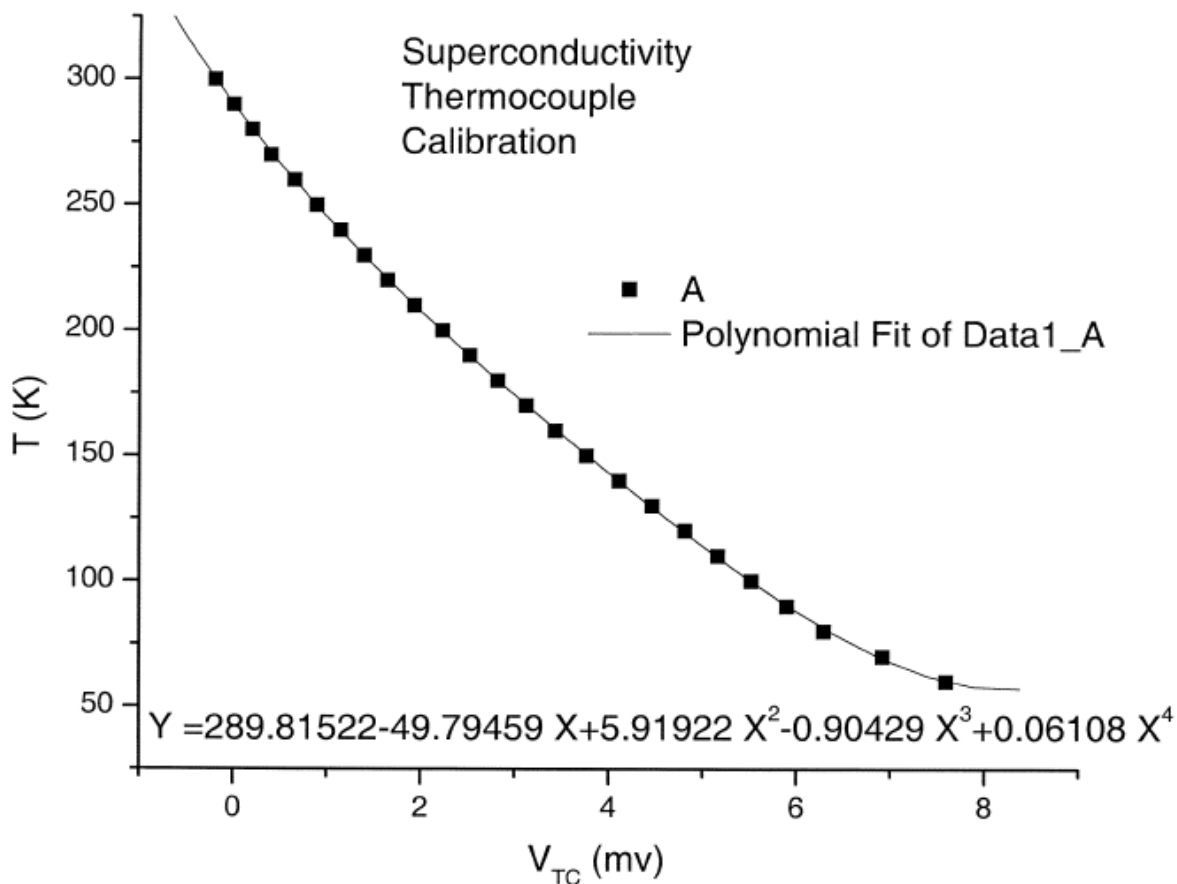
Table 1: Conversion from thermocouple voltage to Kelvin.

The AC voltage will be a $f = 1$ kHz sine wave taken from the low voltage output of a function generator. From intro physics you learned that:

$$\omega L = \sqrt{\left(\frac{V_L}{I}\right)^2 - R_L^2} \quad \text{Eq.13}$$

Where I is current through the inductor (and resistor), R_L is inductor resistance, L is inductance, and $\omega = 2\pi f$.

