## Chapter 5 - Force and Motion I

I. Newton's first law.
II. Newton's second law.
III. Particular forces:
-Gravitational

- Weight
- Normal
- Friction
- Tension
IV. Newton's third law.

Newton mechanics laws cannot be applied when:

1) The speed of the interacting bodies are a fraction of the speed of light $\rightarrow$ Einstein's special theory of relativity.
2) The interacting bodies are on the scale of the atomic structure $\rightarrow$ Quantum mechanics
I. Newton's first law: If no net force acts on a body, then the body's velocity cannot change; the body cannot accelerate $\rightarrow$ $\overrightarrow{\mathrm{v}}=$ constant in magnitude and direction.

- Principle of superposition: when two or more forces act on a body, the net force can be obtained by adding the individual forces vectorially.
- Inertial reference frame: where Newton's laws hold.
II. Newton's second law: The net force on a body is equal to the product of the body's mass and its acceleration.
$\vec{F}_{\text {net }}=m \vec{a}$
(5.1)

$$
\begin{equation*}
F_{n e t, x}=m a_{x}, \quad F_{n e t, y}=m a_{y}, \quad F_{n e t, z}=m a_{z} \tag{5.2}
\end{equation*}
$$

- The acceleration component along a given axis is caused only by the sum of the force components along the same axis, and not by force components along any other axis.
- System: collection of bodies.
- External force: any force on the bodies inside the system.
III. Particular forces:
-Gravitational: pull directed towards a second body, normally the Earth $\rightarrow$

$$
\vec{F}_{g}=m \vec{g}
$$

- Weight: magnitude of the upward force needed to balance the gravitational force on the body due to an astronomical body $\rightarrow$

$$
\begin{equation*}
W=m g \tag{5.4}
\end{equation*}
$$

- Normal force: perpendicular force on a body from a surface against which the body presses.

$$
\begin{equation*}
N=m g \tag{5.5}
\end{equation*}
$$

- Frictional force: force on a body when the body attempts to slide along a surface. It is parallel to the surface and opposite to the motion.

-Tension: pull on a body directed away from the body along a massless cord.

IV. Newton's third law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

$$
\vec{F}_{B C}=-\vec{F}_{C B}
$$

QUESTIONS


Q2. Two horizontal force $\overrightarrow{e s} F_{1}, F_{2}$ pull a banana split across a frictionless counter. Without using a calculator, determine which of the vectors in the free body diagram below best represent: a) $F_{1}$, b) $F_{2}$. What is the net force component along (c) the x-axis, (d) the $y$-axis? Into which quadrant do (e) the net-force vector and (f) the split's acceleration vector point?

$$
\begin{aligned}
& \vec{F}_{1}=(3 N) \hat{i}-(4 N) \hat{j} \\
& \vec{F}_{2}=-(1 N) \hat{i}-(2 N) \hat{j}
\end{aligned}
$$

$$
\vec{F}_{\text {net }}=\vec{F}_{1}+\vec{F}_{2}=(2 N) \hat{i}-(6 N) \hat{j}
$$

Same quadrant, 4


## I. Frictional force

Counter force that appears when an external force tends to slide a body along a surface. It is directed parallel to the surface and opposite to the sliding motion.
-Static: $\left(f_{s}\right)$ compensates the applied force, the body does not move.

$$
\vec{f}_{s}=-\vec{F}_{/ /}
$$

-Kinetic: $\left(f_{k}\right)$ appears after a large enough external force is applied and the body loses its intimate contact with the surface, sliding along it.



Acceleration


Constant velocity


$$
\begin{equation*}
f_{k}<f_{s, \text { max }} \quad f_{s, \text { max }}=\oiiint^{N} N \tag{6.1}
\end{equation*}
$$

Friction coefficients

$$
\text { If } F_{/ /}>f_{s, \text { max }} \rightarrow \text { body slides }
$$

$$
\begin{equation*}
f_{k}=\varkappa_{k} N \tag{6.2}
\end{equation*}
$$

After the body starts sliding, $\mathrm{f}_{\mathrm{k}}$ decreases.

Q1. The figure below shows overhead views of four situations in which forces act on a block that lies on a frictionless floor. If the force magnitudes are chosen properly, in which situation it is possible that the block is (a) stationary and (b) moving with constant velocity?


