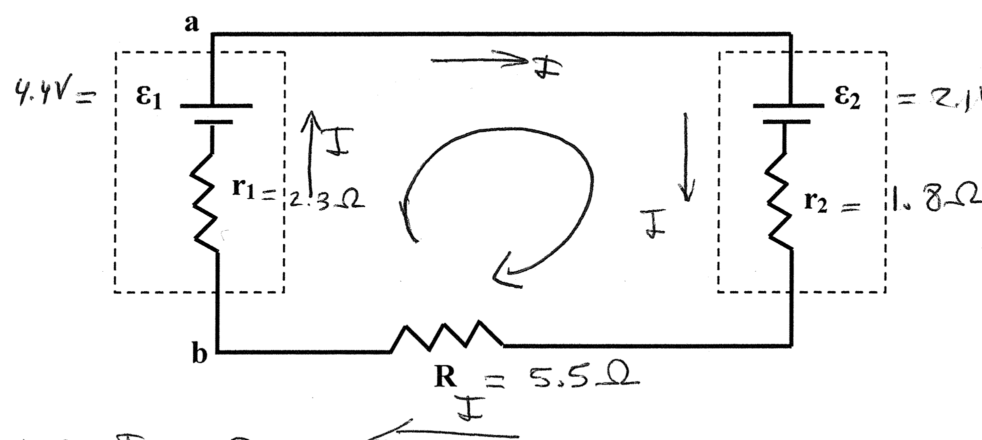
1. The electromotive force sources (emf) and resistances in the circuit below have the following values: $\varepsilon_1 = 4.4$ V, $\varepsilon_2 = 2.1$ V, $r_1 = 2.3\ \Omega$, $r_2 = 1.8\ \Omega$, $R = 5.5\ \Omega$. r_1 and r_2 are the internal resistances of batteries 1 and 2. Calculate:

(25 points)

(a) the current in the circuit. *(8 points)*
(b) the potential difference between the terminals of battery 1. *(6 points)*
(c) power dissipated by all resistors in the circuit. *(6 points)*



(a) $\mathcal{E}_1 - \mathcal{E}_2 - I r_2 - I R - I r_1 = 0$

$4.4V - 2.1V - I\,(1.8\Omega + 2.3\Omega + 5.5\Omega) = 0$

$\boxed{I = 0.24\ A}$

(b) $V_{ab} = V_a - V_b = \mathcal{E}_1 - r_1 \cdot I = 4.4V - (2.3\Omega)(0.24\,A)$

$\boxed{V_{ab} = 3.8V}$

(c) $P = I^2 \cdot R = I^2 (r_1 + r_2 + R) = (0.24\,A)^2 (2.3\Omega + 1.8\Omega + 5.5\Omega) = \boxed{0.55W}$

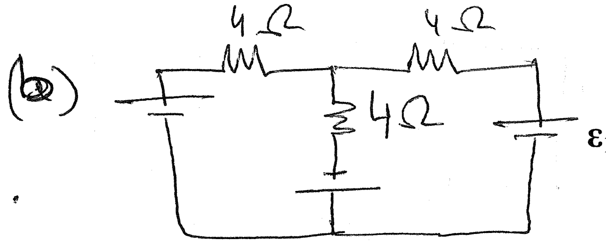
(a) $I = 0.24A$	(b) $V_{ab} = 3.8V$	(c) $P = 0.55\ W$

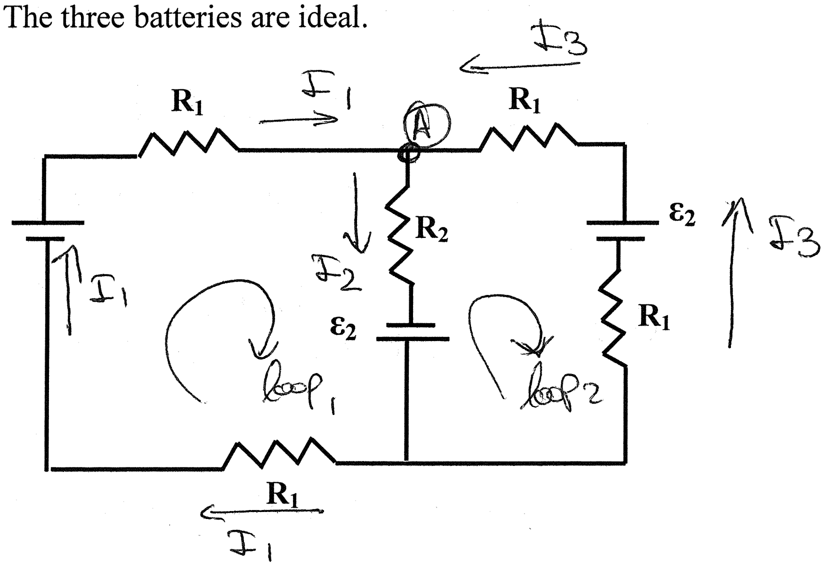
2. (a) Find the magnitude and the direction of the current in each of the three branches of the circuit below. *(15 points)* (25 points)

(b) Find the equivalent resistance of the circuit. *(15 points)*

$\varepsilon_1 = 3$ V, $\varepsilon_2 = 6$ V, $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$. The three batteries are ideal.