STEP 1: Your thermocouple measured temperature by generating a DC voltage (in the millivolts range) that is depending upon temperature. Connect up a DMM so it again reads DC millivolts from your thermocouple (red and blue wires). Make sure you are not on AC mode. You can convert between voltage and temperature using either Table 1 or by using the following calibration curve on the calibration curve (determined by Lindsay Subers and Amanda Dillon in 2005):



where V is in milliVolts. (Note from the table above that the sign of the thermocouple voltage changes close to room temperature, so do not be surprised if your thermocouple voltage goes negative—just change the polarity of the leads.)

STEP 2: Connect up a second DMM so it reads AC volts across the 50 Ohm power resistor. You will use this to measure current through the entire circuit, $I = V/50$ Ohms. (Measure your power resistor's actual resistance first.)

STEP 3: Make a quick reading of your inductor's resistance, R_L , at room temperature to use later; do this using a 4-point Ohm measurement. (See Appendix A.) Ideally you would measure this as a function of temperature, but

that's not feasible here. Instead, you should correct for temperature dependence using the temperature coefficient of copper, which is 0.393 % per degree C. In other words, if the temperature increases by 1 degrees C, the resistance will increase by 0.393%.

STEP 4: Connect up a DMM so it reads AC volts across your inductor. This is V_L .

STEP 5: Carefully place your sample in the "sand cryostat" (foam container filled with sand) and pour sand around it until the sample is at the center of the container. Get liquid nitrogen in a transfer Dewar, being careful to wear eye goggles and gloves to shield your hands from spills or Dewar glass breakage. Your instructor can explain where the nearest liquid nitrogen supply is located. Cool the sample down to 77 K by pouring liquid nitrogen in a little at a time and allowing it to boil off while cooling down the sand a little at a time. You will need to keep adding liquid nitrogen as it boils off; you should try to keep the top of the sand just covered, while avoiding vigorous boiling that spills the sand over the table. Watch the thermocouple voltage to see when it stabilizes at the temperature of liquid nitrogen (77 K). Record a low temperature inductor resistance reading once it has approximately stabilized. Next, allow the sample to warm up slowly while recording temperature, current (resistor voltage) and inductor voltage. Do this by simply stopping adding the liquid nitrogen and allowing the sand cryostat + sample to come back to room temperature. (This will occur more rapidly if you do not have a large puddle of liquid nitrogen on top of the sand.)

STEP 6: Make a clearly marked table to keep track of all these different voltages! Be especially careful to record data near the transition! Remember to use the inductor resistance that varies with temperature, interpolating between your room temperature and liquid nitrogen temperature readings. You will see a dramatic increase in L (and hence inductor voltage) at the transition. Take data for a few degrees above the transition, but you do not have to take data all the way up to room temperature. Plot up your data using a computer so as to display inductance L vs. T [K]. Indicate and comment on the important features on your plot.

In your lab report, be sure to explain how the Meissner effect and the magnetic properties of superconductors can explain your sample's inductance vs. Temperature curve. What would happen to your curve if the superconductor entirely filled the coil?

Experiment B: Interpreting Mr. SQUID[®]'s V-I curve

The Mr. SQUID[®] chip contains a dc SQUID made with thin film high temperature YBCO superconductor. The dc SQUID is a simple circuit that can be schematically represented by our earlier Josephson junction cartoon. (Fig. 11).

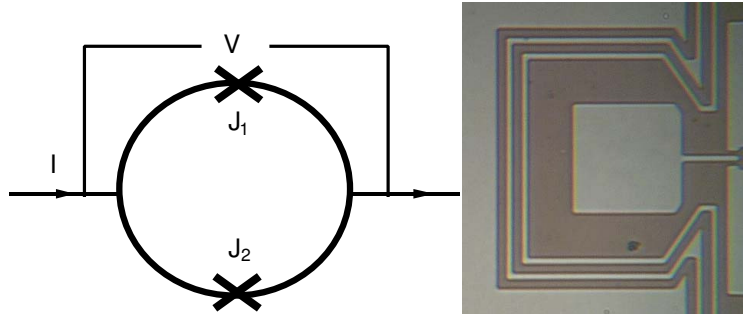


Figure 11: Schematic representation of the dc SQUID in Mr. SQUID and actual Mr. SQUID.

