Damped, driven oscillator

- Start with the case where $q=0$, $F_D=0$

\[
\frac{d^2y}{dt^2} + \omega_0^2 y = 0
\]

- $y(t) = A\cos \omega_0 t + B\sin \omega_0 t$
- Initial conditions, $A=y_0$, $B=v_0/\omega_0$
- Energy (kinetic + potential) should be conserved!

Compare with analytical to verify code, also test energy conservation!
Test with simple harmonic oscillator

\[
\frac{d^2y}{dt^2} + \omega_0^2 y = 0
\]

• Use Verlet algorithm

\[ y_{n+1} = 2y_n - y_{n-1} - \omega_0^2 dt^2 y_n \]

c force/mass from spring

force = \(-\omega_0^2y_{n}\)

c integrate to get y at next time step, use Verlet

ynext = 2.0d0*ynow-ylast+dt**2*force
Initial conditions, analytic solution

- Analytic result computed for comparison
- Verlet algorithm needs position at two previous times
- Translate into initial position and initial velocity

\[ y(t) = A \cos \omega_0 t + B \sin \omega_0 t \]

Initial conditions, \( A = y_0 \), \( B = \frac{v_0}{\omega_0} \)

c velocity at current timestep
\[ v_{\text{now}} = \frac{(y_{\text{next}} - y_{\text{last}})}{(2.0 \times d0 \times dt)} \]
c If \( i = 1 \) (first integration step), determine the initial conditions for an

c Next five lines not used in the case of damped, driven oscillator
\[ \text{if}(i \text{.eq.} 1) \text{ then} \]
\[ A = y_{\text{now}} \]
\[ B = v_{\text{now}} / \omega_0 \]
\[ \text{endif} \]
Energy calculation, analytical, and output

c velocity at current timestep
   vnow = (ynext-ylast)/(2.0d0*dt)

potential = 0.5d0*sk * ynow**2
kinetic = 0.5d0*vnow**2
etot = potential + kinetic
write (6,100) t,ynow,yanalytic,diff,potential,kinetic,
     : etot
100   format(f8.4,6(2x,f12.6))
For $\omega_0 = 1$, $dt=0.05$

Difference between exact and numerical

gnuplot> set term jpeg
Terminal type set to 'jpeg'
Options are 'small size 640,480'

gnuplot> set output 'displace.jpg'

gnuplot> plot 'output','output' using 1:3,'output' using 1:4
Energy conservation, potential, kinetic, total

Total energy conserved!

gnuplot> set output 'energy.jpg'
gnuplot> plot 'output' using 1:5,'output' using 1:6,'output' using 1:7
Damped, driven harmonic oscillator

\[ \frac{d^2y}{dt^2} + 2q \frac{dy}{dt} + \omega_0^2 y = F_D \cos \omega_D t \]

- Have to work out numerical integration using Verlet!
- Case with \( q=0, F_D=0 \) serves as starting point
- Damping, driving force mean energy not conserved
- Can still compare to analytical \( y(t) \) after transient decays

In the underdamped regime, \( q < \omega_0 \)

\[
y(t) = c e^{-qt} \sin(\beta t + \phi)
\]

For \( q=0.01, \omega_0=1 \), transient decays away \( \tau=1/q = 100 \)

After decay of transient, analytical behavior is

\[
y(t) = A \sin(\Omega_D t - \gamma)
\]
Damped, driven oscillator, position vs. time

Terminal type set to 'jpeg'
Options are 'small size 640,480'
gnuplot> set output 'damped1.jpg'
gnuplot> plot [300:400] 'output' using 1:2,'output' using 1:3
Differences between analytic, numerical are due to transients, important for $t<100$

Differences are equal to the transient behavior, which is not included in analytical result in code

```gnuplot
gnuplot> set output 'damped2.jpg'
gnuplot> plot 'output' using 1:4
```
**Code for the analytical result, comparison**

c Next three lines are for the damped, driven harmonic oscillator

\[
A = \frac{F}{dsqrt((\omega_0^2 - \omega^2)^2 + (2.0d0*q*\omega_0)^2)} \quad \text{! used for damped driven oscillator}
\]

\[
\phi = \text{datan}(2.0d0*\omega*q/(\omega_0^2 - \omega^2)) \quad \text{! used for damped driven oscillator}
\]

\[
y_{\text{analytic}} = A*\text{dcos}(\omega*t-\phi) \quad \text{! used for damped driven oscillator}
\]

\[
diff = y_{\text{now}} - y_{\text{analytic}}
\]

Notice the transient behavior, which depends on the initial conditions, is not included here which explains the differences seen in the preceding slide.