PHY 2048-S4, Fall 2009

Examination #3
November 19, 2009
Instructor: Beatriz Roldan Cuenya

Name_______________________________________ID______________

Please answer all questions.

#1______________

#2______________

#3______________

#4______________

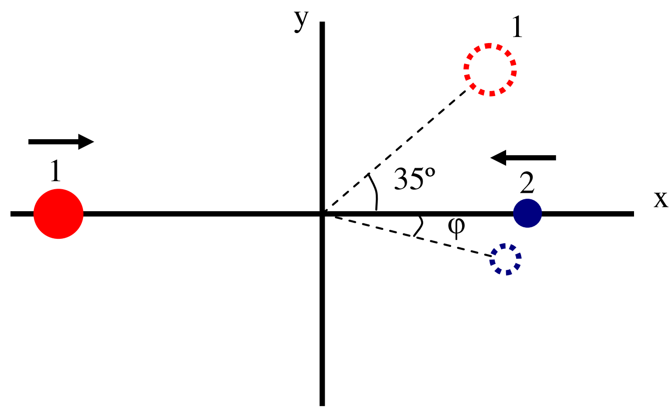
Total: ___________

Show all work and enter answers in boxes, if provided.

1. Two particles of masses $m_1 = 3$ kg and $m_2 = 0.5$ kg slide in opposite directions over a frictionless surface with constant velocities $v_1 = 1.5$ m/s and $v_2 = -5$ m/s, respectively. The particles collide and bounce as it is indicated in the figure below. If the magnitude of the velocity of particle 1 after the collision is 0.4 m/s, determine:

   (a) Velocity (magnitude and direction) of particle 2 after colliding with particle 1. *(10 points)*

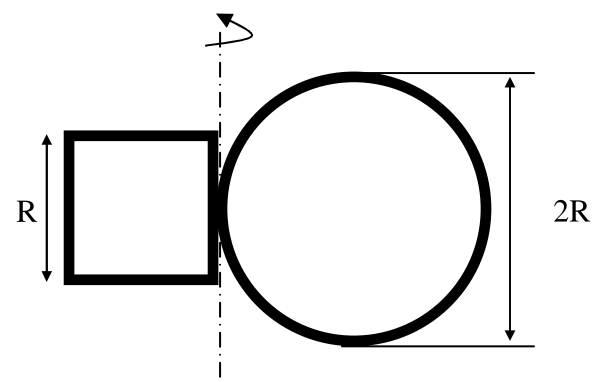
   (b) Kinetic energy lost during the collision. *(10 points)*



(a)	(b)

2. (a) The angular speed of a point on the rim of a rotating wheel is given by: $\omega = bt - ct^2$, where t is in seconds, b= 2 rad/s^2 and c= 5 rad/s^3. Find an expression for the angular acceleration as a function of time. Assume that ω=0 rad/s at t=0 s. *(5 points)*

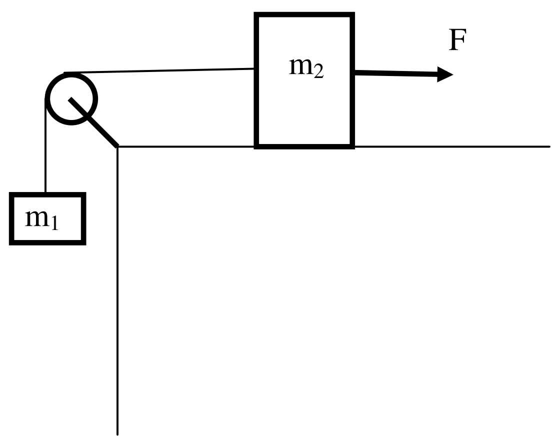
(b) The figure below shows a rigid structure consisting of a circular hoop of radius R and mass "m" and a square made of four thin bars, each of length R and mass "m". The rigid structure rotates at a constant speed about a perpendicular axis at the location shown. Assuming R = 0.2 m and m = 3 kg. Calculate the structure's rotational inertia about the common axis of rotation. The moment of inertia of the hoop about its COM is $I_{hoop} = mR^2$. The moment of inertia of a thin rod about an axis through its COM perpendicular to its length is $I_{rod} = 1/12\ m\ L^2$)
(15 points)



(a)	(b)

3. Two blocks, $m_1 = 0.2$ kg and $m_2 = 7$ kg are connected by a massless string through a pulley of mass M. The rotational inertia of the pulley is $I = ½ MR^2$, with M = 0.3 kg and a radius R= 0.2 m. Block m_2 is pulled rightward by applying a force of 100 N.