The Mr. SQUID™ dc SQUID is simply a ring of superconductor containing two Josephson junctions, which are marked by the X's labeled J_1 and J_2 in the circuit diagram above. The actual device is mounted on a post that allows you to insert it into a bath of liquid nitrogen to cool it to superconducting temperatures. The SQUID is also surrounded by a cylindrical mu-metal shield that partially screens it from outside magnetic fields. The small fraction of the earth's field that is not attenuated by the magnetic shield on the Mr. SQUID® probe is sufficient to shift the $V-\Phi$ curve by several flux quanta!

The simplest experiment to perform with this circuit is to pass current I from left to right across the ring and to measure the voltage V that appears across the ring. If the two junctions in the SQUID are identical, in the absence of any magnetic field, the current will divide evenly and half of it will pass through each junction before recombining at the right side. The Mr. SQUID® control box allows you to perform this experiment quite easily. The instructions in the next section explain how to set up your device so you are ready to try this voltage-current experiment.

Setting Up the Output Device & measuring V-I for an ordinary resistor

We assume that you are familiar with the operation of your oscilloscope. You will be using your oscilloscope in x-y mode. For initial settings, try an x sensitivity of 0.2 volts per division (or whatever value is closest to this on your instrument) and a y sensitivity of around 50 millivolts per division. The Autoscale feature should work once you have a signal. **BE SURE YOU HAVE YOUR OSCILLOSCOPE CHANNELS BOTH SET TO DC COUPLING! Otherwise, your V-I curves will not properly retrace, an effect known as hysteresis.** You may want to adjust these values as needed later. Now refer to the diagram of the Mr. SQUID® control box (Fig. 12).

STEP 1: Since you will be using an oscilloscope in the X-Y mode, set the oscillator frequency toggle switch (7) to the OSC (high-speed) position. Once again check that both 'scope channels are set for DC coupling (NOT AC coupling).

STEP 2: Before turning on the power, make sure the amplitude control (4) is at its minimum (fully counter-clockwise) position and set the flux bias control (2) and the current bias control (3) to their 12-o'clock (straight up) positions.

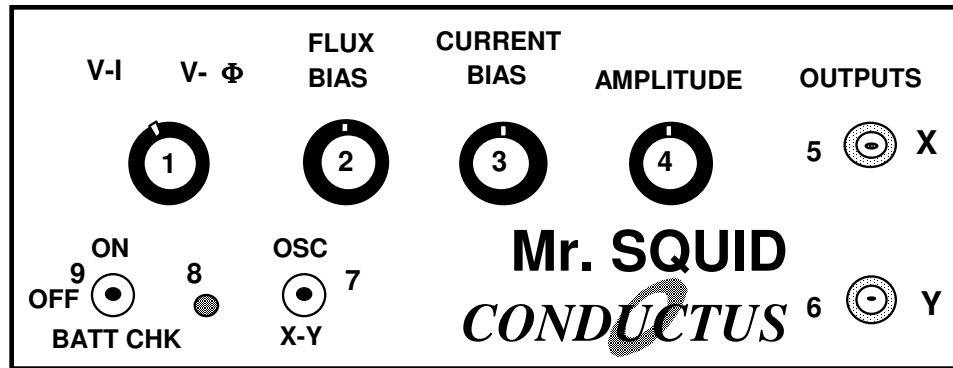


Figure 12: Front panel of the Mr. SQUID® electronics box.

STEP 3: Make sure the function switch (1) is in the V-I position. In this position, the BNC cable connected from the X output on the front panel (5) to the oscilloscope “X” input reads the current being fed through the SQUID. The cable connected from the Y output (6) to the oscilloscope “Y” input reads the voltage across the SQUID. Set the power switch (9) to the ON position. On the oscilloscope, there should be a small bright spot on the center of the screen. You may have to adjust the offset controls to achieve this.

