MECHANICS → Kinematics

Chapter 2 - Motion along a straight line

I. Position and displacement

II. Velocity

III. Acceleration

IV. Motion in one dimension with constant acceleration

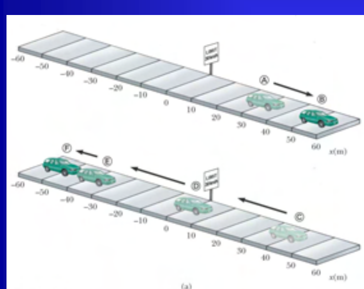
V. Free fall

Particle: point-like object that has a mass but infinitesimal size.

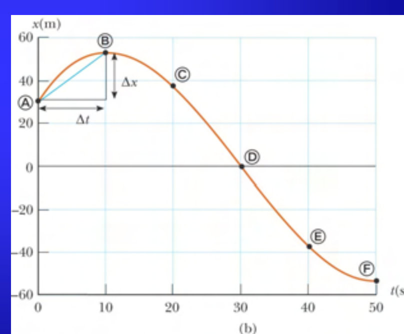
I. Position and displacement

Position: Defined in terms of a frame of reference: x or y axis in 1D.

- The object's position is its location with respect to the frame of reference.



Position-Time graph: shows the motion of the particle (car).



The smooth curve is a guess as to what happened between the data points.

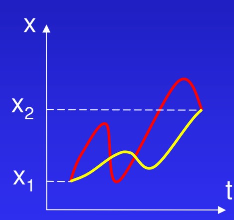
I. Position and displacement

Displacement: Change from position x_1 to x_2 → during a time interval.

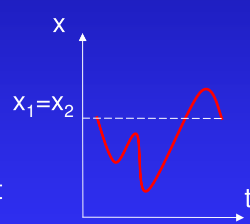
$$\Delta x = x_2 - x_1 \qquad (2.1)$$

- Vector quantity: Magnitude (absolute value) and direction (sign).

- Coordinate (position) ≠ Displacement → $x \neq \Delta x$

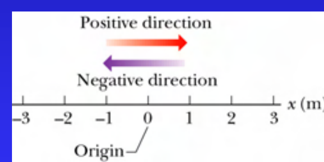


$\Delta x > 0$



$\Delta x = 0$

Coordinate system



Only the initial and final coordinates influence the displacement → many different motions between x_1 and x_2 give the same displacement.

Distance: length of a path followed by a particle.

- Scalar quantity

Displacement ≠ Distance

Example: round trip house-work-house → distance traveled = 10 km
displacement = 0

Review:

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them.

  - We will use + and – signs to indicate vector directions.

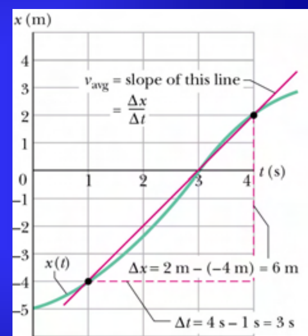
- Scalar quantities are completely described by magnitude only.

II. Velocity

Average velocity: Ratio of the displacement Δx that occurs during a particular time interval Δt to that interval.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (2.2)$$

Motion along x-axis



-Vector quantity → indicates not just how fast an object is moving but also in which direction it is moving.

- SI Units: m/s

- Dimensions: Length/Time [L]/[T]

- The slope of a straight line connecting 2 points on an x-versus-t plot is equal to the average velocity during that time interval.

Average speed: Total distance covered in a time interval.

$$S_{avg} = \frac{\text{Total distance}}{\Delta t} \qquad (2.3)$$

$S_{avg} \neq$ magnitude V_{avg}

S_{avg} always >0

Scalar quantity

Same units as velocity

Example: A person drives 4 mi at 30 mi/h and 4 mi and 50 mi/h → Is the average speed >,<,= 40 mi/h ? <40 mi/h

t_1= 4 mi/(30 mi/h)=0.13 h ; t_2= 4 mi/(50 mi/h)=0.08 h → t_{tot}= 0.213 h

→ S_{avg}= 8 mi/0.213h = 37.5mi/h

Instantaneous velocity: How fast a particle is moving at a given instant.