In which situations does the object acceleration have (a) an x-component, (b) a y component? (c) give the direction of a.
Q. A body suspended by a rope has a weigh of 75 N . Is $T$ equal to, greater than, or less than 75 N when the body is moving downward at (a) increasing speed and (b) decreasing speed?


$$
\vec{F}_{n e t}=\vec{F}_{g}-\vec{T}=m \vec{a} \rightarrow T=m(g-a)
$$

(a) Increasing speed: $\quad \mathrm{v}_{\mathrm{f}}>\mathrm{v}_{0} \rightarrow \mathrm{a}>0 \rightarrow \mathrm{~T}<\mathrm{F}_{\mathrm{g}}$
(b) Decreasing speed: $\mathrm{v}_{\mathrm{f}}<\mathrm{v}_{0} \rightarrow \mathrm{a}<0 \rightarrow \mathrm{~T}>\mathrm{F}_{\mathrm{g}}$

The figure below shows a train of four blocks being pulled across a frictionless floor by force $F$. What total mass is accelerated to the right by (a) F, (b) cord 3 (c) cord 1? (d) Rank the blocks according to their accelerations, greatest first. (e) Rank the cords according to their tension, greatest first.

(a) F pulls $\mathrm{m}_{\text {total }}=(10+3+5+2) \mathrm{kg}=20 \mathrm{~kg}$
(c) Cord $1 \rightarrow \mathrm{~T}_{1} \rightarrow \mathrm{~m}=10 \mathrm{~kg}$
(b) Cord $3 \rightarrow \mathrm{~T}_{3} \rightarrow \mathrm{~m}=(10+3+5) \mathrm{kg}=18 \mathrm{~kg}$
(d) F=ma $\rightarrow$ All tie, same acceleration

$$
\begin{array}{ll}
\text { (e) } \begin{array}{l}
\mathrm{F}-\mathrm{T}_{3}=2 \mathrm{a} \\
\mathrm{~T}_{3}-\mathrm{T}_{2}=5 \mathrm{a} \\
\mathrm{~T}_{2}-\mathrm{T}_{1}=3 \mathrm{a} \\
\mathrm{~T}_{1}=10 \mathrm{a}
\end{array} & \begin{array}{l}
\mathrm{F}-\mathrm{T}_{3}=2 \mathrm{a} \rightarrow \mathrm{~F}=18 \mathrm{a}+2 \mathrm{a}=20 \mathrm{a} \\
\\
\end{array} \quad \begin{array}{l}
3-13 \mathrm{a}=5 \mathrm{a} \rightarrow \mathrm{~T}_{3}=18 \mathrm{a} \\
\mathrm{~T}_{2}-10 \mathrm{a}=3 \mathrm{a} \rightarrow \mathrm{~T}_{2}=13 \mathrm{a} \\
\\
\mathrm{~T}_{1}=10 \mathrm{a}
\end{array}
\end{array}
$$

Q. A toy box is on top of a heavier dog house, which sits on a wood floor. These objects are represented by dots at the corresponding heights, and six vertical vectors (not to scale) are shown. Which of the vectors best represents (a) the gravitational force on the dog house, (b) on the toy box, (c) the force on the toy box from the dog house, (d) the force on the dog house from the toy box, (e) force on the dog house from the floor, (f) the force on the floor from the dog house? (g) Which of the forces are equal in magnitude? Which are (h) greatest and (i) least in magnitude?
(a) $F_{g}$ on dog house: or 5 (h) Greatest: 6,3
(b) $\mathrm{F}_{\mathrm{g}}$ on toy box: 2
(i) Smallest: 1,2,5
(c) $F_{\text {toy }}$ from dog house: 1
(d) $F_{\text {dog-house }}$ from toy box: 4 or
(e) $F_{\text {dog-house }}$ from floor: 3
(f) $F_{\text {floor }}$ from dog house: 6
(g) Equal: 1=2, 1=5, 3=6
5. There are two forces on the 2 kg box in the overhead view of the figure below but only one is shown. The figure also shows the acceleration of the box. Find the second force (a) in unit-vector notation and as (b) magnitude and (c) direction.