STEP 4: For starters, rather than looking at the complicated SQUID outputs, we will just connect a simple resistor to your circuit and study its V-I behavior. Find the DB-9 connect with the resistor soldered to it, and measure the resistance using a DMM. Connect this to the end of your Mr. SQUID™ box’s cable in place of Mr. SQUID™ before proceeding.

Understanding the current and amplitude controls in VI mode

First, we will understand what is necessary to measure a V-I curve. The current bias control (3) varies the current being sent through the SQUID. Slowly turn this knob in either direction. The spot on the oscilloscope screen will move in response to the changing current. As you turn the knob back and forth, you will trace out a V-I curve representing the relationship between the current fed through the attached device and the voltage. This curve is called the *V-I* curve. Since you have attached a simple resistor for now, you will see this as a straight line. You ideally would like to automatically collect a V-I curve without sweeping the current bias knob by hand. To do so, return the current bias control to the 12 o'clock position and now slowly turn the amplitude control (4) in the clockwise direction. This function sweeps the current high and low between preset values, thus automating the procedure you performed by hand using the current bias control. A solid curve should now appear on the oscilloscope screen. (see Fig. 13) This is basically the V-I curve for Ohm’s law for a resistor; the line’s slope is proportional to resistance.

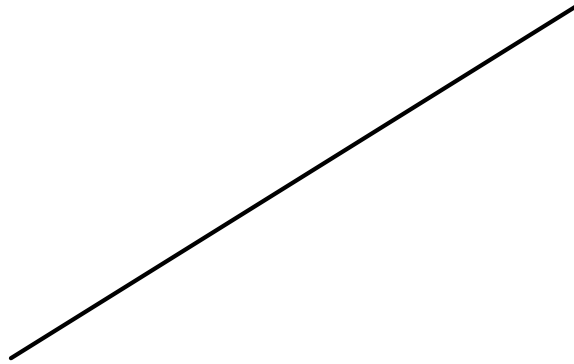


Figure 13: A linear (Ohmic) V - I characteristic.

If you now turn the current bias control, the center point of the curve being traced on the screen will move. The current bias control sets a single value of current being passed through the SQUID and the amplitude control sweeps the current back and forth about that set value.

Interpreting your V - I curve

Your output device acts like a voltmeter. The sensitivity settings on it determine how much voltage corresponds to a division on the screen. The current output on the Mr. SQUID[®] box (5) actually represents the voltage across a 10,000 Ω resistor in the electronics box. According to Ohm's Law ($I = V/R$), the current flowing through the resistor is therefore equal to the voltage across it divided by 10,000 Ω .

The typical voltage levels from the SQUID are small enough that the Mr. SQUID[®] control box amplifies the small signals. Thus, to calculate the actual voltages, the measured value on the oscilloscope should also be divided by 10,000. You can determine resistance by measuring the slope of the V - I curve. To obtain a resistance, you must convert the x-axis value into amperes from volts and take into account the amplification of the y-axis signal. **This is all very simple since the conversion factor for each axis is the same value of 10,000, so these factors cancel—resistance is just the slope of the measured curve on your oscilloscope.** Measure resistance for your simple resistor, and check that you are getting the same value as the resistance you measure direction using a DMM. Be sure to check with your instructor if you have any problems at this point!

Now, disconnect the resistor and connect the cable to your SQUID device. It is at room temperature, so the entire SQUID is normal and hence resistive. Look at the SQUID's room temperature V - I curve and measure its resistance.