(a) Write down, using symbols, the equations of motion of the masses and the pulley. *(5 points)*
(b) Obtain the acceleration of the masses (the same for both masses). *(7.5 points)*
(c) Calculate the forces of tension in the two sides of the pulley. *(7.5 points)*

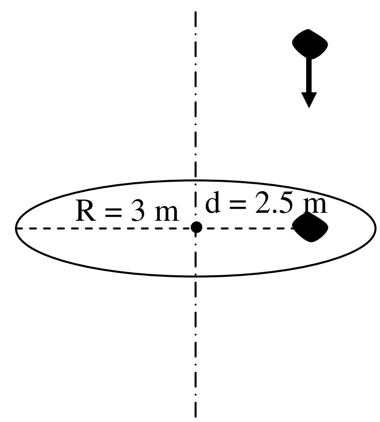


(b) a =	(c) T_1 = T_2 =

4. A uniform circular platform of mass M = 40 kg and radius R = 3 m rotates in a horizontal plane about a vertical axis through its center at 50 rad/s. The moment of inertia of the platform is ½ M R^2. A 8 kg wad of wet putty drops at negligible speed (v_0=0) onto the platform hitting it at a point 2.5 m from the platform's center and then sticking to it.

(a) What is the angular velocity of the platform-putty system immediately after the impact? *(10 points)*

(b) If the platform's rotational axis was located 0.3 m away from its geometrical center, what will be the angular velocity of the system after the impact? *(7.5 points)*



(a)	(b)

Formula sheet **PHY 2048**

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + 0.5\vec{a}t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\vec{F} = m \cdot \vec{a}$$

$$F_s = -kx$$

$$a_c = \frac{v^2}{r}$$

$$E_{mec} = K + U$$

$$W = \vec{F} \cdot \vec{d}$$

$$W = \int_{x1}^{x2} F(x)dx$$

$$W_{net} = \Delta K$$

$$W_{net} = -\Delta U$$

$$F = -\frac{dU(x)}{dx}$$

$$\Delta E = \Delta K + \Delta U$$

$$\Delta E = \Delta K + \Delta U + \Delta E_{th}$$

$$\Delta E_{th} = f_k d$$

$$K = \frac{1}{2}mv^2$$

$$U(y) = mgy$$

$$U(x) = \frac{1}{2}kx^2$$

$$\vec{p} = m\vec{v}$$

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{ext} = M\vec{a}_{COM}$$

$$\vec{a}_{COM} = \frac{d^2\vec{R}_{COM}}{dt^2}$$

$$\vec{R}_{COM} = \frac{1}{M_{tot}}\sum_i m_i \vec{r}_i$$

$$\vec{R}_{COM} = \frac{1}{M_{tot}}\int_V \vec{r}\, dm$$

$$\tau = I\alpha = r_\perp F = rF_\perp$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$K_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$I = \sum_i m_i r^2$$

$$I = \int r^2 dm$$

$$s = \theta \cdot r$$

$$\omega = \omega_0 + \alpha \cdot t$$

$$a_t = \alpha \cdot r$$

$$v = \omega \cdot r$$

$$I = I_{COM} + Mh^2$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$I_{disk}(COM) = \frac{1}{2}mR^2 = I_{cylinder}(COM)$$