$$
a=12 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
\begin{gathered}
\vec{a}=\left(12 \cos 240^{\circ} \hat{i}+12 \sin 240^{\circ} \hat{j}\right) \mathrm{m} / \mathrm{s}^{2}=(-6 \hat{i}-10.39 \hat{j}) \mathrm{m} / \mathrm{s}^{2} \\
\vec{F}_{T}=m \vec{a}=2 \mathrm{~kg}(-6 \hat{i}-10.39 \hat{j}) \mathrm{m} / \mathrm{s}^{2}=(-12 \hat{i}-20.78 \hat{j}) \mathrm{N} \\
\vec{F}_{T}=\vec{F}_{1}+\vec{F}_{2}=20 \hat{i}+\vec{F}_{2}
\end{gathered}
$$

$$
F_{T x}=-12 N=F_{2 x}+20 N \rightarrow F_{2 x}=-32 N
$$

$$
F_{T y}=-20.78 \mathrm{~N}=F_{2 y}
$$

$$
F_{2}=(-32 \hat{i}-20.78 \hat{j}) N
$$

$$
F_{2}=\sqrt{32^{2}+21^{2}}=38.27 N
$$

$$
\tan \theta=\frac{-20.78}{-32}=33^{\circ} \text { or } 180^{\circ}+33^{\circ}=213^{\circ}
$$

## Rules to solve Dynamic problems

- Select a reference system.
- Make a drawing of the particle system.
- Isolate the particles within the system.
- Draw the forces that act on each of the isolated bodies.
- Find the components of the forces present.
- Apply Newton's second law (F=ma) to each isolated particle.

9. (a) A 11 kg salami is supported by a cord that runs to a spring scale, which is supported by another cord from the ceiling. What is the reading on the scale, which is marked in weigh units? (b) Here the salami is supported by a cord that runs around a pulley and to a scale. The opposite end of the scale is attached by a cord to a wall. What is the reading on the scale? (c) The wall has been replaced by a second salami on the left, and the assembly is stationary. What is the reading on the scale now?


$$
\begin{aligned}
& W=\left|F_{g}\right|=m g=(11 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=107.8 \mathrm{~N} \\
& \text { (a) } a=0 \rightarrow T=F_{g}=107.8 \mathrm{~N}
\end{aligned}
$$


(b) $a=0 \rightarrow T=F_{g}=107.8 \mathrm{~N}$

(c) $a=0 \rightarrow T=F_{g}=107.8 \mathrm{~N}$

In all three cases the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weigh of the salami because the salami is not accelerating.
23. An electron with a speed of $1.2 \times 10^{7} \mathrm{~m} / \mathrm{s}$ moves horizontally into a region where a constant vertical force of $4.5 \times 10^{-16} \mathrm{~N}$ acts on it. The mass of the electron is $\mathrm{m}=9.11 \times 10^{-31} \mathrm{~kg}$. Determine the vertical distance the electron is deflected during the time it has moved 30 mm horizontally.


$$
\begin{aligned}
& \quad d_{x}=v_{x} t=0.03 \mathrm{~m}=\left(1.2 \cdot 10^{7} \mathrm{~m} / \mathrm{s}\right) t \rightarrow t=2.4 \mathrm{~ns} \\
& F_{\text {net }}=m a_{y}=F-F_{g}=4.5 \cdot 10^{-16} \mathrm{~N}-\left(9.11 \cdot 10^{-31} \mathrm{~kg}\right)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& F_{\text {net }}=\left(9.11 \cdot 10^{-31} \mathrm{~kg}\right) a_{y} \rightarrow a_{y}=4.94 \cdot 10^{14} \mathrm{~m} / \mathrm{s}^{2} \\
& d_{y}=v_{o y} t+0.5 a_{y} t^{2}=0.5 \cdot\left(4.94 \cdot 10^{14} \mathrm{~m} / \mathrm{s}^{2}\right) \cdot\left(2.5 \cdot 10^{-9} \mathrm{~s}\right)^{2}=0.0015 \mathrm{~m}
\end{aligned}
$$

13. In the figure below, $\mathrm{m}_{\text {block }}=8.5 \mathrm{~kg}$ and $\theta=30^{\circ}$. Find (a) Tension in the cord. (b) Normal force acting on the block. (c) If the cord is cut, find the magnitude of the block's acceleration.

(a) $a=0 \rightarrow a_{x}=0 \rightarrow T-F_{g x}=0 \rightarrow$

$$
T=F_{g x}=m g \sin 30^{\circ}=(8.5 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) 0.5=41.65 \mathrm{~N}
$$

(b) $a_{y}=0 \rightarrow N-F_{g y}=0 \rightarrow N=F_{g y}=m g \cos 30^{\circ}=72.14 N$
(c) $T=0 \rightarrow F_{g x}=m a=41.65 \mathrm{~N}=8.5 a \rightarrow a=4.9 \mathrm{~m} / \mathrm{s}^{2}$
55. The figure below gives as a function of time $t$, the force component $F_{x}$ that acts on a 3 kg ice block, which can move only along the $x$ axis. At $t=0$, the block is moving in the positive direction of the axis, with a speed of $3 \mathrm{~m} / \mathrm{s}$. What are (a) its speed and (b) direction of travel at $\mathrm{t}=11 \mathrm{~s}$ ?

