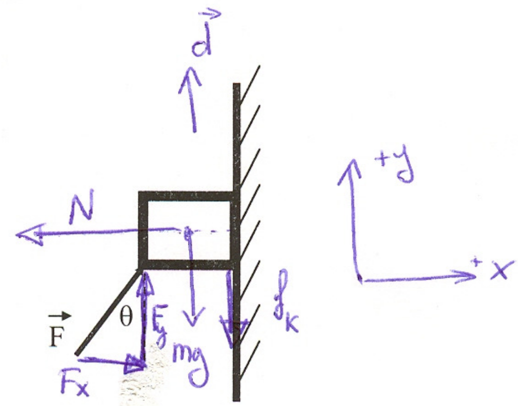
$a_y = 0$

1. A 2 kg block is pushed 3 m up a vertical wall with constant speed by a constant force of magnitude F applied at an angle θ = 30° with the horizontal as shown in the figure below. If the coefficient of kinetic friction between the block and the wall is 0.3, determine: *(25 points)*

(a) The magnitude of the force F *(7 points)*
(b) Normal force between the block and the wall *(6 points)*
(c) Work done by the force F. (6 *points)*
(d) Work done by the gravitational force. *(6 points)*



(a) $\sum_i \vec{F}_{y_i} = \vec{0} \Rightarrow F_y - f_k - mg = 0$

$$\boxed{F\cos 30° - \mu_K \cdot N - mg = 0} \quad (1)$$

$\sum_i \vec{F}_{x_i} = \vec{0} \Rightarrow N - F_x = 0 \quad ; \rightarrow \boxed{N = F\sin 30°} \quad (2)$

(1) $\Rightarrow F\cos 30° - \mu_K \cdot F\sin 30° - mg = 0 \Rightarrow \underline{\underline{F}} = \dfrac{mg}{[\cos 30° - \mu_K \sin 30°]} = \boxed{27.37N}$

(b) (2) $\underline{\underline{N}} = F \cdot \sin 30° = (27.37N)\sin 30° = \boxed{13.68N}$

(c) $W_F = \vec{F} \cdot \vec{d} = (F\sin 30\hat{i} + F\cos 30°\hat{j}) \cdot (3\hat{j}) = 3F\cos 30° = \boxed{71.1\ J}$

(d) $W_{Fg} = mgd\cos(180°) = -mgd = -(2kg) \cdot (9.8\frac{m}{s^2}) \cdot (3m) = \boxed{-58.8\ J}$

(a) F = 27.37 N	(b) N = 13.68N	(c) W_F = 71.1 J	(d) W_{mg} = -58.8 J

2. The force acting on an object is given by $F(x) = \frac{a}{x^5} - \frac{b}{x^2}$, where a = 6 Nm^5 and b = 2 Nm^2. *(25 points)*

(a) Calculate the work done by this force in moving the object from x_1= 1 m to x_2 = 5 m. *(12 points)*

(b) Find an expression for the potential energy associated with this force and locate the positions at which a particle will be in equilibrium. *(13 points)*

$$(a)\ W_F = \int_{x_1}^{x_2} F(x)dx = \int_1^5 \left(\frac{a}{x^5} - \frac{b}{x^2}\right)dx = a\int_1^5 x^{-5}dx - b\int_1^5 x^{-2}dx =$$

$$W_F = -a x^{-4}\Big|_1^5 + b x^{-1}\Big|_1^5 = -\frac{6}{4}[5^{-4} - 1] + 2[5^{-1} - 1] = \boxed{-0.1\ J}$$

$$(b)\ W_F = -\Delta U$$

$$F = -\frac{dU}{dx} \Rightarrow -F dx = dU \Rightarrow U = -\int F(x)dx$$

$$U(x) = -\left[-\frac{a x^{-4}}{4} + b x^{-1}\right] + \underset{\text{constant}}{C} = \boxed{\frac{a x^{-4}}{4} - b x^{-1} + C}$$

Stable equilibrium $\Rightarrow$ $K = 0$ $F = 0$

$$F = 0 \Rightarrow \frac{a - bx^3}{x^5} = 0 \Rightarrow a - bx^3 = 0 \Rightarrow x = \sqrt[3]{\frac{a}{b}}$$

$\boxed{x \neq 0}$!

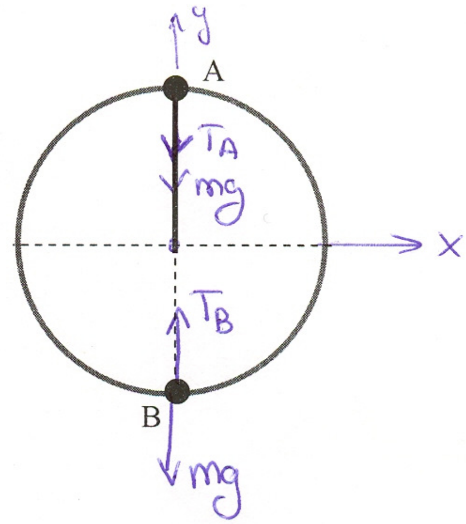
$$\Rightarrow x = \sqrt[3]{\frac{6}{2}} \simeq \boxed{1.44\,m}$$