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.4)$$

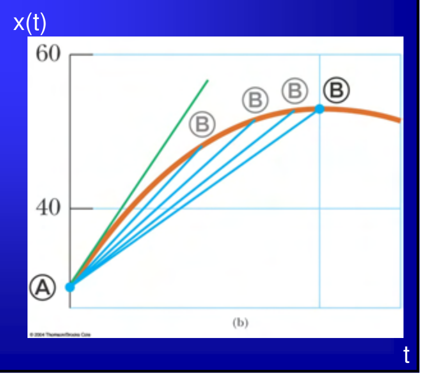
- Vector quantity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero.

- The instantaneous velocity indicates what is happening at every point of time.

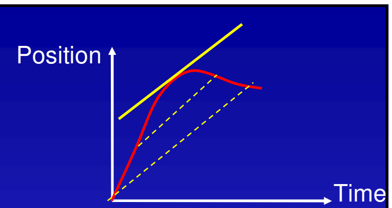
- Can be positive, negative, or zero.

- The instantaneous velocity is the slope of the line tangent to the x vs. t curve (green line).



Instantaneous velocity:

Slope of the particle's position-time curve at a given instant of time. V is tangent to x(t) when $\Delta t \rightarrow 0$



When the velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.

Instantaneous speed: Magnitude of the instantaneous velocity.

Example: car speedometer.

- Scalar quantity

Average velocity (or average acceleration) always refers to an specific time interval.

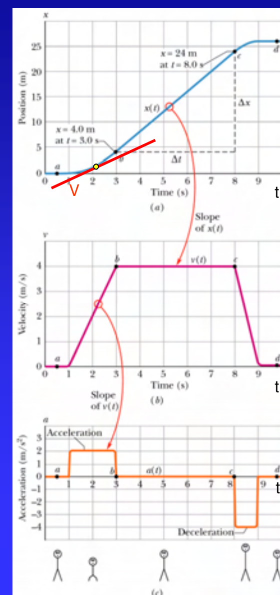
Instantaneous velocity (acceleration) refers to an specific instant of time.

III. Acceleration

Average acceleration: Ratio of a change in velocity Δv to the time interval Δt in which the change occurs.

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad (2.5)$$

- Vector quantity
- Dimensions $[L]/[T]^2$, Units: m/s^2
- The average acceleration in a "v-t" plot is the slope of a straight line connecting points corresponding to two different times.

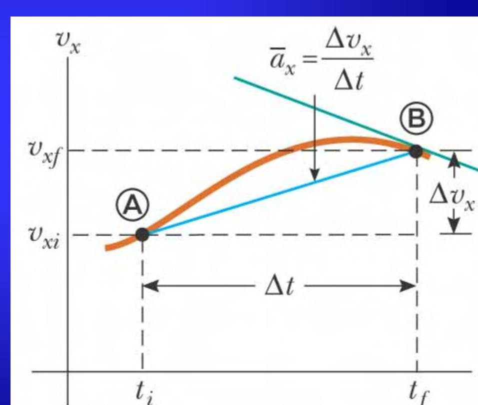


Instantaneous acceleration: Limit of the average acceleration as Δt approaches zero.

- Vector quantity

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2} \qquad (2.6)$$

- The instantaneous acceleration is the slope of the tangent line (v-t plot) at a particular time. (green line in B)
- Average acceleration: blue line.
- When an object's velocity and acceleration are in the same direction (same sign), the object is speeding up.
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down.



- *Positive acceleration does not necessarily imply speeding up, and negative acceleration slowing down.*

Example (1): v_1= -25m/s ; v_2= 0m/s in 5s → particle slows down, a_{avg}= 5m/s^2

- *An object can have simultaneously v=0 and a≠0*

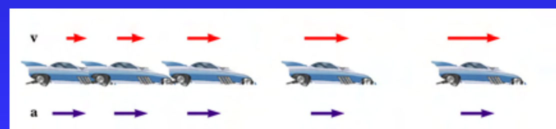
Example (2): $x(t)=At^2$ → v(t)=2At → a(t)=2A ; At t=0s, v(0)=0 but a(0)=2A

Example (3):

- The car is moving with constant positive velocity (red arrows maintaining same size) → Acceleration equals zero.