$$
\begin{aligned}
& t=0 \rightarrow v_{0}=3 \mathrm{~m} / \mathrm{s} \\
& t=11 \mathrm{~s} \rightarrow v_{f}=? \\
& a_{x}=\frac{F_{x}}{m}=\frac{d v_{x}}{d t} \rightarrow \int \frac{d v_{x}}{d t} d t=v_{f}-v_{0}=\int_{0}^{11 s} \frac{F_{x}}{m} d t
\end{aligned}
$$

Total graph area $=15 N s=\int_{0}^{11 \mathrm{~s}} F_{x} d t=\left(v_{f}-v_{0}\right) m=\left(v_{f}-3 m / s\right) 3 \mathrm{~kg}$
$\rightarrow v_{f}=\frac{15 \mathrm{kgm} / \mathrm{s}}{3 \mathrm{~kg}}+3 \mathrm{~m} / \mathrm{s}=8 \mathrm{~m} / \mathrm{s}$


Midterm1_extra_Spring04. Two bodies, $m 1=1 \mathrm{~kg}$ and $\mathrm{m} 2=2 \mathrm{~kg}$ are connected over a massless pulley. The coefficient of kinetic friction between m 2 and the incline is 0.1 . The angle $\theta$ of the incline is $20^{\circ}$. Calculate:
(a) Acceleration of the blocks.
$F_{2 g, x}=m_{2} g \sin 20^{\circ}=6.7 \mathrm{~N}$
$N_{2}=F_{2 g, y}=m_{2} g \cos 20^{\circ}=18.42 \mathrm{~N}$
$f=\mu_{k} N_{2}=\mu_{k} m_{2} g \cos 20^{\circ}=1.84 \mathrm{~N}$

Block 1: $m_{1} g-T=m_{1} a$

$$
\rightarrow 9.8-T=a
$$

Block 2: $T-f-F_{2 g, x}=m_{2} a \rightarrow T-1.84-6.7=2 a$
Adding $3 a=1.26 \rightarrow a=0.42 \mathrm{~m} / \mathrm{s}^{2}, T=9.38 \mathrm{~N}$
(b) Tension of the cord.


Midterm1_Spring04. The three blocks in the figure below are connected by massless cords and pulleys. Data: $m_{1}=5 \mathrm{~kg}, \mathrm{~m}_{2}=3 \mathrm{~kg}, \mathrm{~m}_{3}=2 \mathrm{~kg}$. Assume that the incline plane is frictionless.
(i) Show all the forces that act on each block.
(ii) Calculate the acceleration of $m_{1}, m_{2}, m_{3}$.
(iii) Calculate the tensions on the cords.
(iv) Calculate the normal force acting on $\mathrm{m}_{2}$
$\mathrm{Fg}_{2 \mathrm{y}}=\mathrm{m}_{2} \mathrm{~g} \cos 30^{\circ}$
$\mathrm{Fg}_{2 \mathrm{x}}=\mathrm{m}_{2} \mathrm{~g} \sin 30^{\circ}$


Block 1: $m_{1} g-T_{1}=m_{1} a$
Block 2: $\mathrm{m}_{2} \mathrm{~g}\left(\sin 30^{\circ}\right)+\mathrm{T}_{1}-\mathrm{T}_{2}=\mathrm{m}_{2} \mathrm{a}$
Block 3: $\mathrm{T}_{2}-\mathrm{m}_{3} \mathrm{~g}=\mathrm{m}_{3} \mathrm{a}$
(i) Adding (1)+(2)+(3) $\rightarrow \mathrm{g}\left(\mathrm{m}_{1}+0.5 \mathrm{~m}_{2}-\mathrm{m}_{3}\right)=\mathrm{a}\left(\mathrm{m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{3}\right) \rightarrow \mathrm{a}=4.41 \mathrm{~m} / \mathrm{s}^{2}$
(ii) $\mathrm{T}_{1}=\mathrm{m}_{1}(\mathrm{~g}-\mathrm{a})=5 \mathrm{~kg}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}-4.41 \mathrm{~m} / \mathrm{s}^{2}\right)=26.95 \mathrm{~N}$
(iii) $\mathrm{T}_{2}=\mathrm{m}_{3}(\mathrm{~g}+\mathrm{a})=2 \mathrm{~kg}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}+4.41 \mathrm{~m} / \mathrm{s}^{2}\right)=28.42 \mathrm{~N}$
(iv) $\mathrm{N}_{2}=\mathrm{Fg}_{2 \mathrm{y}}=\mathrm{m}_{2} \mathrm{gcos} 30^{\circ}=25.46 \mathrm{~N}$

