CHAPTER 7

P9

(a) $\vec{A} = 3\hat{i} - 2\hat{j}$, $\vec{B} = 4\hat{i} - 4\hat{j}$

$\vec{A}\cdot\vec{B} = AB\cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{A\cdot B}\right)$

$\vec{A}\cdot\vec{B} = 12+8 = 20$

$|\vec{A}| = A = \sqrt{3^2+2^2} = 3.6$

$|\vec{B}| = B = \sqrt{4^2+4^2} = 5.65$

$\rightarrow \boxed{\theta = 11.3^\circ}$

(b) $\vec{A} = -2\hat{i} + 4\hat{j}$; $\vec{B} = 3\hat{i} - 4\hat{j} + 2\hat{k}$

$\vec{A}\cdot\vec{B} = -6 - 16 + 0 = -22$

$|\vec{A}| = \sqrt{2^2+4^2} = \sqrt{20}$

$|\vec{B}| = \sqrt{3^2+4^2+2^2} = \sqrt{29}$

$\rightarrow \theta = \cos^{-1}\left(\frac{-22}{\sqrt{20}\cdot\sqrt{29}}\right)$

$\boxed{\theta \simeq 156^\circ}$

(c) $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{B} = 3\hat{j} + 4\hat{k}$ $\Big\}$ $\vec{A}\cdot\vec{B} = 0 - 6 + 8 = 2$

$|\vec{A}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$|\vec{B}| = \sqrt{9+16} = \sqrt{25} = 5$

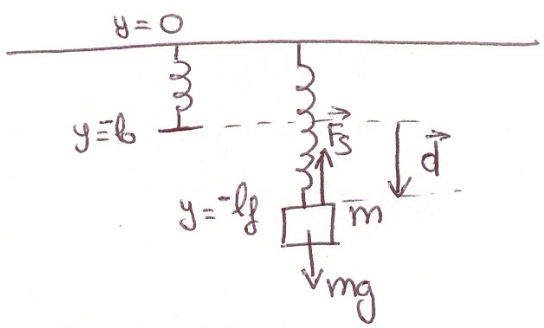
$\rightarrow \theta = \cos^{-1}\left(\frac{2}{3\times 5}\right) = \boxed{82.3^\circ}$

P14 $\vec{F} = (4x\hat{i} + 3y\hat{j})$ N $\quad (x=0 \text{ to } x=5\text{m})$

$W = \int \vec{F}\cdot d\vec{r}$

$W = \int_0^{5m} (4x\hat{i} + 3y\hat{j})^N \cdot (dx\,\hat{i})^m = \int_0^{5m} 4x\,dx = 4\frac{x^2}{2}\Big|_0^5 = 2\times 5^2 - 0 = \boxed{50\text{ J}}$

P18



$b = 0.35\,m$
$l_f = 0.415\,m$
$m = 7.5\,kg$

(a) K ?

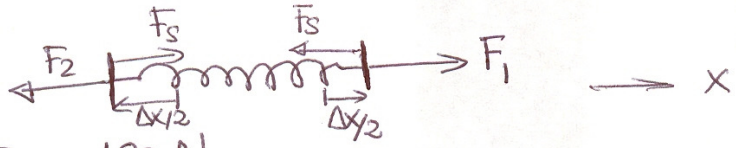
$|\vec{d}| = -l_f - (-b) = -l_f + b = -0.415m + 0.35m = -0.065\,m$

$\Sigma F = 0 \Rightarrow F_s - mg = 0 \Rightarrow \boxed{F_s = mg}$ (1)

$\boxed{\vec{F_s} = -k \cdot \vec{d}}$ (2)

(1),(2) $\Rightarrow mg = -k \cdot d \Rightarrow k = \dfrac{-mg}{d} = \dfrac{-(7.5kg)(9.8m/s^2)}{(-0.065m)} = \boxed{1.13\,\dfrac{kN}{m}}$

(b)



$F_1 = F_2 = 190\,N$

length of spring ?

