Operational Amplifiers

• The three rules of the op amp
• The inverting and the non-inverting configuration.
• The (weighted) summer.
• The difference amplifier. The CMRR (common mode rejection ratio).
• An instrumentation amplifier.
• Frequency effects on open and closed loop amplifiers. The gain-bandwidth theorem.
• The real op amp. Large signal limitations and DC imperfections.
• Integrators and differentiators.
Operational Amplifier is a device that can amplify the difference between two input signals. Among its pins, there are two signal inputs (the inverting and the non inverting input), a ground, an output pin and two dc power supply pins that provide the necessary power for the op amp IC chip to work.

\[
 v_o = A(v_2 - v_1)
\]

The amplification factor $A$ (or open loop gain) depends highly on temperature, varies a lot from op amp to op amp and depends a lot on frequency.
The open loop op amp acts as a voltage (differential) amplifier. It has huge input and tiny output resistances.

\[ v_o = A(v_2 - v_1) \]

It can be modeled as:

When op amps are being employed in circuits they are being used almost exclusively with negative feedback (closed loops). The gain of the configuration is reduced but the system is much more predictable, stable and with improved bandwidth.
The closed loop op amp.

When op amps are being employed in circuits they are being used almost exclusively with negative feedback (closed loops). The gain of the configuration is reduced but the system is much more predictable, stable and with improved bandwidth.

Negative feedback means that a portion of the output is being fed back at the inverting input of the op amp.

When we analyze circuits employing closed loop op amps we are using the three (approximate) rules of the ideal op amp.
The rules of an ideal op-amp connected in a closed loop.

1) The voltage gain $A$ of the ideal open loop op-amp is infinitely large.

2) The current through the ideal op-amp is zero. That is, the ideal op-amp has infinite input resistance.

3) Both terminals of the ideal op-amp are at the same voltage. (Consequence of rule #2)
The inverting amplifier.

Input resistance:
\[ v_i = i R_{in} \Rightarrow R_{in} = R_1 \]

may be too small

output resistance \( R_{out} \approx 0 \)

\[ -v_o = i R_2 \]

\[ v_i = i R_1 \]

\[ \frac{v_o}{v_i} = -\frac{R_2}{R_1} = A \]
An improved inverting amplifier.

Input resistance:

\[ R_{in} = R_1 \]

can be chosen to be large

amplifier gain:

\[
\frac{v_o}{v_i} = -\frac{R_2}{R_1} \left( 1 + \frac{R_4}{R_2} + \frac{R_4}{R_3} \right)
\]

Choose \( R_1 = R_2 = R_4 = R \) as large as possible to maximize the input resistance and minimize its effects on the closed loop voltage gain. Choose \( R_3 \) as small as possible to obtain the desired voltage gain.
An improved inverting amplifier.

\[ v_i = iR_1, \quad iR_2 = -iR_2 \]

\[-v_o = iR_2 + (i - i_2)R_4 \quad \Rightarrow \quad v_o = -i \left( R_2 + R_4 + \frac{R_2R_4}{R_3} \right) \Rightarrow\]

\[ v_o = -v_i \frac{R_2}{R_1} \left( 1 + \frac{R_4}{R_2} + \frac{R_4}{R_3} \right) \]
The weighted summer.

\[ v_o = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2 - \cdots - \frac{R_f}{R_N} v_N \]
Effects of finite open loop gain $A$ on the inverting amplifier.

$$G = \frac{v_o}{v_i} = -\frac{\frac{R_2}{R_1}}{1 + \frac{R_2}{R_1}} = \frac{R_2}{1 + \frac{R_2}{R_1}A}$$

Since $A \gg 1 + \frac{R_2}{R_1}$, $G = \frac{v_o}{v_i} \approx -\frac{R_2}{R_1}$
The non-inverting amplifier.

\[v_o - v_i = iR_2\]
\[v_o = i(R_1 + R_2)\]
\[\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} = A\]

The input resistance is infinite and the output resistance is 0.
Effects of finite open loop gain $A$ on the non-inverting amplifier.

\[
G = \frac{v_o}{v_i} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_2}{R_1} \frac{1}{A}}
\]

Since $A \gg 1 + \frac{R_2}{R_1}$, $G = \frac{v_o}{v_i} \approx 1 + \frac{R_2}{R_1}$
The voltage follower.

The voltage follower has infinite input resistance 0 output resistance and voltage gain unity (the output voltage is equal, follows the input voltage). It is used as a “buffer” a “matching” element between different stages of electronic instrumentation.
The difference amplifier.