1B. (a) What should be the magnitude of $F$ in the figure below if the body of mass $\mathrm{m}=10 \mathrm{~kg}$ is to slide up along a frictionless incline plane with constant acceleration $a=1.98 \mathrm{~m} / \mathrm{s}^{2}$ ? (b) What is the magnitude of the Normal force?

$$
F \cos 20^{\circ}-m g \sin 30^{\circ}=m a \rightarrow F=\frac{m(a+0.5 g)}{\cos 20^{\circ}}=73.21 \mathrm{~N}
$$

$$
N-m g \cos 30^{\circ}-F \sin 20^{\circ}=0 \rightarrow N=109.9 N
$$



2B. Given the system plotted below, where $m_{1}=2 \mathrm{~kg}$ and $m_{2}=6 \mathrm{~kg}$, calculate the force F necessary to lift up $m_{2}$ with a constant acceleration of $0.2 \mathrm{~m} / \mathrm{s}^{2}$. The pulleys and cords are massless, and the table surface is frictionless.


$$
\begin{aligned}
& d_{1}=\frac{1}{2} a_{1} t^{2} \\
& d_{2}=\frac{d_{1}}{2}=\frac{1}{2} a_{2} t^{2} \rightarrow 0.5 a_{1} t^{2}=a_{2} t^{2} \rightarrow a_{2}=0.5 a_{1}=0.2 \mathrm{~m} / \mathrm{s}^{2} \rightarrow a_{1}=0.4 \mathrm{~m} / \mathrm{s}^{2} \\
& 2 T-m_{2} g=m_{2} a_{2} \rightarrow T=0.5 m_{2}\left(a_{2}+g\right)=0.5(6 \mathrm{~kg})(0.2+9.8) \mathrm{m} / \mathrm{s}^{2}=30 \mathrm{~N} \\
& F-T=m_{1} a_{1} \rightarrow F=T+m_{1} a_{1}=30 \mathrm{~N}+(2 \mathrm{~kg})\left(0.4 \mathrm{~m} / \mathrm{s}^{2}\right)=30.8 \mathrm{~N}
\end{aligned}
$$

## Chapter 5 - Force and Motion II

I. Drag forces and terminal speed.
II. Uniform circular motion.
III. Non-Uniform circular motion.

## I. Drag force and terminal speed

-Fluid: anything that can flow. Example: gas, liquid.
-Drag force: $\vec{D}$

- Appears when there is a relative velocity between a fluid and a body.
- Opposes the relative motion of a body in a fluid.
- Points in the direction in which the fluid flows.

Assumptions:

* Fluid = air.
* Body is blunt (baseball).
* Fast relative motion $\rightarrow$ turbulent air.

$$
\begin{equation*}
D=\frac{1}{2} C \rho A v^{2} \tag{6.3}
\end{equation*}
$$

$\mathrm{C}=\mathrm{drag}$ coefficient (0.4-1).
$\rho=$ air density (mass/volume).


A= effective body's cross sectional area $\rightarrow$ area perpendicular to $\vec{v}$
-Terminal speed: $\mathbf{v}_{\mathrm{t}}$

- Reached when the acceleration of an object that experiences a vertical movement through the air becomes zero $\rightarrow \mathrm{F}_{\mathrm{g}}=\mathrm{D}$

$$
\begin{equation*}
D-F_{g}=m a \rightarrow \text { if } a=0 \rightarrow \frac{1}{2} C \rho A v^{2}-F_{g}=0 \tag{6.4}
\end{equation*}
$$

$$
v_{t}=\sqrt{\frac{2 F_{g}}{C \rho A}}
$$

## II. Uniform circular motion

-Centripetal acceleration:

$$
\begin{equation*}
a=\frac{v^{2}}{r} \tag{6.5}
\end{equation*}
$$

$\mathrm{v}, \mathrm{a}=$ constant, but direction changes during motion.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing its speed.
-Centripetal force:

$$
\begin{equation*}
F=m \frac{v^{2}}{R} \tag{6.6}
\end{equation*}
$$

$\overrightarrow{\mathrm{a}}, \overrightarrow{\mathrm{F}}$ are directed toward the center of curvature of the particle's path.