Cooling down Mr. SQUID

Make sure the mu-metal magnetic shield is in place around your SQUID, and that you have the Lakeshore temperature probe connected to the SQUID (This allows you to measure your sample's temperature later.) Connect your temperature probe to the display and make sure it reads room temperature (in Kelvin!) Get a dewar of liquid

nitrogen (your instructor can help you) and prepare to cool down your device. First, make sure you have turned off all electronic devices around the SQUID, including your oscilloscope, the temperature display and the Mr. SQUID control box itself. (This eliminates the major sources of stray magnetic flux near your SQUID, to help avoid trapping so much flux you kill its superconductivity.) Next, gradually lower your SQUID into the liquid nitrogen a small section at a time. We have found a lowering rate of about 0.5 cm per 30 seconds is reasonable to avoid temperature shocks that can damage the device. Once your SQUID is immersed in liquid nitrogen, you can lower the device more rapidly until it is close to the bottom of the dewar. Once the temperature sensor has stabilized at 77K, proceed to turn on your electronics, reconnect the SQUID cable to the SQUID and try to get a V-I curve for the SQUID itself. If the settings on the Mr. SQUID[®] box and your output device are correct, and if the SQUID is behaving properly, you will see a curve that looks more-or-less like Fig. 14 on the screen.



Figure 14: Typical Mr. SQUID[®] V-I characteristic. Compare with Fig. 6. The central region has low resistance because the current is less than the Josephson junctions' critical current there. For currents greater than the critical current, the SQUID is Ohmic, with a normal state resistance given by the slope.

It is important that there is a flat, low resistance region in the center of the curve as shown above, although its width may vary from device to device. (Your SQUID may have a noticeable slope in the central section, even though superconductors really do have zero resistance. This is due to a nonzero contact resistance between the superconductor and its leads.) If what you see looks like a straight line, as in Fig. 13, then either you don't have enough liquid nitrogen in the dewar, you have not let the probe get cold, or there may be trapped magnetic flux in the SQUID. The latter is a very common occurrence because the SQUID is very sensitive to external magnetic fields. You may need to bring the SQUID back to normal and recool it. Refer to the discussion on trapped flux in Section 6.3 of the full documentation, Magnetic Flux Trapping in SQUIDS, if this appears to be the problem. Assuming you see the proper curve, how can we understand its shape?

What you are looking at is the V-I characteristic of two Josephson junctions connected in parallel with one another. Assuming they are identical junctions (in practice, they are at least very similar), the V-I characteristic you see is the same as for a single junction. The most important feature of the curve is the flat spot in the middle, corresponding to

currents less than the critical current. In this region, there is current flowing with no voltage – it is a supercurrent. This is the dc Josephson effect: a resistanceless current that flows through a superconductor Josephson junction.

At this point, you might want to adjust the flux bias control (2) on the control box. This control feeds current into a small gold coil placed above the SQUID and this current applies a magnetic field upon the loop the SQUID. As explained earlier, a magnetic field will modulate the critical current in the SQUID in a very specific manner. By turning the flux bias control knob, the critical current in the junction will change visibly on the oscilloscope screen or on your plotter page. At this point, try to adjust the flux bias current such that the flat region of the $V-I$ curve is widest. The response to these changes may be quite sensitive; it may take some practice to tune the critical current to its maximum value. This procedure may be especially useful if some small amount of magnetic field was already present in the SQUID loop. A zero applied field will yield the largest critical current through the junction.

Determining the critical current of your device

You can **determine the critical current** of the junctions in Mr. SQUID[®] by measuring the width (in volts) of the flat part of the $V-I$ curve and dividing that number by 10,000 Ω to convert your answer into amperes of current. (See Fig. 15.) If the flat region is 1 volt wide, for example, then the corresponding current is 100 microamps. However, you are looking at the current through both of the junctions in the SQUID, not just one. Therefore, assuming the junctions are identical, the current through one junction is half the value you are measuring (in our example, 50 microamps). Is this the critical current of the junction? Not quite. The curve you are looking at drives the current symmetrically about zero (marked in Fig. 15—compare Fig. 6). The current you have measured is actually composed of a contribution in the positive direction and a contribution in the negative direction. If the curve is symmetric, they are equal. As a result, the real critical current of the junction is half the value you infer from the measurement. Thus, in our example, the critical current is 25 microamps, 1/4 of the original measurement. The flat region on the curve corresponds to 4 times the junction critical current.