(a) $-0.1\ J$	(b) $U(x) = \frac{a}{4x^4} - \frac{b}{x} + C$

$x = 1.44\,m$

3. A stone attached to the end of a non-elastic cord describes a vertical circular trajectory of 2 m radius. When the stone is at its highest point, the tension in the cord T_A=15 N, when the stone is at the lowest position of its trajectory, the tension T_B=33 N. Calculate: (Use energetic considerations). *(25 points)*

(a) Stone's mass *(9 points)*
(b) Magnitude of the stone's velocity in A *(8 points)*
(c) Magnitude of the stone's velocity in B *(8 points)*



A $F_c = \boxed{\frac{m V_A^2}{R} = T_A + mg}$ (1)

B $F_c = \boxed{\frac{m V_B^2}{R} = T_B - mg}$ (2)

$\Delta E = 0 = \Delta K + \Delta U \rightarrow K_A + U_A = K_B + U_B$

$\rightarrow \boxed{\frac{1}{2} \cancel{m} V_A^2 + \cancel{m} g R = \frac{1}{2} \cancel{m} V_B^2 - \cancel{m} g R}$ (3)

(3) $\boxed{V_A^2 + 4 g R = V_B^2}$ Substitute this in (2)

Combining (1) and (2) you have 2 eq. + 2 unknowns.

(1) $m \frac{V_A^2}{R} = T_A + mg$

(2) $\frac{m}{R} (V_A^2 + 4 g R) = T_B - mg$

$\rightarrow \boxed{V_A^2 = \frac{R}{m}(T_A + mg)}$ Input this in (2)

$\rightarrow$ (2) $\Rightarrow \frac{m}{R}\left(\frac{R}{m} T_A + \frac{R}{m} mg + 4 g R\right) = T_B - mg$

(2) $T_A + mg + 4 g m = T_B - mg \rightarrow T_A - T_B = -6mg \Rightarrow m = \frac{-T_A + T_B}{+6g}$

$\rightarrow \boxed{m = 0.3 \text{ Kg}}$

$V_A^2 = \frac{R}{m}(T_A + mg) \rightarrow \boxed{V_A = 10.94 \text{ m/s}}$

$V_B^2 = V_A^2 + 4gR \rightarrow \boxed{V_B = 14.07 \text{ m/s}}$

(a) 0.3 Kg = m	(b) V_A = 10.94 m/s	(c) V_B = 14.07 m/s

4. A block is released from rest at height h = 2.5 m and slides down a frictionless ramp onto a plateau, which has a length L = 9 m and where the coefficient of kinetic friction is 0.4. *(25 points)*

(a) What is the block's speed at the end of the incline plane (B)? *(12 points)*
(b) Can the block reach the plateau's end (C)? If the block cannot reach C, how far from B will it stop? *(13 points)*

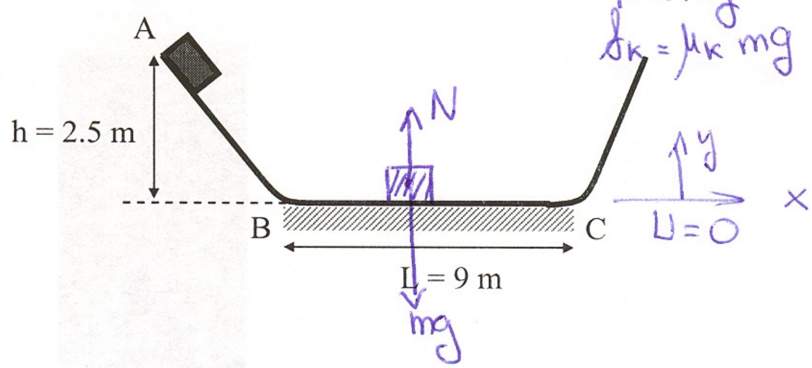
$N = mg$
$f_K = \mu_K mg$

$\boxed{A-B}$

(a) $\Delta E = 0 \Rightarrow \cancel{K_A} + U_A = K_B + \cancel{U_B}$

$mg h_A = \frac{1}{2} m V_B^2$

$V_B = \sqrt{2g h_A} = \boxed{7 \text{ m/s}}$



(b) $\boxed{B-C} \rightarrow \Delta E = 0 = \Delta K + \cancel{\Delta U} + \Delta E_{th}$

$0 = \cancel{K_f} - K_B + \Delta E_{th} \Rightarrow K_B = \Delta E_{th} = f_K \cdot d = \mu_K mgd$

Block Stops

$\Rightarrow \frac{1}{2} m V_B^2 = \mu_K mgd \rightarrow d = \frac{1/2\, m V_B^2}{\mu_K mg} = 6.25 \text{ m} < 9\text{m}$

$\rightarrow$ The block will stop after $\boxed{6.25\text{m}}$ $\leftarrow$ (away from B)

(a) $V_B = 7$ m/s	(b) $d = 6.25$ m