Example (4):

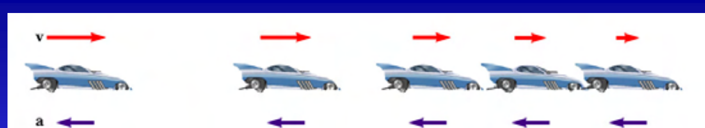
+ acceleration
+ velocity



- Velocity and acceleration are in the same direction, "a" is uniform (blue arrows of same length) → Velocity is increasing (red arrows are getting longer).

Example (5):

- acceleration
+ velocity



- Acceleration and velocity are in opposite directions.
- Acceleration is uniform (blue arrows same length).
- Velocity is decreasing (red arrows are getting shorter).

IV. Motion in one dimension with constant acceleration

- Average acceleration and instantaneous acceleration are equal.

$$a = a_{avg} = \frac{v - v_0}{t - 0}$$

- Equations for motion with constant acceleration:

$$v = v_0 + at \qquad (2.7)$$

$$v_{avg} = \frac{x - x_0}{t} \rightarrow x = x_0 + v_{avg}\, t \qquad (2.8)$$

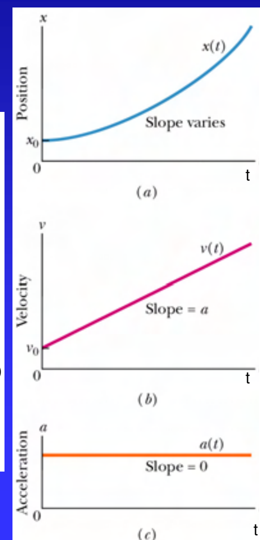
$$v_{avg} = \frac{v_0 + v}{2} \quad and \quad (2.7) \rightarrow v_{avg} = v_0 + \frac{at}{2} \qquad (2.9)$$

$$(2.8), (2.9) \rightarrow x - x_0 = v_0 t + \frac{at^2}{2} \qquad (2.10)$$

$$(2.7), (2.10) \rightarrow v^2 = v_0^2 + a^2t^2 + 2a(v_0 t) = v_0^2 + a^2t^2 + 2a(x - x_0 - \frac{at^2}{2})$$

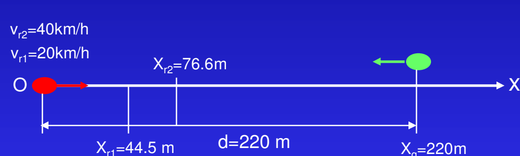
$$\rightarrow v^2 = v_0^2 + 2a(x - x_0) \qquad (2.11)$$

t missing



PROBLEMS - Chapter 2

P1. A red car and a green car move toward each other in adjacent lanes and parallel to The x-axis. At time t=0, the red car is at x=0 and the green car at x=220 m. If the red car has a constant velocity of 20km/h, the cars pass each other at x=44.5 m, and if it has a constant velocity of 40 km/h, they pass each other at x=76.6m. What are (a) the initial velocity, and (b) the acceleration of the green car?



$$\left(40\frac{km}{h}\right)\cdot\left(\frac{1h}{3600s}\right)\cdot\left(\frac{10^3 m}{1km}\right) = 11.11 m/s$$

