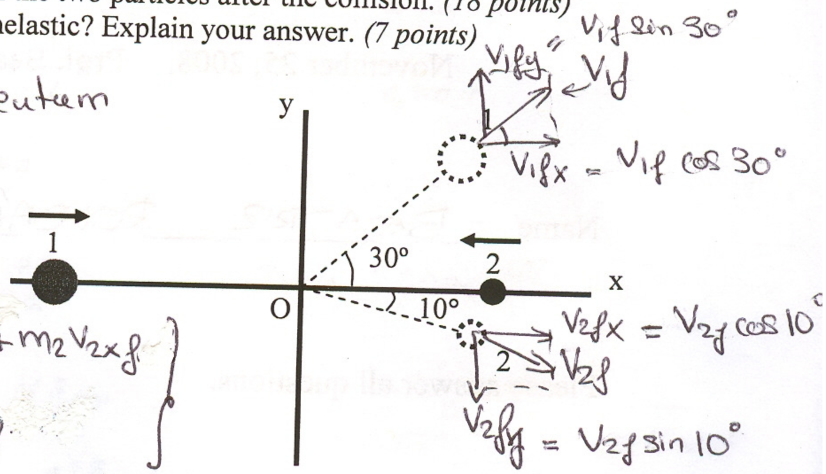
1. Two particles of masses $m_1 = 2$ kg and $m_2 = 0.4$ kg slide in opposite directions over a frictionless surface with initial constant velocities $v_1 = 2$ m/s and $v_2 = 4$ m/s, respectively. The particles collide at point O and bounce as it is indicated in the figure below.

   a) Determine the velocities of the two particles after the collision. *(18 points)*
   b) Is the collision elastic or inelastic? Explain your answer. *(7 points)*

Conservation of linear momentum

$$\vec{P}_i = \vec{P}_f$$



a)

x-Axis: $m_1 V_{1x_i} - m_2 V_{2x_i} = m_1 V_{1xf} + m_2 V_{2xf}$

y-Axis: $0 = m_1 V_{1yf} - m_2 V_{2yf}$

$$(2\,\text{kg})(2\,\tfrac{m}{s}) - (0.4\,\text{kg})(4\,\text{m/s}) = (2\,\text{kg}) V_{1f} \cdot \cos 30° + (0.4\,\text{kg}) V_{2f} \cdot \cos 10°$$

$$0 = (2\,\text{kg}) V_{1f} \sin 30° - (0.4\,\text{kg}) V_{2f} \cdot \sin 10°$$

$$2.4\ \text{kg}\tfrac{m}{s} = 1.73 V_{1f} + 0.39 V_{2f}$$

$$0 = V_{1f} - 0.069 V_{2f} \longrightarrow V_{1f} = 0.069 V_{2f}$$

$$2.4 = 1.73 V_{1f} + 0.39 V_{2f} \Rightarrow 2.4 = 1.73(0.069 V_{2f}) + 0.39 V_{2f}$$

$$\rightarrow \boxed{V_{2f} = 4.71\ \text{m/s}}$$

$$V_{1f} = 0.069 \cdot (4.71\ \text{m/s}) = \boxed{0.325\ \text{m/s}}$$

b) $KE_i = \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = 7.2\ \text{J}$

$KE_f = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 = 0.106\ \text{J} + 4.44\ \text{J} = 4.54\ \text{J}$