There is also a parameter of the junction known as the **normal-state resistance or R_N** . **You can determine it by measuring the slope of the $V-I$ curve out at the ends where it is essentially a straight line.** To obtain a resistance, you must convert the x-axis value into amperes from volts and take into account the amplification of the y-axis signal. This is all very simple since the conversion factor for each axis is the same value of 10,000. Remember that the dc SQUID contains two junctions in parallel, so that the measured resistance corresponds to half the resistance of a single junction (assuming they are identical.) Thus, simply taking the slope numbers in volts off your oscilloscope will give you one-half the normal-state resistance in ohms. The product of the critical current and the normal state resistance ($I_c R_N$) is a voltage that is an important parameter for the operation of a SQUID. **Make a note of it now for your report.** This value sets the maximum voltage change in the SQUID by an individual magnetic flux quantum, and is discussed later in this section; to see why this is so, look at Figs. 8 & 9. Note that $I_c R_N$ for one of the junctions has the same value for the SQUID, which has two junctions in parallel. For the

junctions in Mr. SQUID[®] operating in liquid nitrogen, you will probably obtain a value between 10 and 100 microvolts; this is just a ball park number that can vary from device to device.

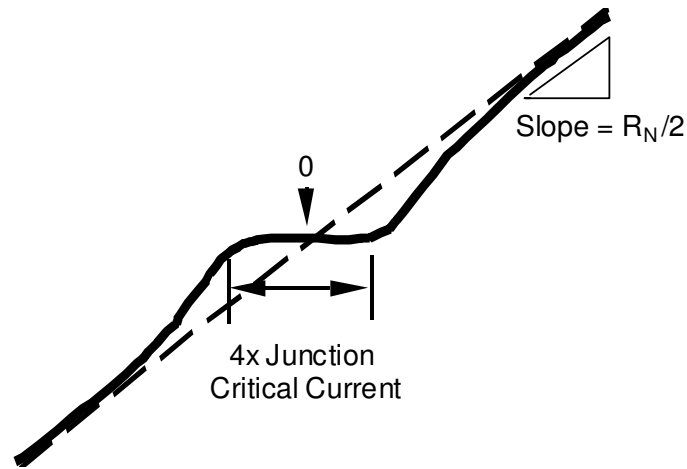


Figure 15 Determining the junction parameters from the V - I characteristic.

Experiment C: Observing the V - Φ curve of Mr. SQUID[®]

Up to this point, we have been looking at the properties of Josephson junctions. Now we will turn our attention to the flux-measuring of the dc SQUID itself. The Mr. SQUID[®] control box will allow you to observe the periodic oscillations in the V - Φ curve on your oscilloscope and hence measure the flux quantum. You have already observed the effects of a magnetic field on the V - I characteristics of the SQUID by adjusting the flux bias control current. The V - Φ characteristics are basically an automatically plotted version of this behavior.

As we saw before, in the V - I operating mode one can apply a magnetic field to the SQUID using the flux bias control. This dial controls a current that is applied to the “internal” modulation coil, simply a 3/4 turn gold or silver coil that creates a magnetic field inside the loop of the SQUID, as shown in Fig. 16. (The second coil shown in the photograph is the “external” modulation coil that allows you to couple other sources of current to the SQUID.) In this experiment, you will use your SQUID control box to send a time-varying current through this internal coil. That in turn exposes your SQUID to a time-varying magnetic flux. Your control box fixes the bias current flowing through the SQUID at a set fixed value, and measures the behavior of SQUID voltage, V , vs. magnetic flux, Φ .

If you slowly turn the flux bias control knob, you will see the change in the critical current and the changing V - I curve that occurs as the magnetic flux threading the loop of the SQUID is varied. If you experiment carefully with the flux bias control, you will see that the critical current of the SQUID oscillates between a maximum value (the flat region of the V - I curve is at its widest) and a minimum value at which point the V - I curve may be more-or-less linear.

The Mr. SQUID[®] control box allows you to view this periodic behavior of the SQUID's V curve as a function of magnetic flux in a convenient, automated way. To obtain the V - Φ plot, the bias current is set so that the SQUID voltage is most sensitive to changes in applied magnetic field. This occurs at the "knee" of the V - I curve, the area highlighted in Figure below.