## III. Non-Uniform circular motion

- A particle moves with varying speed in a circular path.
- The acceleration has two components: - Radial $\rightarrow a_{r}=v^{2} / R$
- Tangential $\rightarrow a_{t}=d v / d t$
- $a_{t}$ causes the change in the speed of the particle.

$$
a=\sqrt{a_{r}^{2}+a_{t}^{2}}
$$



$$
\vec{a}=\vec{a}_{t}+\vec{a}_{r}=\frac{d|\vec{v}|}{d t} \hat{\theta}-\frac{v^{2}}{r} \hat{r} \quad \sum \vec{F}=\sum \vec{F}_{r}+\sum \vec{F}_{t}
$$

- In uniform circular motion, $\mathrm{v}=\mathrm{constant} \rightarrow \mathrm{a}_{\mathrm{t}}=0 \rightarrow \mathrm{a}=\mathrm{a}_{\mathrm{r}}$

49. A puck of mass $m$ slides on a frictionless table while attached to a hanging cylinder of mass $M$ by a cord through a hole in the table. What speed keeps the cylinder at rest?


$$
\text { For } M \rightarrow T=M g \rightarrow a_{c}=0
$$

$$
\text { For } m \rightarrow T=m \frac{v^{2}}{r} \rightarrow M g=m \frac{v^{2}}{r} \rightarrow v=\sqrt{\frac{M g r}{m}}
$$

Calculate the drag force on a missile 53 cm in diameter cruising with a speed of $250 \mathrm{~m} / \mathrm{s}$ at low altitude, where the density of air is $1.2 \mathrm{~kg} / \mathrm{m}^{3}$.
Assume C=0.75

$$
D=\frac{1}{2} C \rho A v^{2}=0.5 \cdot 0.75 \cdot\left(1.2 \mathrm{~kg} / \mathrm{m}^{3}\right) \cdot \pi \cdot(0.53 \mathrm{~m} / 2)^{2}(250 \mathrm{~m} / \mathrm{s})^{2}=6.2 \mathrm{kN}
$$

32. The terminal speed of a ski diver is $160 \mathrm{~km} / \mathrm{h}$ in the spread eagle position and $310 \mathrm{~km} / \mathrm{h}$ in the nosedive position. Assuming that the diver's drag coefficient C does not change from one point to another, find the ratio of the effective cross sectional area A in the slower position to that of the faster position.

$$
v_{t}=\sqrt{\frac{2 F_{g}}{C \rho A}} \rightarrow \frac{160 \mathrm{~km} / \mathrm{h}}{310 \mathrm{~km} / \mathrm{h}}=\frac{\sqrt{\frac{2 F_{g}}{C \rho A_{E}}}}{\sqrt{\frac{2 F_{g}}{C \rho A_{D}}}}=\frac{\sqrt{A_{D}}}{\sqrt{A_{E}}} \rightarrow \frac{A_{E}}{A_{D}}=3.7
$$

11P. A worker wishes to pile a cone of sand onto a circular area in his yard. The radius of the circle is $R$, and no sand is to spill into the surrounding area. If $\mu_{s}$ is the static coefficient of friction between each layer of sand along the slope and the sand beneath it (along which it might slip), show that the greatest volume of sand that can be stored in this manner is $\pi \mu s R^{3} / 3$. (The volume of a cone is Ah/3, where A is the base area and $h$ is the cone's height).

- To pile the most sand without extending the radius, sand is added to make the height " $h$ " as great as possible.
- Eventually, the sides become so steep that sand at the surface begins to slip.

Goal: find the greatest height (greatest slope) for which the sand does not slide


Cross section of sand's cone
Static friction $\rightarrow$ grain does not move


$$
\begin{aligned}
& N=F_{g y}=m g \cos \theta \\
& f=F_{g x}=m g \sin \theta
\end{aligned}
$$

If grain does not slide $\downarrow$

$$
F_{g x}=m g \sin \theta \leq f_{s, \max }=\mu_{s} N=\mu_{s} m g \cos \theta \rightarrow \mu_{s} \geq \tan \theta
$$

The surface of the cone has the greatest slope and the height of the cone is maximum if :

$$
\begin{aligned}
& \mu_{s}=\tan \theta=\frac{h}{R} \rightarrow h=R \mu_{s} \\
& V_{\text {cone }}=\frac{A \cdot h}{3}=\frac{\pi R^{2}\left(R \mu_{s}\right)}{3}=\frac{\pi \mu_{s} R^{3}}{3}
\end{aligned}
$$

