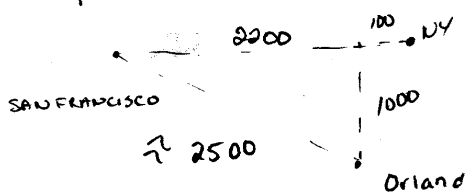
Test 1 solutions: Problem 2 (Estimation Problem: 15 points)

You and a friend are planning a two-week vacation out to the West Coast for a wedding in San Francisco next summer. However you're both on a tight budget. Your friend thinks it would be cheaper to drive his car than fly. A cheap plane fare from Orlando International Airport to San Francisco is $400 round-trip. Realistically estimate your travel expenses to and from the West Coast to see if your friend is right. What would your average speed be going from Orlando to San Francisco if you drove? Assume you will have free room and board at a relative's house once you arrive. Be sure to explicitly state all assumptions.

plane far = $400 round trip



Expenses

gas

food

hotel

Assumtions

- Car can go 250 miles on one tank of gas
- $15 per day of travel per person
- It costs $20 to fill the gas tank.
- Distance from Orlando to San Francisco 2,500 miles
- Hotel cost is $30 per night, but it is shared by both people.

1) We will travel for 8 hours per day, going on average 60 mph.

Distance traveled each day = 480 miles

2) Number of days to get to San Francisco:

2500 miles ÷ 480 miles = 5.2 days

* They would arrive at their relatives house around lunch time.

3) Money Spent

(a.) gas: car can go 250 miles on one tank

$2500 \text{ miles} \div 250 \frac{\text{miles}}{\text{tank}} = 10 \text{ tanks}$

Costs $20 per tank $\$20 \times 10 = \$200$

(b.) food: $15 per day for 5.2 days

3 meals per day plus breakfast on day 6.

$15 \times 5 + \frac{15}{3} = \$80$

(c.) hotel: $30 per night shared by both people for 5 nights $\frac{30}{2} \times 5 = \$75$

total gas money: $200 split by 2 people

total gas money per person = $100

total food money per person $80

total hotel costs per person = $75

Total travel expenses: gas + food + hotel

$100 + $80 + $75 = $255.00 → one way

$255 x 2 = $510.00 round trip

look on back ↓

Average Speed

average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{2500 \text{ miles}}{5.2 \cancel{\text{days}}} \times \frac{1 \cancel{\text{day}}}{24 \text{ hrs}} = 20 \text{ mph}$

* It would be cheaper to fly if you take into account the round trip costs.

driving roundtrip = \$510
flying roundtrip = \$400

you would save \$110

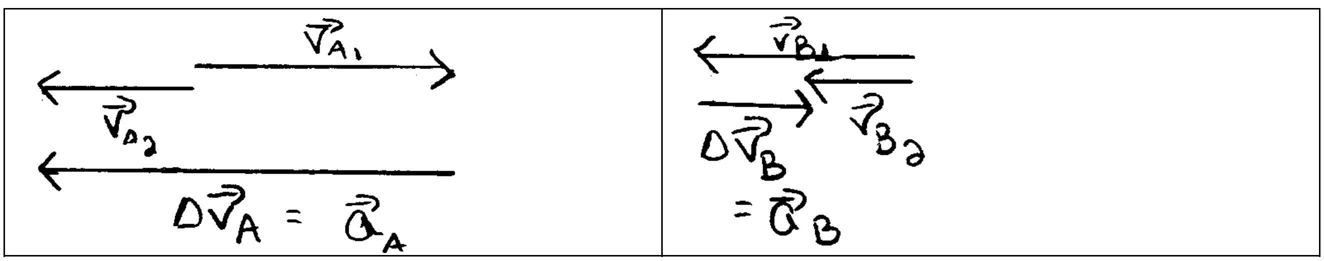
Instructor's Note: This is an excellent solution. The only thing that could be improved would be more use of symbol formulas on the previous page and better attention to units, i.e. 480 miles/day.

For estimates of the distance from Orlando to San Francisco, acceptable values were between 2,000 and 6,000 miles. Speeds of up to 80 mph were allowed but to receive full credit you need to account for tickets and lower speed limits (sections of the road are limited to 55 MPH).

