Homework 1

PHZ 5156

Due Thursday, August 31

1. Consider a particle of mass $m$ trapped in a two-dimensional box with infinitely high walls at $x = 0$, $x = a$, $y = 0$, and $y = b$. Inside the box, the potential is zero and the Hamiltonian is given by

$$\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$$  \hspace{1cm} (1)

The infinitely high potential walls that trap the particle result in the boundary conditions for the wave function $\psi(x = 0, y) = 0$, $\psi(x = a, y) = 0$, $\psi(x, y = 0) = 0$, and $\psi(x, y = b) = 0$.

a) Use the method of separation of variables to write the time-independent Schrodinger equation

$$\hat{H} \psi(x, y) = E \psi(x, y) \hspace{1cm} (2)$$

as two ordinary differential equations.

b) Determine the solutions to the two equations obtained in part a).

c) What are the allowed energies $E$?

2. The torsion of a bar is described by the fourth-order equation,

$$\frac{d^4 \theta}{dx^4} + \tau \theta = 0 \hspace{1cm} (3)$$

Show how this can be expressed as a system of coupled first-order differential equations.

3. Consider the equation of a damped, driven simple harmonic oscillator,

$$m \frac{d^2 y}{dt^2} + \gamma \frac{dy}{dt} + ky = F \cos(\omega t) \hspace{1cm} (4)$$

a) Find the particular solution $y(t)$ to this equation. Write your answer in the form $y(t) = |A| \cos(\omega t + \delta)$, and determine expressions for $|A|$ and the phase angle $\delta$. To keep your work simple, use the definition for the natural frequency of the oscillator $\omega_0 = \sqrt{\frac{k}{m}}$. 

1
b) Determine the condition for resonance. In other words, find an expression for the driving frequency \( \omega \) which results in a maximum amplitude \( |A| \) for the response.

4. Write down first-order finite difference approximations for each of the following:
   a) \( \frac{df}{dx} \)
   b) \( \frac{d^2f}{dx^2} \)
   c) \( \frac{d^3f}{dx^3} \)
   d) \( \frac{\partial}{\partial t} \frac{\partial f}{\partial x} \)