21. Block B weighs 711 N . The coefficient of static friction between the block and the table is 0.25 ; assume that the cord between $B$ and the knot is horizontal. Find the maximum weight of block $\mathbf{A}$ for which the system will be stationary.

$$
\text { System stationary } \rightarrow f_{s, \max }=\mu_{s} N
$$

Block $B \rightarrow N=m_{B} g$

$$
T_{1}-f_{s, \max }=0 \rightarrow T_{1}=0.25 \cdot 711 \mathrm{~N}=177.75 \mathrm{~N}
$$

Knot $\rightarrow T_{1}=T_{2 x}=T_{2} \cos 30^{\circ} \rightarrow T_{2}=\frac{177.75 \mathrm{~N}}{\cos 30^{\circ}}=205.25 \mathrm{~N}$

$$
T_{2 y}=T_{2} \sin 30^{\circ}=T_{3}
$$

Block $A \rightarrow T_{3}=m_{A} g=T_{2} \sin 30^{\circ}=0.5 \cdot 205.25 \mathrm{~N}=102.62 \mathrm{~N}$


23 P. Two blocks of weights 3.6 N and 7.2 N , are connected by a massless string and slide down a $30^{\circ}$ inclined plane. The coefficient of kinetic friction between the lighter block and the plane is 0.10; that between the heavier block and the plane is 0.20 . Assuming that the lighter block leads, find (a) the magnitude of the acceleration of the blocks and (b) the tension in the string. (c) Describe the motion if, instead, the heavier block leads.


Light block A leads


## Light block A leads

$$
\begin{aligned}
& \text { Block } A \rightarrow N_{A}=F_{g y A}=m_{A} g \cos 30^{\circ}=3.12 \mathrm{~N} \\
& \\
& f_{k A}=\mu_{k A} N_{A}=(0.1)(3.12 \mathrm{~N})=0.312 \mathrm{~N} \\
& F_{g \times A}-f_{k A}-T=m_{A} a \rightarrow(3.6 \mathrm{~N}) \sin 30^{\circ}-0.312 \mathrm{~N}-T=0.37 a \rightarrow 1.49-T=0.37 \mathrm{a} \\
& \text { Block } B \rightarrow N_{B}=F_{g y B}=m_{B} g \cos 30^{\circ}=6.23 \mathrm{~N} \\
& \quad f_{k B}=\mu_{k B} N_{B}=(0.2)(6.23 \mathrm{~N})=1.25 \mathrm{~N} \\
& F_{g \times B}+T-f_{k B}=m_{B} a \rightarrow(7.2 N) \sin 30^{\circ}+T-1.25 \mathrm{~N}=0.73 a \rightarrow 2.49 \mathrm{~m} / \mathrm{s}^{2} \\
& \\
& \\
& T=\left(\frac{W_{A} W_{B}}{W_{A}+W_{B}}\right)\left(\mu_{k B}-\mu_{k A}\right) \cos \theta=0.2 \mathrm{~N}
\end{aligned}
$$

## Heavy block B leads

Reversing the blocks is equivalent to switching the labels. This would give $\mathrm{T} \sim\left(\mu_{\mathrm{kA}}-\mu_{\mathrm{kB}}\right)<0$ impossible!!!
The above set of equations is not valid in this circumstance $\rightarrow a_{A} \neq a_{B} \rightarrow$ The blocks move independently from each other.
74. A block weighing 22 N is held against a vertical wall by a horizontal force F of magnitude 60 N . The coefficient of static friction between the wall and the block is 0.55 and the coefficient of kinetic friction between them is 0.38 . A second $\mathbf{P}$ acting parallel to the wall is applied to the block. For the following magnitudes and directions of P, determine whether the block moves, the direction of motion, and the magnitude and direction of the frictional force acting on the block: (a) 34 N up (b) 12 N up, (c) 48 N up, (d) 62 N up, (e) 10 N down, (f) 18 N down.
(a) $P=34 N$, up

(b) $P=12 N$, up

$$
\begin{array}{ll}
\mathbf{f} \left\lvert\, \begin{array}{ll}
\mathbf{P} & P+f-m g=m a=0 \\
& f=22 N-12 N=10 N \text { up } \\
& f<f_{s, \max }=33 N \rightarrow \text { Not moving }
\end{array}\right.
\end{array}
$$