An amplifier that responds to the difference $\nu_d = \nu_2 - \nu_1$ between two input signals $\nu_1$ and $\nu_2$. The output voltage $\nu_o$ (response) of the amplifier can be written as:

$$\nu_o = A_d \nu_d + A_{cm} \nu_{cm} \text{ with } \nu_d = \nu_2 - \nu_1 \text{ the difference input signal}$$

and $$\nu_{cm} = \frac{\nu_2 + \nu_1}{2} \text{ the common mode input signal}$$

$A_{cm}$ is the common mode gain and ideally must be 0

$A_d$ is the difference gain and ideally must be very large.

The common mode rejection ratio CMRR of a difference amplifier (ideally infinite) is defined as:

$$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| (\text{in dB})$$
The difference amplifier.

straightforward analysis gives:

\[ v_o = \frac{v_2 R_4 (R_1 + R_2)}{R_1 (R_3 + R_4)} - \frac{v_1 R_2}{R_1} \]

define

\[ v_d = v_2 - v_1 \quad v_{cm} = \frac{v_2 + v_1}{2} \]

and therefore:

\[ v_2 = v_{cm} + \frac{v_d}{2}, \quad v_1 = v_{cm} - \frac{v_d}{2} \]

The output voltage now can be expressed as a function of the common mode and difference voltages.
The difference amplifier.

By choosing $R_1 = R_3$ and $R_2 = R_4$ the common mode gain becomes 0. The difference gain becomes $R_2/R_1$.

$$v_o = \frac{v_d}{2} \left( \frac{R_2}{R_1} + \frac{R_4(R_1 + R_2)}{R_1(R_3 + R_4)} \right) + v_{cm} \left( \frac{R_4(R_1 + R_2)}{R_1(R_3 + R_4)} - \frac{R_2}{R_1} \right)$$
The difference amplifier.

\[ v_o = v_d \frac{R_2}{R_1} \]

\[ v_d = i2R_1 \Rightarrow R_{in} = 2R_1 \]

The input resistance of this difference amplifier is relatively low (since we want high difference gain). The instrumentation amplifier is a superior difference amplifier with infinite input resistance.
A major advantage of this configuration is that the first stage op amps do not amplify the common mode signal therefore are not prone to saturation. It is a superior difference amplifier.
The open loop gain of an op amp has a very strong dependence on frequency. It reaches its 3dB attenuation very quickly at 10 Hz very low value.

We can model this behavior with the function:

\[ A(f) = \frac{A_0}{1 + j \frac{f}{f_b}} \]

\( A_0 \) is the dc open loop gain and \( f_b \) is the 3 dB frequency. The product \( f_t = A_0 f_b \) is called the “unity-gain bandwidth” and it is specified in the data sheet by the manufacturer of the op amp.
Frequency dependence of closed loop op amps

Let’s examine the following question. How does the external feedback affect the frequency response of the gain of an inverting amplifier?

\[
G(f) = -\frac{R_2}{R_1} = -\frac{R_2}{1 + \frac{R_2}{R_1}} + \frac{R_2}{1 + \frac{R_2}{R_1}} + j\left(\frac{f}{A(f)} + \frac{f}{A_0 + j \left(\frac{f}{1 + \frac{R_2}{R_1}}\right)}\right)
\]

where \(A_0\) is very large
The gain bandwidth theorem

Since is $A_0$ very large we can write

$$G(f) \approx -\frac{R_2}{R_1} \frac{1}{1 + j \frac{f}{f_t}} \left( \frac{f_t}{1 + \frac{R_2}{R_1}} \right)$$

We see that the new 3dB frequency of the closed loop is:

$$f_{3dB} = \frac{f_t}{1 + \frac{R_2}{R_1}} \Rightarrow f_{3dB} (1 + K) = f_t = \text{const.}$$

This is the gain bandwidth theorem. It can also be proven for a non inverting amplifier.
Frequency response of closed loop op amps

\[ 20 \log |G(f)| \]

\[ v_o = |G(f)| v_i \]

3dB

Amplifier bandwidth

\[ f_{3dB} \]
Op Amp large signal limitations.

Output voltage saturation.
The output voltage of an op amp cannot be larger than a specified value, the rated output voltage of the op amp.

Maximum output current.
The output current supplied by an op amp is limited to a specified maximum value. (about 20 mA for an741C op amp).

Slew Rate (SR).
There is a maximum rate of change of the output voltage that an op amp can handle. It is called the slew rate (SR) of the op amp.

\[ SR = \left( \frac{d\nu_o}{dt} \right)_{\text{max}} \]
Op Amp full power bandwidth

For a sinusoidal input signal to an op amp. \( v_i(t) = V_i \sin(\omega t) \)

\[
\frac{dv_i}{dt}(t) = \omega V_i \cos(\omega t)
\]

The full power bandwidth \( f_M \) of an op amp is defined by the relationship:

\[
SR = \omega_M V_{omax} = 2\pi f_M V_{omax}
\]

where \( V_{omax} \) is the rated output voltage of the op amp (the maximum output voltage the op amp can deliver without clipping).