$$x_r = x_{r0} + v_r t \qquad (1)$$

$$x_g = x_{g0} + v_{g0} t + \frac{1}{2} a t^2 \qquad (2)$$

$$x_{r1} = v_{r1} t_1 \rightarrow t_1 = \frac{44.5m}{5.55 m/s} = 8s$$

$$x_{r2} = v_{r2} t_2 \rightarrow t_2 = \frac{76.6m}{11.11 m/s} = 6.9s$$

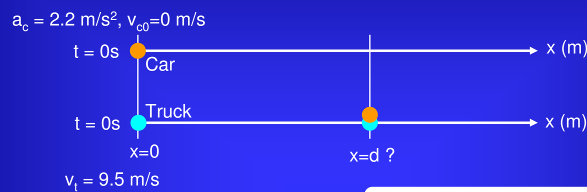
$$x_{r2} - x_g = \square v_{g0} t_2 \square 0.5 \cdot a_g t_2^2 \rightarrow 76.6 - 220 = -v_{g0} \cdot (6.9s) - 0.5 \cdot (6.9s)^2 a_g$$

$$x_{r1} - x_g = \square v_{g0} t_1 \square 0.5 \cdot a_g t_1^2 \rightarrow 44.5 - 220 = -v_{g0} \cdot (8s) - 0.5 \cdot (8s)^2 a_g$$

The car moves to the left (-) in my reference system → a<0, v<0

a_g = 2.1 m/s²
v_{0g} = 13.55 m/sc

P2: At the instant the traffic light turns green, an automobile starts with a constant acceleration *a* of 2.2 m/s^2. At the same instant, a truck, traveling with constant speed of 9.5 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be traveling at that instant?



$x_T = d = v_T t = 9.5\ t \rightarrow$ (1) Truck

$x_c = d = v_{C0} t + \frac{1}{2} a_c t^2 \rightarrow d = 0 + 0.5 \cdot (2.2 m/s^2) \cdot t^2 = 1.1 t^2$ (2) Car

$(a)\ 9.5 \cdot t = 1.1 \cdot t^2 \rightarrow t = 8.63\ s \rightarrow d = (9.5 m/s)(8.63 s) \approx 82 m$

$(b)\ v_f^2 = v_0^2 + 2 \cdot a_c \cdot d = 2 \cdot (2.2 m/s^2) \cdot (82 m) \rightarrow v_f = 19 m/s$

P3: A proton moves along the x-axis according to the equation: x = 50t+10t^2, where *x* is in meters and *t* is in seconds. Calculate (a) the average velocity of the proton during the first 3s of its motion.

$$v_{avg} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80\ \mathrm{m/s}.$$

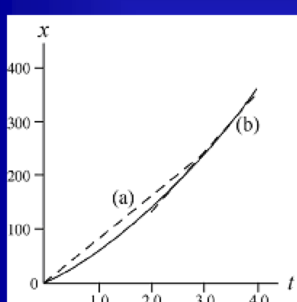
(b) Instantaneous velocity of the proton at t = 3s. $v(t) = \frac{dx}{dt} = 50 + 20\ t \rightarrow v(3s) = 50 + 20 \cdot 3 = 110\ m/s$

(c) Instantaneous acceleration of the proton at t = 3s. $a(t) = \frac{dv}{dt} = 20\ m/s^2 = a(3s)$

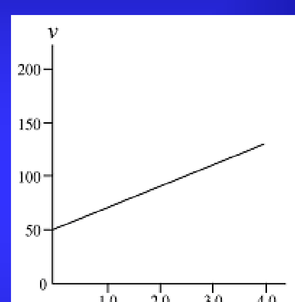
(d) Graph x versus t and indicate how the answer to (a) (average velocity) can be obtained from the plot.

(e) Indicate the answer to (b) (instantaneous velocity) on the graph.

(f) Plot v versus t and indicate on it the answer to (c).



x = 50t + 10t^2



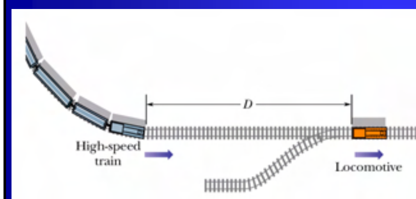
v = 50 + 20t

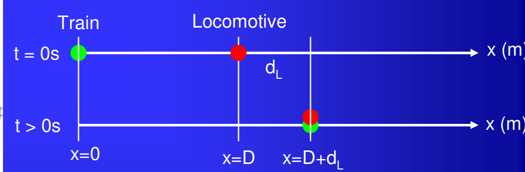
P4. An electron moving along the x-axis has a position given by: x = 16t·exp(-t) m, where t is in seconds. How far is the electron from the origin when it momentarily stops?

x(t) when v(t)=0??

