1. The Lotka-Volterra model is often also referred to as the “predator-prey equations”. These are two coupled nonlinear differential equations that can only be solved numerically. The equations for this model are given below:

\[
\frac{dx}{dt} = \alpha_1 x - \alpha_2 xy
\]

\[
\frac{dy}{dt} = \beta_1 xy - \beta_2 y
\]

a) Find the stable critical points where \( \frac{dx}{dt} = 0 \) and \( \frac{dy}{dt} = 0 \) in terms of the coefficients \( \alpha_1, \alpha_2, \beta_1, \) and \( \beta_2 \). The trivial stable point is \( x = 0, y = 0 \).

b) Write a computer code in Fortran 77 or Fortran 90 to numerically solve the Lotka-Volterra model. Use two approaches and compare. First, try the ordinary Euler algorithm discussed in class. Second, try the “midpoint method”, which is essentially a 2nd order Runge-Kutta approach (see Eq. A.15 and A.16 in Giordano). In particular, you can find \( x_{n+1} \) and \( y_{n+1} \) from,

\[
x_{n+1} = x_n + \Delta t (\alpha_1 x' - \alpha_2 x' y')
\]

\[
y_{n+1} = y_n + \Delta t (\beta_1 x' y' - \beta_2 y')
\]

where \( x' \) and \( y' \) are given by,

\[
x' = x_n + \frac{\Delta t}{2} (\alpha_1 x_n - \alpha_2 x_n y_n)
\]

\[
y' = y_n + \frac{\Delta t}{2} (\beta_1 x_n y_n - \beta_2 y_n)
\]
Make a picture of the populations \( x \) and \( y \) vs. time. Have both populations plotted on the same graph, with the plot extending through at least 5 oscillations. You can pick \( \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1 \), and start with any initial populations \( x_0 \) and \( y_0 \) that are away from the stationary points.

2. Consider the equation of a damped, driven simple harmonic oscillator

\[
\frac{d^2 y}{dt^2} + 2q \frac{dy}{dt} + \omega_0^2 y = A \cos \omega t
\]

a) Find the analytical solution for \( y(t) \), neglecting the homogeneous solution describing transient behavior.

b) Write a Fortran 77 or Fortran 90 code that uses the Verlet algorithm to determine \( y(t) \). Also in your code, compute the analytical solution \( y(t) \) at each time step for comparison. Determine a suitable time step so that the numerical and analytical solutions agree. Determine the resonant frequency where the amplitude is maximum. Run your simulation at exactly the resonant frequency.

Hand in a plot of \( y(t) \) from your code that includes both the analytical and numerical results.