The physical meaning of the op amp’s full power bandwidth \( f_M \) is the maximum frequency a harmonic input to the op amp can have when the output amplitude is equal to the op amp’s rated voltage. If the frequency is increased the output signal is distorted due to slew rate limitation of the op amp. The full power bandwidth \( f_M \) of an op amp is specified by the manufacturer in the data sheet of the device.
DC imperfections of an op amp

Offset voltage
If the op amp is supplied with two equal voltages at its inputs the output voltage is not zero. The non ideal op amp is equivalent to an ideal op amp with a small dc offset voltage connected to one of its terminals.

Input bias and offset currents.
In order for a non ideal op amp to operate its two input terminals are supplied with two small input bias currents $I_{B1}$ and $I_{B2}$
Op Amp Offset Voltage

In order to account for the offset voltage effect a non ideal op amp can be modeled as shown

The small dc voltage at the positive terminal of the op amp is called the “offset voltage” of the op amp and it is specified by the manufacturer.
Op Amp Offset Voltage

The offset voltage of the op amp appears amplified at its output. Consider the inverting amplifier below where both its terminals are connected to ground. What is its output voltage?

\[ v_o = V_{os} \left(1 + \frac{R_2}{R_1}\right) \]

The offset dc voltage of the op amp appears amplified at the output.
Op Amp Offset Voltage

By connecting a capacitor at the input of the inverting amplifier, the offset voltage appears with unity gain at the output. Useful, but only if we want to amplify ac signals.

\[ v_o = V_{os}\left(1 + \frac{R_2}{\infty}\right) = V_{os} \]
Op Amp Offset Voltage

Some op amps have two additional terminals to which a specified circuit should be connected to trim the output voltage due to $V_{os}$.
Op Amp input bias and offset currents

In order for a non ideal op amp to operate its two input terminals are supplied with two small input bias currents $I_{B1}$ and $I_{B2}$

Input offset current

$$I_{OS} = |I_{B1} - I_{B2}|$$

typical value 10 nA

Input bias current

$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

typical value 100 nA

A non ideal op amp with input bias and offset currents is modeled as
Effect of op amp input offset and bias currents to a closed loop amplifier

By introducing the appropriate resistance $R_3$ to the non-inverting input of the op amp we can fix the dc output voltage due to the bias current.
Effect of op amp input offset and bias currents to a closed loop amplifier

We can get a more meaningful expression for the output voltage if we do the substitution:

\[ v_o = I_{B1} R_2 - I_{B2} R_3 \left(1 + \frac{R_2}{R_1}\right) \]

\[ I_{B1} = I_B + \frac{I_{OS}}{2} \quad I_{B2} = I_B - \frac{I_{OS}}{2} \]
Effect of op amp input offset and bias currents to a closed loop amplifier

\[ \nu_o = I_B R_2 \left[ 1 - \frac{R_3 (R_1 + R_2)}{R_1 R_2} \right] + \]
\[ + I_{OS} \frac{R_2}{2} \left[ 1 + \frac{R_3 (R_1 + R_2)}{R_1 R_2} \right] \]

By choosing \( R_3 = \frac{R_1 R_2}{R_1 + R_2} \) we get \( \nu_o \approx I_{OS} R_2 \)
Op Amp differentiators

\[ v_i = \frac{q}{C} \]

\[ -v_o = iR \Rightarrow v_o(t) = -R \frac{dq}{dt} \Rightarrow \]

\[ v_o(t) = -RC \frac{dv_i}{dt}(t) \]
Op Amp integrators

\[ I = \frac{v_i}{R} = \frac{dq}{dt} \Rightarrow q(t) = \frac{1}{R} \int_0^t v_i(t') dt' \]

\[-v_o = \frac{q}{C} \Rightarrow v_o(t) = -\frac{1}{RC} \int_0^t v_i(t') dt' \]
Effect of op-amp offset voltage on integrators

Because of the offset voltage the integrator will saturate soon after its power on.
A remedy for the offset voltage effect on integrators

The presence of a small $R_F$ will limit the output voltage at an asymptotic value a little larger than $V_{os}$. 
Effect of op-amp offset current on integrators

\[ \frac{dq}{dt} = I_{B1} \Rightarrow q = I_{B1}t \]

\[ v_o = \frac{q}{C} = \frac{I_{B1}}{C}t \]
Remedy of op-amp offset current effect on integrators

\[ -I_{B2}R \]

\[ I_{B2} \]

\[ R \]

\[ R \]

\[ I_{B1} \]

\[ C \]

\[ I_{B2} - I_{B1} \]

\[ \nu_o \]