$$I_{ring}(COM) = mR^2$$

$$I_{rod}(COM) = \frac{1}{12}ML^2$$

$$I_{sphere}(COM) = \frac{2}{5}mR^2$$

$$I_m = md^2$$

$$Rv_{rel} = Ma$$

$$v_f - v_i = v_{rel}\ln\frac{M_i}{M_f}$$

$$L = I \cdot \omega$$

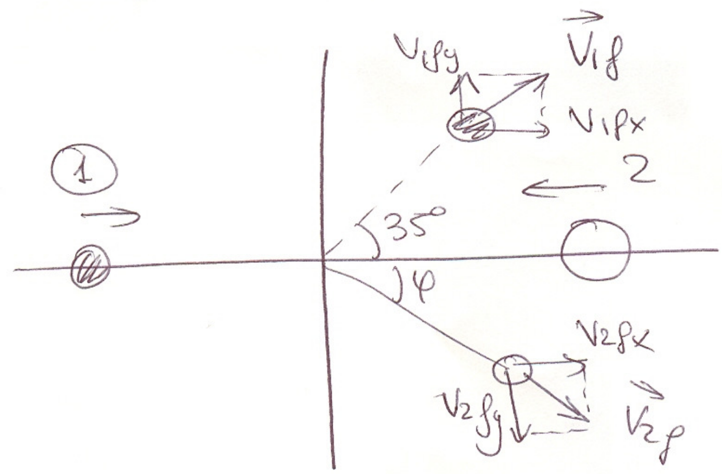
$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

MIDTERM 3

(1) $m_1 = 3\,kg$ $\quad V_{1ix} = 1.5\,m/s$ $\quad V_{1f} = 0.4\,m/s$

$m_2 = 0.5\,kg$ $\quad V_{2ix} = (-)5\,m/s$ $\quad V_{2f}\,?$



$V_{1fx} = V_{1f}\cos 35°$
$V_{1fy} = V_{1f}\sin 35°$

a) $\vec{P}$ conservation

$\vec{P}_i = \vec{P}_f$

x-AXIS: $m_1 V_{1xi} - m_2 V_{2xi} = m_1 V_{1fx} + m_2 V_{2fx}$

(1) $(3\,kg)(1.5\,m/s) - (0.5\,kg)(5\,m/s) = (3\,kg)(\overset{0.4\,m/s}{V_{1f}}\cos 35°) + (0.5\,kg)\,V_{2fx}$

$\Rightarrow \boxed{V_{2fx} = 2.03\,m/s} = V_{2f}\cos\varphi$

y-AXIS: $0 = m_1 V_{1fy} - m_2 V_{2fy} \Rightarrow 0 = (3\,kg)(0.4\,m/s)(\sin 35°) - (0.5)\,V_{2fy}$

(2) $\Rightarrow \boxed{V_{2fy} = 1.376\,m/s} = V_{2f}\sin\varphi$

$V_{2f} = \sqrt{V_{2fx}^2 + V_{2fy}^2} = \boxed{2.453\,m/s}$

$\tan\varphi = \dfrac{V_{2fy}}{V_{2fx}} \Rightarrow \boxed{\varphi = 34.1°}$

(b) $\Delta K = K_f - K_i = \frac{1}{2}m_1 V_{1f}^2 + \frac{1}{2}m_2 V_{2f}^2 - \frac{1}{2}m_1 V_{1i}^2 - \frac{1}{2}m_2 V_{2i}^2$

$\Delta K = \frac{1}{2}(3\,kg)(0.4\,m/s)^2 + \frac{1}{2}(0.5\,kg)\overset{(2.453\,m/s)^2}{V_{2f}^2} - \frac{1}{2}(3\,kg)(1.5\,m/s)^2 - \frac{1}{2}\overset{(0.5\,kg)}{m_2}(5\,m/s)^2$

$\boxed{\Delta K = -7.9\,J}$

(2) (a) $\omega = bt - ct^2 = 2t - 5t^2$

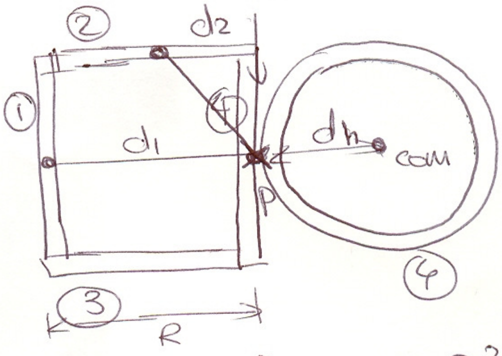
$$\alpha = \frac{d\omega}{dt} = b - 2ct = \boxed{2 - 10t}$$