(a) v_{1f} = 0.325 m/s v_{2f} = 4.71 m/s	(b) INELASTIC

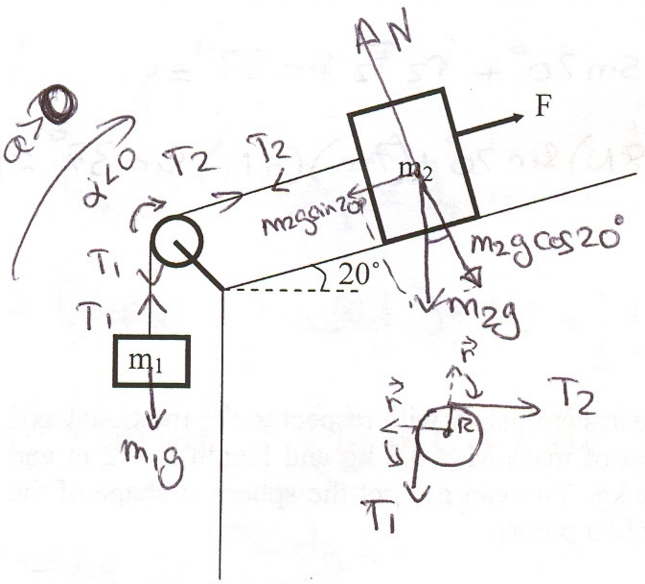
KE not conserved

2.Two blocks, $m_1 = 0.1$ kg and $m_2 = 6$ kg are connected by a massless string through a pulley of mass M. The rotational inertia of the pulley is $I_{COM} = ½ MR^2$, with M = 0.3 kg and a radius R= 0.2 m. Block m_2 lies on a 20° incline plane and is pulled upward by applying a force of 90 N. The incline is frictionless.

a) Write down, using symbols, the equations of motion of the masses and the pulley. *(8 points)*

b) Obtain the acceleration of the masses. *(8 points)*

c) Calculate the forces of tension in the two sides of the pulley. *(9 points)*



$m_1 \Rightarrow \boxed{T_1 - m_1 g = m_1 a}$ (1)

$m_2 \Rightarrow \boxed{F - T_2 - m_2 g \sin 20° = m_2 a}$

$M \Rightarrow \tau_{ext} = I\alpha = I\left(-\frac{a}{R}\right) = -\frac{1}{2}MR$

$\tau_{ext} = -\frac{1}{2}MRa$

$\tau_{ext} = \tau_{T_1} + \tau_{T_2} = RT_1 - RT$

$\Rightarrow -\frac{1}{2}M\cancel{R}a = -\cancel{R}(T_2 - T_1)$

$\Rightarrow \boxed{T_2 - T_1 = \frac{1}{2}Ma}$ (3)

(1) + (2) + (3)

$\cancel{T_1} - m_1 g + F - \cancel{T_2} - m_2 g \sin 20° + \cancel{T_2} - \cancel{T_1} = (m_1 + m_2 + \frac{1}{2}M)a$

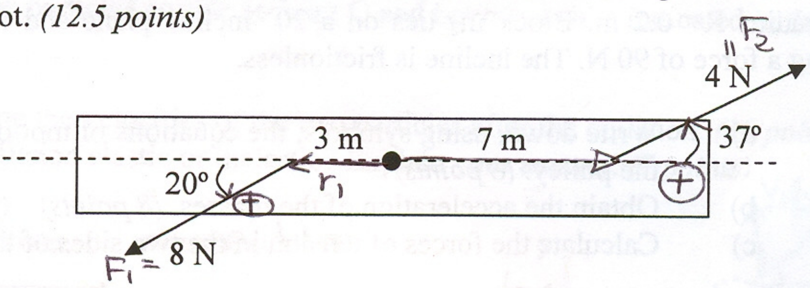
$$\frac{-m_1 g + F - m_2 g \sin 20°}{m_1 + m_2 + \frac{1}{2}M} = a \Rightarrow \boxed{a = 11.02 \text{ m/s}^2}$$

(1) $T_1 = m_1(g + a) = \boxed{2.08 \text{ N}}$

(3) $T_2 = T_1 + \frac{1}{2}Ma = \boxed{3.73 \text{ N}}$

(b) a= 11.02 m/s²	(c) T_1= 2.08 N T_2 = 3.73 N

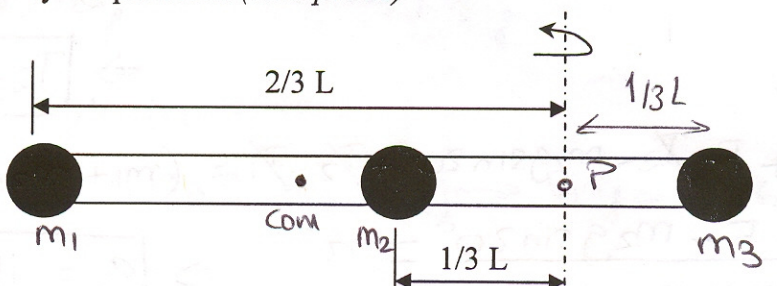
3. (a) A thin rod is pivoted about its center. A 8-N force is applied 3 m from the pivot and a 4-N force is applied 7 m from the pivot, as shown. Calculate the magnitude of the total torque about the pivot. *(12.5 points)*