$\frac{dx}{dt} = v = 16e^{-t} - 16te^{-t} = 16e^{-t}(1-t)$ ➡ $v = 0 \rightarrow (1-t) = 0;\ (e^{-t} > 0) \rightarrow t = 1s$ ➡ $x(1) = 16/e = 5.9m$

P5. When a high speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered into the track from a siding and is a distance D= 676 m ahead. The locomotive is moving at 29 km/h. The engineer of the high speed train immediately applies the brakes. (a) What must be the magnitude of the resultant deceleration if a collision is to be avoided? (b) Assume that the engineer is at x=0 when at t=0 he first spots the locomotive. Sketch x(t) curves representing the locomotive and high speed train for the situation in which a collision is just avoided and is not quite avoided.





v_T=161km/h = 44.72 m/s = v_{T0} → 1D movement with a<0=cte

v_L=29 km/h = 8.05 m/s is constant

$d_L = v_L t = 8.05\ t \rightarrow t = \frac{d_L}{8.05}$ (1) Locomotive

$d_L + D = v_{T0} t + \frac{1}{2} a_T t^2 \rightarrow d_L + 676 = 44.72\ t + \frac{1}{2} a_T t^2$ (2) Train

P5.

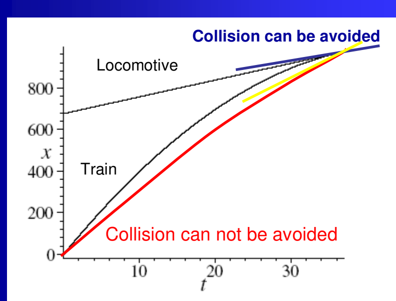
$$v_{Tf} = v_{T0} + a_T t = 0 \rightarrow a_T = \frac{-44.72 m/s}{t} = (eq.\ 1) = \frac{(-44.72 m/s)(8.05 m/s)}{d_L} = \frac{-360 m^2/s^2}{d_L} \quad (3)$$

$$v^2_{Tf} = v^2_{T0} + 2a_T(D + d_L) = 0 \rightarrow a_T = \frac{-(44.72 m/s)^2}{2(676 m + d_L)} \quad (4)$$

$$(3) = (4) \rightarrow d_L = 380.3 m$$

$$from\ (1) \rightarrow t = \frac{d_L}{8.05} = 47.24 s$$

$$(1)+(3) \rightarrow a_T = \frac{-360 m^2/s^2}{380.3 m} = -0.947 m/s^2$$



$x_L = 676 + 8.05\, t$

$x_T = 44.72\, t + 0.5\, a_T t^2$

- Collision can be avoided:

Slope of x(t) vs. t locomotive at t = 47.24 s (the point were both Lines meet) = v instantaneous locom > Slope of x(t) vs. t train

- Collision cannot be avoided:

Slope of x(t) vs. t locomotive at t = 47.24 s < Slope of x(t) vs. t train

- The motion equations can also be obtained by indefinite integration:

$$dv = a\ dt \rightarrow \int dv = \int a\ dt \rightarrow v = at + C; \qquad v = v_0 \quad at \quad t = 0 \rightarrow v_0 = (a)(0) + C \rightarrow v_0 = C \rightarrow v = v_0 + at$$

$$dx = v\ dt \rightarrow \int dx = \int v\ dt \rightarrow \int dx = \int (v_0 + at)dt \rightarrow \int dx = v_0 \int dt + a \int t\ dt \rightarrow x = v_0 t + \frac{1}{2} at^2 + C';$$