(c) $P=48 \mathrm{~N}$, up

(d) $P=62 N$, up


$$
\begin{aligned}
& P-f-m g=0\left(^{*}\right) \rightarrow f=62 N-22 N=40 N \text { up } \\
& f>f_{s, \max }=33 N \rightarrow \text { Block moves up } \rightarrow \text { Assumption }\left(^{*}\right) \text { wrong } \\
& \rightarrow P-f-m g=m a \text { with } f=f_{k}=22.8 N \text { down }
\end{aligned}
$$

(e) $P=10 \mathrm{~N}$, down

$$
\begin{array}{rl}
f & f-P-m g=m a=0 \\
\mathbf{2 2 N} & f=22 N+12 N=32 N \text { up } \\
f<f_{s, \max }=33 N \rightarrow \text { Not moving }
\end{array}
$$

## (f) $\mathrm{P}=18 \mathrm{~N}$, down

$$
\begin{aligned}
& f-P-m g=m a=0 \\
& f=18 N+22 N=40 N \text { up } \\
& f>f_{s, \max }=33 N \rightarrow \text { moves } \\
& f=f_{k}=22.8 N \text { up }
\end{aligned}
$$

28. Blocks A and B have weights of 44 N and 22 N , respectively. (a) Determine the minimum weight of block $C$ to keep $A$ from sliding if $\mu_{s}$ between $A$ and the table is 0.2 . (b) Block $C$ suddenly is lifted of A. What is the acceleration of block $A$ if $\mu_{k}$ between $A$ and the table is 0.15 ?

(a) $\quad f=f_{s, \max }=\mu_{s} N$

Block $A \rightarrow a=0 \rightarrow T-f_{s, \max }=0 \rightarrow T=\mu_{s} N$
Block $B \rightarrow-T+m_{B} g=0 \rightarrow T=22 N$
(1) + (2) $N=\frac{T}{\mu_{s}}=\frac{22 N}{0.2}=110 \mathrm{~N}$

Blocks $A, B \rightarrow N=W_{A}+W_{C} \rightarrow W_{C}=110 N-44 N=66 N$
(b) C disappears $\rightarrow N=m_{A} g=44 N$

$$
\begin{array}{lll}
T-\mu_{k} N=m_{A} a & & \\
m_{B} g-T=m_{B} a & T-6.6=4.5 a & \rightarrow a=2.3 \mathrm{~m} / \mathrm{s}^{2} \\
& 22-T=2.2 a & \rightarrow T \approx 17 \mathrm{~N}
\end{array}
$$

29. The two blocks (with $m=16 \mathrm{~kg}$ and $\mathrm{m}=88 \mathrm{~kg}$ ) shown in the figure below are not attached. The coefficient of static friction between the blocks is: $\boldsymbol{\mu}_{\mathrm{s}}=\mathbf{0 . 3 8}$ but the surface beneath the larger block is frictionless. What is the minimum value of the horizontal force F required to keep the smaller block from slipping down the larger block?


Movement
$F_{\text {min }}$ required to keep $m$ from sliding down?
Treat both blocks as a single system sliding across a frictionless floor

$$
\begin{align*}
F=m_{\text {total }} a \rightarrow a= & \frac{F}{m+M} \\
\text { Small block } \rightarrow F-F^{\prime}= & m a=m\left(\frac{F}{m+M}\right)  \tag{1}\\
& f_{s}-m g=0 \rightarrow \mu_{s} F^{\prime}-m g=0 \tag{2}
\end{align*}
$$

(1) + (2) $\quad \mu_{s} M\left(\frac{F}{m+M}\right)=m g \rightarrow F=\frac{m g}{\mu_{s}}\left(\frac{m+M}{M}\right)=488 \mathrm{~N}$
44. An amusement park ride consists of a car moving in a vertical circle on the end of a rigid boom of negligible mass. The combined weigh of the car and riders is 5 kN , and the radius of the circle is 10 m . What are the magnitude and the direction of the force of the boom on the car at the top of the circle if the car's speed is (a) $5 \mathrm{~m} / \mathrm{s}$ (b) $12 \mathrm{~m} / \mathrm{s}$ ?


The force of the boom on the car is capable of pointing any direction

$$
F_{B}-W=m\left(-\frac{v^{2}}{R}\right) \rightarrow F_{B}=W\left(1-\frac{v^{2}}{R g}\right)
$$

$$
\text { (a) } v=5 \mathrm{~m} / \mathrm{s} \rightarrow F_{B}=3.7 \mathrm{~N} \text { up } \quad \text { (b) } v=12 \mathrm{~m} / \mathrm{s} \rightarrow F_{B}=-2.3 \text { down }
$$