(b) $R = 0.2\,m$

$m = 3\,kg$

$L = R$

Rotational axis ⊥ paper



$$I_{system_P} = I_{hoop_P} + I_{1P} + I_{2P} + I_{3P} + I_{4P}$$

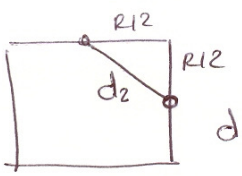
$$I_{hoop_P} = I_{com_{hoop}} + m \cdot d_h^2 = mR^2 + m \cdot R^2 = 2mR^2$$

$$I_{1P} = I_{1com} + m \cdot d_1^2 = \frac{1}{12} m L^2 + m \cdot R^2 = \frac{1}{12} m R^2 + mR^2$$

($L^2 = R^2$)

$$I_{4P} = \frac{1}{12} MR^2$$

$$I_2 = I_3 = I_{com} + m \cdot d_2^2 = \frac{1}{12} mR^2 + m \cdot \frac{R^2}{2}$$

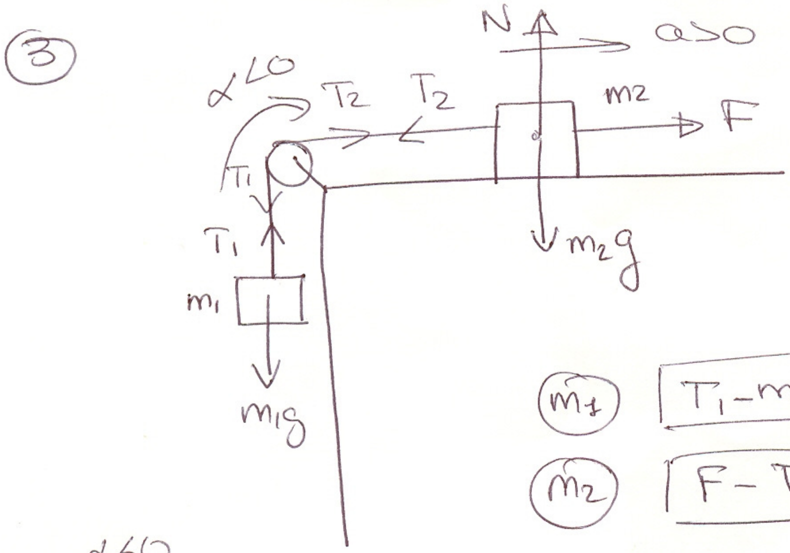


$$d_2^2 = \left(\frac{R}{2}\right)^2 + \left(\frac{R}{2}\right)^2 = 2\frac{R^2}{4} = \frac{R^2}{2} \Rightarrow d_2 = \frac{R}{\sqrt{2}}$$

$$I_{TOTAL} = \underbrace{2mR^2}_{I_{hoop}} + \underbrace{\frac{1}{12} mR^2 + mR^2}_{I_1} + \underbrace{\frac{1}{12} mR^2}_{I_4} + \underbrace{\frac{2}{12} mR^2 + mR^2}_{2I_2}$$

$$I_{TOTAL} = 4mR^2 + \frac{1}{3} mR^2 = \frac{13}{3} mR^2 = \boxed{0.52\ kg\,m^2}$$

③



$m_1 = 0.2\,kg$ $\quad M = 0.3\,kg$

$m_2 = 7\,kg$

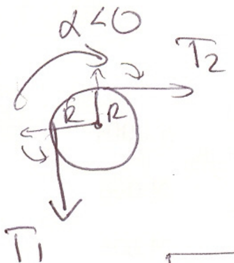
$F = 100\,N$

$I_{disk} = \frac{1}{2} M R^2$

$\alpha = -\frac{a}{R}$

(m_1) $\boxed{T_1 - m_1 g = m_1 a}$ (1)

(m_2) $\boxed{F - T_2 = m_2 a}$ (2)



$\left.\begin{array}{l}\vec{\tau} = \vec{r} \times \vec{F} \\ \tau = I\alpha\end{array}\right\} \Rightarrow$ $-RT_2 + RT_1 = I \cdot \alpha = I \cdot \left(-\frac{a}{R}\right)$