Right end : $-F_1 = +F_s = -190N = -K \cdot \left(\dfrac{2\Delta x}{2}\right) = -\left(1.13\,\dfrac{kN}{m}\right) \cdot \Delta x$

$\Delta x = \dfrac{+190\,N}{+1.13\,\dfrac{kN}{m}} = 0.168\,m$

Total length of spring $= 0.35m + (0.168m) = \boxed{0.518\,m}$

P22 $m = 0.1\,kg$

$x_f = 0.6\,m$

$\vec{F} = (15000 + 10000x - 25000x^2)\,\hat{i}$; $d\vec{r} = dx\,\hat{i}$

(a) $W = \int_0^{x_f} \vec{F}\cdot d\vec{r} = \int_0^{0.6m} (15000 + 10000x - 25000x^2)\,dx =$

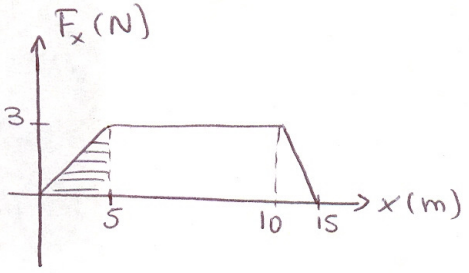
$W = \left[15000x + 10000\frac{x^2}{2} - 25000\frac{x^3}{3}\right]_0^{0.6m} = 9000 + 1800 - 1800 = \boxed{9kJ}$

(b) $x_f = 1\,m$?

$W = \left[15\times10^3 x + 10^4\frac{x^2}{2} - 25\times10^3\frac{x^3}{3}\right]_0^{1} = 15\times10^3 + 5\times10^3 - 8.33\times10^3 = \boxed{11.67k}$

P32 $x=0$, $V_i = 0\ m/s$
$m = 4\,kg$

(a) Speed at $x = 5\,m$?
(b) " " $x = 10\,m$?
(c) " " $x = 15\,m$?



$W_{net} = \Delta K = \frac{1}{2}m(V_f^2 - V_i^2) \Rightarrow \boxed{V_f = \sqrt{\frac{2W_{net}}{m}}}$

W_{net} = area under the curve F_x, x $\Leftarrow$ $W_{net} = \int_{x_0}^{x_f} F_x\cdot dx$ = Area under curve

(a) $\Delta K = \frac{1}{2}mV_f^2 - 0 = \frac{(3N)\cdot(5m)}{2} = 7.5\,(N\cdot m)$

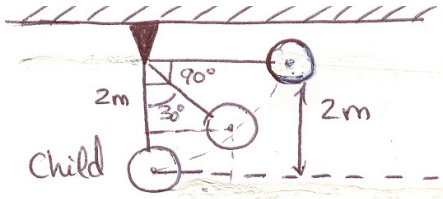
$V_f = \sqrt{\frac{2*7.5\,N\cdot m}{4\,kg}} = \boxed{1.94\,m/s}$

(b) $V_f = \sqrt{\frac{2*(7.5Nm+15Nm)}{4\,kg}} = \boxed{3.35\,m/s}$ ← "Net work" W_{net} area under curve from $x=0$ to $x=10\,m$.

(c) $V_f = \sqrt{\frac{2*(7.5+15+7.5)Nm}{4\,kg}} = \boxed{3.87\,m/s}$

(P38) $\ell = 2\,m$

$W = 400\,N$

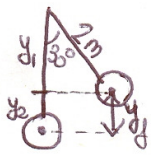


(a) Horizontal ropes

$U = mgy = (400N)\cdot(2m) = \boxed{800\,J}$

(b) Ropes make 30° with vertical.

$U = mgy = (400N)\cdot(0.27m) \simeq \boxed{108\,J}$



$y_1 + y_2 = 2m$

$\cos 30° = \frac{y_1}{2} \Rightarrow y_1 = 1.73m$

$y_f = y_2 = (2m - y_1) = (2m - 1.73m)$

$y_f = 0.27m$

(c) The child is at the bottom of the circular arc $\Rightarrow y = 0 \rightarrow \boxed{U = 0}$

(P44) $\vec{F} = (-Ax + Bx^2)\,\hat{i}\ N$

(a) Potential energy function $U(x)$?

$U = 0$ at $x = 0$

$$U = -\int F_x\,dx = -\int(-Ax + Bx^2)\,dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$$

(b) $x = 2m$ to $x = 3m$

ΔU ? ΔK?

$$\Delta U = -\int_2^3 (-Ax + Bx^2)\,dx = \left[\frac{Ax^2}{2} - \frac{Bx^3}{3}\right]_2^3 = A\frac{9}{2} - 9B - 2A + \frac{8}{3}B$$

$$\Delta U = \boxed{2.5A - 6.33B}$$

$$\Delta K + \Delta U = 0 \Rightarrow \boxed{\Delta K = -\Delta U = -2.5A + 6.33B}$$