Test 1 solution: Problem 3 (Essay 11 points)
You may use diagrams and equations but no calculations in your response for this problem.
USE WHAT YOU'VE LEARNED FROM CLASS SO FAR TO GIVE A CONVINCING EXPLANATION OF YOUR ANSWER.

Two carts roll toward each other on a level table. The vectors represent the velocities of the carts just before and just after they collide.

A. Draw and label a vector for <u>each cart</u> to represent the *change in velocity* from before to after the collision. Make your vectors consistent with the vectors drawn above.



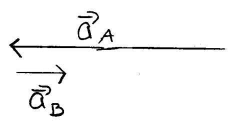
B. How do the magnitude and direction of Cart A's average acceleration compare with Cart B's average acceleration over the time interval shown? Explain your reasoning well enough to convince a classmate who disagrees with you.

The average acceleration can be defined as change velocity over change in time, because we do not know a set value for change in time we can represent acceleration as a relative comparison of changes in velocity. The change of velocity in cart A is towards the left while the change of velocity of cart B is to the right. The magnitude of the ΔV vector of A is about 3 times the size of the ΔV vector of B. So acceleration is approximately 3x to the left for cart A and 1x to the right for cart B.

Instructor's note: Essay questions are graded by listing all the key points that need to be included for a complete solution. Full credit is given if the essay is coherent and includes 80% of the key points.

The key point in the 2nd part of the essay is that average acceleration is proportional to delta v and points in the same direction. Thus looking directly at the Δv vectors lets us compare the direction and relative magnitude of the average acceleration vectors for carts A and B. A complete solution would also include a comment on why the average acceleration vectors look the way they do. Here follows two examples:

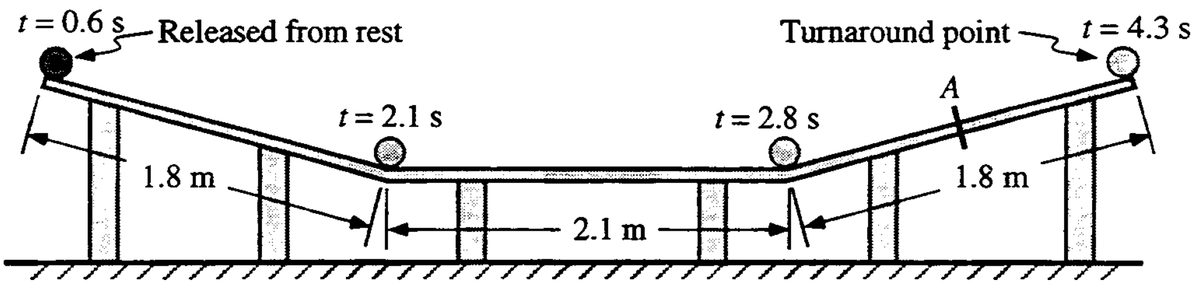
Cart A's average acceleration is about 4 times larger than cart B's average acceleration. Cart A's average acceleration vector points in the opposite direction of Cart B's average acceleration vector.



This makes sense because cart A must have an acceleration acting on it that causes it not only to stop but to turn around and begin moving in the opposite direction. Cart B only needs an acceleration vector to slow it down some.

Average acceleration is ΔV, the change in velocity, by definition. Cart A's acceleration vector looks that way because we went from a very positive to a negative velocity. the change in velocity should be large because we change the velocity a lot. The direction of Cart A's ΔV and Cart B's ΔV are opposite because in the before, the carts traveled in opposite directions and in the after, the directions are the same. So one of the cards changed direction. So their change in velocities direction is shown by opposing ΔV vectors.

Test 1 Solution: Problem 4 (16 points)



A. Determine the speed of the ball at t = 2.1 s. Show your work.

$S = \frac{2.1m}{2.8s - 2.1s} = \boxed{3m/s}$

I used the time interval t = 2.1s to t = 2.8s because velocity & speed is constant.

Determine the magnitude of the acceleration of the ball at point A (halfway up the second incline). Show your work.

$|\vec{v}|$ at t = 4.3s is 0

$|\vec{a}| = \frac{0 - 3m/s}{4.3s - 2.8s} = |-2m/s^2| = \boxed{2m/s^2}$

On the diagram above, draw an arrow indicating the direction of the acceleration of the ball at point A. Explain why you drew the arrow the way you did.