$\not{R}(T_1 - T_2) = -\frac{1}{2} M \not{R}^{\not{2}} \left(\frac{a}{\not{R}}\right)$

$\boxed{T_2 - T_1 = \frac{1}{2} M a}$ (3)

$+$

(1)+(2) $-T_2 + T_1 - m_1 g + F = (m_1 + m_2) a$

$-m_1 g + F = (m_1 + m_2 + 0.5 M) a \Rightarrow a = 13{,}34\ m/s^2$

(1) $T_1 = m_1 (a + g) = \boxed{4.63\,N}$

(2) $T_2 = F - m_2 a = \boxed{6.62\,N}$

(4) $R = 3m$

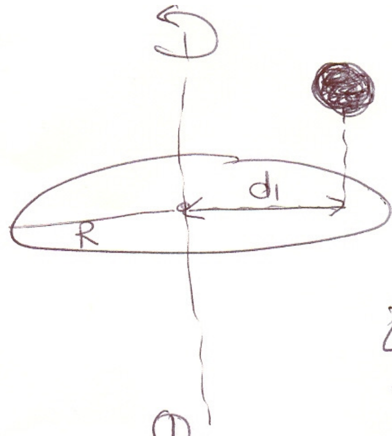
$d_1 = 2.5m$

$\omega_i = 50 \text{ rad/s}$

$M = 40 \text{ kg}$

$m = 8 \text{ kg}$

$I_{disk} = \frac{1}{2} MR^2$



$v_i = 0 \text{ m/s}$

$\vec{L} = \vec{r} \times \vec{p}$

$L = I \cdot \omega$

$\vec{L}$ conservation

$\vec{L_i} = \vec{L_f}$

(a) ω_f ?

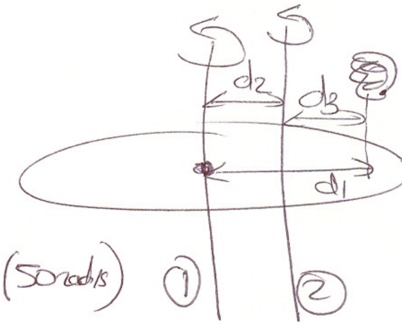
$L_i = I_i \omega_i = \left(\frac{1}{2} MR^2\right) \cdot \omega_i = (0.5)(40 kg)(3m)^2 \cdot (50 \text{ rad/s}) = 9000 \text{ kg m}^2/\text{s}$

$L_f = L_{f\,platform} + L_{f\,putty} = I_f \cdot \omega_f = \omega_f \cdot \left(\frac{1}{2} MR^2 + m \cdot d_1^2\right)$

$L_f = \omega_f \cdot \left(0.5 * 40kg * 3^2 m^2 + 8kg \cdot 2.5^2 m^2\right) = \omega_f (230)$

$L_i = L_f \Rightarrow 9000 = 230\, \omega_f \Rightarrow \omega_f = 39.13 \text{ rad/s}$

(b) New rotational axis (2)



$d_2 = 0.3m$

$L_i = I_i \omega_i = \left(\frac{1}{2} MR^2 + M \cdot d_2^2\right) \omega_i$

$L_i = \left(0.5 * 40kg * 9 m^2 + 40kg \cdot 0.3^2 m^2\right) \cdot (50 \text{ rad/s})$

$L_i = 9180 \frac{\text{kg m}^2}{\text{s}}$

$L_f = I_f \omega_f = \left(\frac{1}{2} MR^2 + M d_2^2 + m d_3^2\right) \omega_f$

$L_f = \left(0.5 * 40kg * 9m^2 + 40kg \cdot (0.3m)^2 + (8kg) \cdot (2.2m)^2\right) \omega_f$

$d_1 = d_2 + d_3$

$d_3 = d_1 - d_2 =$

$d_3 = 2.5m - 0.3m$

$d_3 = 2.2m$

$L_f = 222.32\, \omega_f$

$L_i = 9180 = L_f = 222.32\, \omega_f \Rightarrow \omega_f = 41.29 \text{ rad/s}$