$$\tau_{net} = \tau_{F_1} + \tau_{F_2} = r_1 \cdot F_1 \cdot \sin 20^\circ + r_2 F_2 \sin 37^\circ =$$

$$= (3m)(8N)\sin 20^\circ + (7m)(4N)\sin 37^\circ = \boxed{25\ Nm}$$

(b) Calculate the moment of inertia of the system below with respect to the rotational axis shown. The system is made of a thin rod of mass M = 0.2 kg and length L = 2 m and three balls, each of them of mass m = 2 kg. You can neglect the spherical shape of the balls and assume that they are particles. *(12.5 points)*



$$I_{TOTAL_P} = I_{ROD_P} + I_{m_1 P} + I_{m_2 P} + I_{m_3 P}$$

$$I_{ROD\,P} = I_{ROD\,COM} + M\cdot\left(\frac{2}{3}L - \frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{6}\right)^2 = \frac{\overset{1/9}{4}}{36}ML^2$$

$$I_{m_1} = m\cdot(2/3\,L)^2 = \frac{4m}{9}L^2$$

$$I_{m_2} = m\left(\frac{1}{3}L\right)^2 = \frac{1}{9}mL^2$$

$$I_{m_3} = I_{m_2}$$

$$I_{TOTAL_P} = \frac{1}{9}ML^2 + \frac{4}{9}mL^2 + \frac{1}{9}mL^2 + \frac{1}{9}mL^2$$

$$= \frac{1}{9}ML^2 + \frac{6}{9}mL^2 = L^2\left(\frac{1}{9}M + \frac{2}{3}m\right)$$

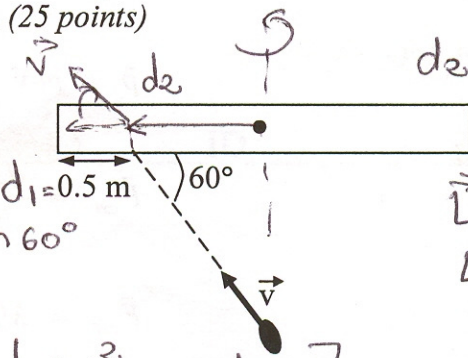
$$= \boxed{5.42\ Kg\,m^2}$$

(a) $\tau = 25\ Nm$	(b) $5.42\ Kg\,m^2$

4. A uniform thin rod of length 2.5 m and mass 1.5 kg can rotate in a horizontal plane about a vertical axis through its center. The rod was initially at rest when a 10 g bullet traveling in the horizontal plane of the rod is fired into the rod. The bullet lodges in the rod at a distance of 0.5 m from its left end. As can be seen in the picture below, the direction of the bullets velocity makes an angle of 60° with the rod. If the velocity of the bullet was 250 m/s before the impact with the rod, determine:

(a) What is the angular velocity of the rod/bullet assembly after the collision? TIP: Think about the conservation of the angular momentum. *(25 points)*

$d_2 = \frac{L}{2} - d_1 = 0.75m$



$L_i = L_f$

$L_i = L_{bullet} + \cancel{L_{ROD}} = -d_2 \cdot m \cdot V \sin 60°$

$\vec{L} = \vec{r} \times \vec{p}$

$L = I\omega$

$L_f = (I_{ROD} + I_{bullet}) \cdot \omega_f = \left[\frac{1}{12}ML^2 + m d_2^2\right] \cdot \omega_f$

$-d_2 m V \sin 60° = \left(\frac{1}{12}ML^2 + md_2^2\right)\omega_f$

$\rightarrow \omega_f = \frac{-d_2 m V \sin 60°}{\frac{1}{12}ML^2 + md_2^2} = \boxed{-2.06 \text{ rad/s}}$ clockwise rotation

(a)	$\omega_f = -2.06$ rad/s