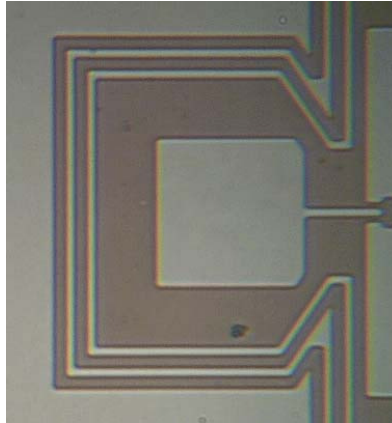


Figure 16: Photograph of the Mr. SQUID[®] version 4 chip showing the two modulation coils. See full documentation for Mr. SQUID[®] dimensions and specifications.

To find this point, first adjust the flux bias control so that the critical current is at its largest value. Then, turn down the amplitude control all the way so that only a point is visible on the oscilloscope screen or so that the pen is stationary on the plotter page. In this mode, you can then sweep the point up and down the V - I curve by adjusting the current bias control, as you did initially. Set the current bias level so that the point rests at either knee in the V - I curve. Now turn the flux bias control, which controls the amount of magnetic flux through the hole in the SQUID loop. You should see the point move back and forth vertically on the screen or page. This periodic motion arises because the screening currents in the SQUID body depend on the applied magnetic flux in a periodic manner. The period is determined by the magnetic flux quantum (Φ_0). This phenomenon is a manifestation of the macroscopic quantum nature of superconductivity.

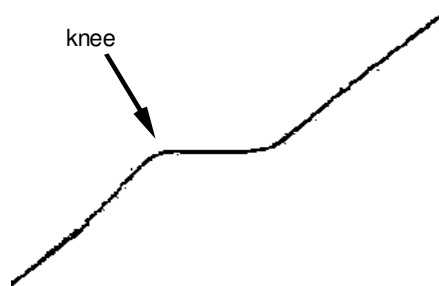


Figure 17: The "knee" in the V - I characteristic.

To automate this procedure, switch the function switch from $V-I$ to $V-\Phi$. In this new mode, all of the controls on the Mr. SQUID[®] control box work the same way as before, except for the amplitude control. Instead of controlling the sweep of the drive current through the SQUID, it now controls the amplitude of the current **through the modulation coil not through the SQUID itself**, which is linearly related to the magnetic field applied to the SQUID. Thus, amplitude now sweeps magnetic flux, Φ . To view as many $V-\Phi$ periods as possible, turn the amplitude control in the clockwise direction. Your oscilloscope's Autoscale feature may be able to find this curve for you. If not, you can use your previously computed value of $I_c R_N$ to judge how large ΔV should be, using the amplification factor explained below. The voltage amplification provided by the Mr. SQUID[®] box is unchanged from before. To determine the actual voltage across the SQUID, you must divide by 10,000. However, since the SQUID modulation signals are smaller than the junction voltages you examined earlier, you will need to increase the sensitivity of the vertical scale on your display device. The maximum peak-to-peak voltage swing of the SQUID modulation in Mr. SQUID[®] (ΔV) typically is between 10 and 30 μV (or 100 to 300 mV output from the front panel of the Mr. SQUID[®] control box). Depending on the setting of the amplitude control, you may wish to change the x sensitivity to more conveniently view the oscillations of the SQUID.

In addition, it may be necessary to use the position knob on your output device to center the curve. If your oscilloscope can be operated in an AC-coupled mode on the vertical channel, you can use this mode for more convenient viewing of the $V-\Phi$ curve (never use the AC mode to look at the $V-I$ curve, or it will be completely distorted on the oscilloscope screen.) You should see a curve that resembles the one shown in Fig. 18. Depending on the particular device and on the settings on the control box, you may see many more periods than shown here. At this point, you can try to maximize the signal by fine-tuning the current bias control (3) to the most sensitive part of the $V-I$ curve (just keep your eye on the $V-\Phi$ curve as you "tweak" the current bias control until you get the maximum modulation). There will be some setting of the current bias control that gives the largest modulation amplitude.