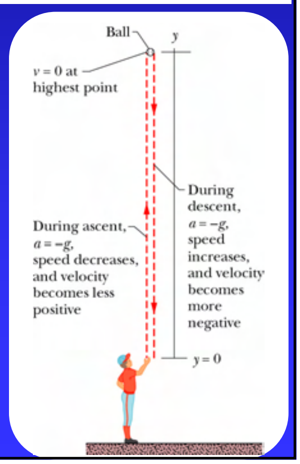
$$x = x_0 \quad at \quad t = 0 \rightarrow x_0 = v_0(0) + \frac{1}{2} a(0) + C' \rightarrow x_0 = C' \rightarrow x = x_0 + v_0 t + \frac{1}{2} at^2$$

V. Free fall

Motion direction along y-axis (y >0 upwards)

Free fall acceleration: (near Earth's surface)
a= -g = -9.8 m/s^2 (in mov. eqs. with constant acceleration)

Due to gravity → downward on y, directed toward Earth's center



Approximations:

- Locally, Earth's surface essentially flat → free fall "a" has same direction at slightly different points.

- All objects at the same place have same free fall "a" (neglecting air influence).

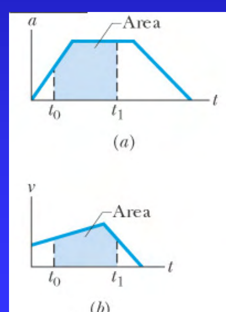
VI. Graphical integration in motion analysis

From a(t) versus t graph → integration = area between acceleration curve and time axis, from t_0 to t_1 → v(t)

$$v_1 - v_0 = \int_{t_0}^{t_1} a \cdot dt$$

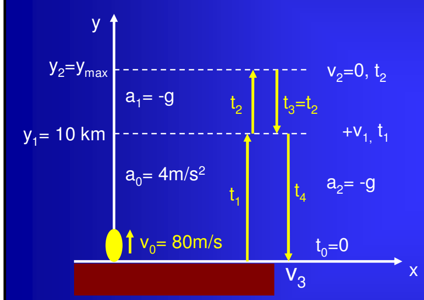
Similarly, from v(t) versus t graph → integration = area under curve from t_0 to t_1 → x(t)

$$x_1 - x_0 = \int_{t_0}^{t_1} v \cdot dt$$



P6: A rocket is launched vertically from the ground with an initial velocity of 80m/s. It ascends with a constant acceleration of 4 m/s^2 to an altitude of 10 km. Its motors then fail, and the rocket continues upward as a free fall particle and then falls back down.

(a) What is the total time elapsed from takeoff until the rocket strikes the ground?
(b) What is the maximum altitude reached?
(c) What is the velocity just before hitting ground?



1) Ascent → $a_0= 4m/s^2$

$$y_1-y_0=v_0t_1+0.5\cdot a_0t_1^2 \to 10^4=80t_1+2t_1^2 \to \boxed{t_1=53.48s}$$

$$a_0=\frac{v_1-v_0}{t_1}\to v_1=(4m/s^2)\cdot(53.48s)+80m/s=294m/s$$

2) Ascent → a= -9.8 m/s^2

$$a_1=-g=\frac{0-v_1}{t_2}\to t_2=\frac{-294m/s}{-9.8m/s^2}=\boxed{29.96s}$$

Total time ascent = t_1+t_2= 53.48 s+29.96 s= 83.44 s

3) Descent → a= -9.8 m/s^2

$$0-y_1=-v_1t_4+0.5\cdot a_0t_4^2 \to -10^4=-294t_4-4.9t_4^2 \to \boxed{t_4=24.22s}$$

$t_{total}=t_1+2t_2+t_4$= 53.48 s + 2·29.96 s + 24.22 s= $\boxed{137.62\ s}$

$h_{max}= y_2$ → y_2-10^4 m = $v_1t_2-4.9t_2^2$= (294 m/s)(29.96s)-(4.9m/s^2)$(29.96s)^2$ = 4410 m → $\boxed{h_{max}=14.4\ km}$

$$a_2=-g=\frac{v_3-(-v_1)}{t_4}\to v_3=-g\cdot t_4-v_1=\boxed{-531.35m/s}$$