I drew the acceleration vector pointing down the ramp because the ball is slowing down.

If object slows down, velocity and acceleration vectors must point in opposite directions.

On the diagram above, draw an arrow indicating the direction of the acceleration of the ball at t = 4.3 s (the turnaround point). If the acceleration at the turnaround point is zero, state that explicitly. Explain why you drew the arrow the way you did.

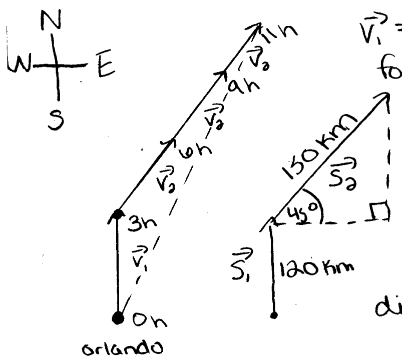
I drew the acceleration vector pointing down the ramp be cause just before t = 4.3s the ball is slowing down and just after t = 4.3s the ball is speeding up indicating that the acceleration vector must be pointing down the ramp.

Instructor's note: This is a pretty good solution. There were only two things that kept it from receiving full credit. One was lack of symbolic solutions in parts A and B. The other was the reasoning in part D. A complete solution to part D would describe how the ball slows down going up the ramp, comes to a stop, and then speeds up down the ramp back the way it came. Thus the ball has an acceleration down the ramp right before turn around and right after. If the ball had zero acceleration at the turn around point, it would stop and just stay there.

Test 1 Solution: Problem 5 (18 points)

One day in the early 1960's, at 11 AM the eye of hurricane David passed over Orlando heading due North at a speed of 40.0 km/h. Three hours later, the course of the hurricane shifted to Northeast towards the Florida Atlantic coast and its speed decreased to 30.0 km/h. David continued on this course at this speed for eight hours before turning due north again.

A. How far from Orlando was the hurricane at 7 PM on the same day?



$\vec{v}_1 = 40$ km/h for 3hrs. $\vec{v}_2 = 30$ km/h for 5 hrs.

$\vec{s}_1 = 0i + 120j$

$\vec{s}_2 = 150\cos 45\, i + 150\sin 45\, j$

$\vec{s}_2 = 106.066\, i + 106.066\, j$

$\vec{s}_1 + \vec{s}_2 = 106.066\, i + 226.066\, j$

displacement $= \sqrt{(106.066)^2 + (226.066)^2}$

$= 249.711$ km $= 250.$ km

direction $= 90° - \tan^{-1}\left(\frac{226.066}{106.066}\right) = 25.1°$ East of north

B. What was David's average speed during this time?

$\langle s \rangle = \frac{d}{\Delta t}$ $\quad d$ = total distance traveled

Δt = time interval = 11AM to 7PM

$\langle s \rangle = \frac{(120\text{ km} + 150\text{ km})}{8\text{ h}} = \frac{270\text{ km}}{8\text{h}} = 33.8$ km/h

C. What was David's average velocity during this time?

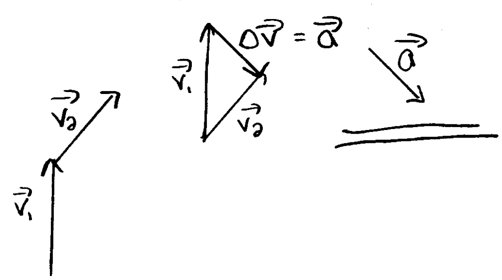
$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$ $\quad \Delta\vec{r}$ = displacement = 250 km

Δt = time interval = 11AM to 7PM

$\langle \vec{v} \rangle = \frac{250\text{ km}}{8\text{h}} = 31.3$ km/h

D. Sketch a vector representing hurricane David's average acceleration during this time.

$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t}$



Instructor's note: A very good solution with two minor omissions. First the solution does not show the calculation for distance or displacement for s1 and s2. Second, part C with average velocity should indicate the direction of the velocity explicitly. A note saying the direction of the velocity is in the direction of the displacement calculated in part A would have been sufficient.