Notice the effect that the flux bias control has on the $V-\Phi$ curve. It allows you to set the central value of applied flux about which the amplitude control sweep varies. In other words, this control allows you to apply a static magnetic field to the SQUID on top of the oscillating field applied with the amplitude control. Turning the flux bias control will therefore allow you to move left and right along the $V-\Phi$ curve and thereby explore more of it than the amplitude sweep permits.

The voltage change that occurs due to the influence of the magnetic field appears now on the vertical axis of your display device. **Sketch your voltage-flux curve and measure ΔV ; compare to your measured $I_c R_N$.**

Now measure the flux quantum using your voltage-flux curve. The SQUID control box is sending current through the internal coil surrounding the SQUID. If you flow electrical current, I_{coil} , through the internal coil, you create magnetic flux through your device according to the equation:

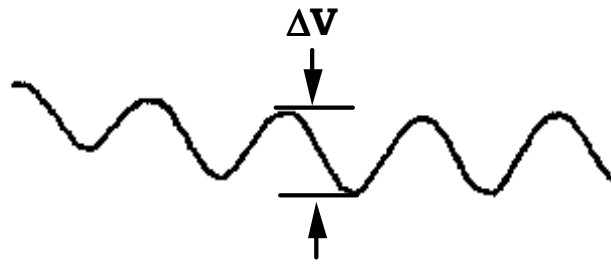


Figure 18: A typical Mr. SQUID[®] V - Φ characteristic. The maximum ΔV may range (approximately) from 10 to 30 μV , depending upon your value of $I_c R_N$. The voltage V is displayed on the y-axis, flux on the x-axis.

$$\Phi = M I_{\text{coil}}$$

Eq. 15

where M is the mutual inductance of the internal coil and SQUID. (Computing this value is pretty complicated because of the many contributions from the device geometry. The SQUID manufacturer's manual quotes the mutual inductance of the internal coil as 37 pH (that's pico-henry, the unit of inductance) on page 19.) This allows you to compute the change in magnetic flux that corresponds to one cycle of voltage change in the measured V - Φ curve in Fig. 18. First, however, you need to know how to interpret the Φ axis (x-axis on your display and their output). The output in the V - Φ mode has the x-axis displaying the current, ΔI , through the internal coil. The conversion between voltage out and current is: 10,000 V/A for this measurement. So, you can compute the change in magnetic flux through the device using this conversion and the mutual inductance. Use this to compute the flux quantum for your device to undergo one voltage cycle.

Experiment D. Resistance vs. Temperature of Mr. SQUID (OPTIONAL: DO ONLY IF THE SQUID IS NOT WORKING!)

This experiment will allow you to observe the superconducting transition of the YBCO film that forms the SQUID in Mr. SQUID[®]. By tracking the resistance of the SQUID while measuring its temperature, you can watch the YBCO undergo its superconducting transition. You will start at the temperature of liquid nitrogen, 77K, and allow your SQUID to slowly warm up to room temperature while recording both T and R , the resistance in the superconducting region. The transition temperature for YBCO superconductors is very near 77K, so take as much data as possible as the sample begins to warm up. You do not have to continue taking data once your sample has heated up to several degrees above its transition temperature. You should see a very noticeable increase in resistance once you hit the transition temperature (since your sample has nonzero resistance at the contacts between the ceramic high-T

superconductor and its metal leads, your lowest temperature resistance won't be truly zero here.) Also, given the nature of the V-I curve, you will have trouble measuring the resistance to high accuracy. Once your sample has gone normal (that is, the entire V-I curve becomes Ohmic) you may use a computer to plot up your data as a graph of R vs. T, indicating the superconducting transition temperature. This graph and an explanation of your experiment should form part of your final report.

ⁱAn article which describes this is Ryhänen *et al*, "SQUID Magnetometers for Low-Frequency Applications," *Journal of Low Temperature Physics* **76**, 287 (1989).