(b)





$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\Rightarrow R_{eq} = \frac{4}{3} = \boxed{1.33\ \Omega}$$

(a) (A) $\boxed{I_1 + I_3 = I_2}$ (1)

loop 1: $\mathcal{E}_1 - I_1 R_1 - I_2 R_2 + \mathcal{E}_2 - I_1 R_1 = 0 \Rightarrow 3 - 4I_1 - 4I_2 + 6 = 0$

(2) $\boxed{9 - 4I_1 - 4I_2 = 0}$

loop 2: $-\mathcal{E}_2 + R_1 I_3 - \mathcal{E}_2 + I_2 R_2 + I_3 R_1 = 0 \Rightarrow \boxed{-12 + 4I_3 + 4I_2 = 0}$ (3)

$I_3 = (I_2 - I_1)$

$-12 + 4I_2 - 4I_1 + 4I_2 = 0$

$-12 + 8I_2 - 4I_1 = 0$

(2) $9 - 4I_1 - 4I_2 = 0$

(3) $-12 - 4I_1 + 8I_2 = 0$

$+21 - 12I_2 = 0 \Rightarrow I_2 = \frac{21}{12} = \boxed{1.75\text{ A}}$

From (2) $9 - 4I_1 - 4I_2 = 0 \Rightarrow I_1 = \frac{+4I_2 - 9}{-4} = \frac{+4(1.75\text{A}) - 9}{-4} = \boxed{0.5\text{ A}}$

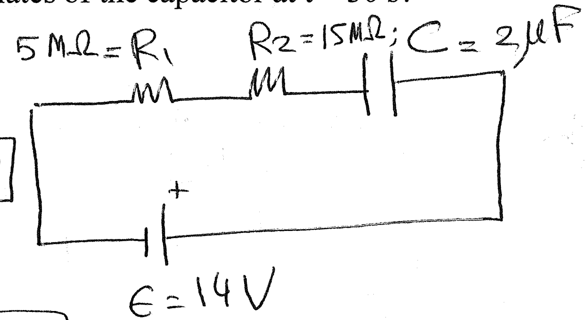
(1) $I_3 = I_2 - I_1 = 1.75\text{A} - 0.5\text{A} = \boxed{1.25\text{ A}}$

(a) I_1 =	I_2 = 1.75A	I_3 =	(b) R_{eq} = 1.33 Ω

3. Two resistors ($R_1 = 5$ MΩ, $R_2 = 15$ MΩ) are connected in series with a capacitor with capacitance C = 2 μF and a battery with emf = 14 V. Before the switch is closed at time t = 0, the capacitor is uncharged. *(25 points)*

(a) What is the time constant of this circuit?

(b) What fraction of the final charge is on the plates of the capacitor at t = 30 s?



(a) $R_{eq} = R_1 + R_2 = 20\,M\Omega$

$\tau = R \cdot C = (20\,M\Omega)\cdot(2\,\mu F) = \boxed{40 s}$

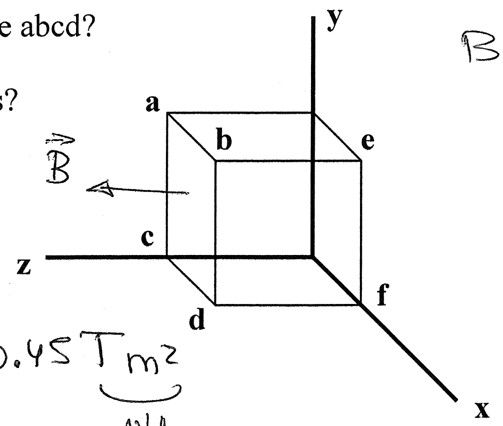
(b) $\frac{q}{Q_f} = (1 - e^{-t/RC}) = 1 - e^{-30/40} = \boxed{0.53}$

Capacitor is 53 % charged after t = 30 s

(a) RC = 40 s	(b) $q/Q_f = 0.53$

4. The magnetic field B in a certain region is 0.2 T and its direction is that of the +z axis in the figure below. The sides of the cube are 1.5 m long. (25 points)

(a) What is the magnetic flux across the surface abcd?
(b) What is the magnetic flux across bedf?
(c) What is the net flux through all six surfaces?



$B = 0.2\,T$, $+z$

$\Phi_B = B_\perp \cdot A = BA\cos\theta$

(a) $\Phi_{B_{abcd}} = (0.2T)\cdot(1.5m)^2 \cdot \cos 0^\circ = 0.45\,T m^2$ (Wb)

(b) $\Phi_{bedf} = BA\cos 90^\circ = \boxed{0}$

(c) $\Phi_{TOTAL} = \boxed{0} = BA - BA + 0 + 0 + 0 + 0$ (4 surfaces)

(a)	(b)	(c)