

Chapter 28 – Sources of Magnetic Field

- Magnetic Field of a Moving Charge
- Magnetic Field of a Current Element
- Magnetic Field of a Straight Current-Carrying Conductor
- Force Between Parallel Conductors
- Magnetic Field of a Circular Current Loop
- Ampere's Law
- Applications of Ampere's Law
- Magnetic Materials

1. Magnetic Field of a Moving Charge

- A charge creates a magnetic field only when the charge is moving.

Source point: location of the moving charge.

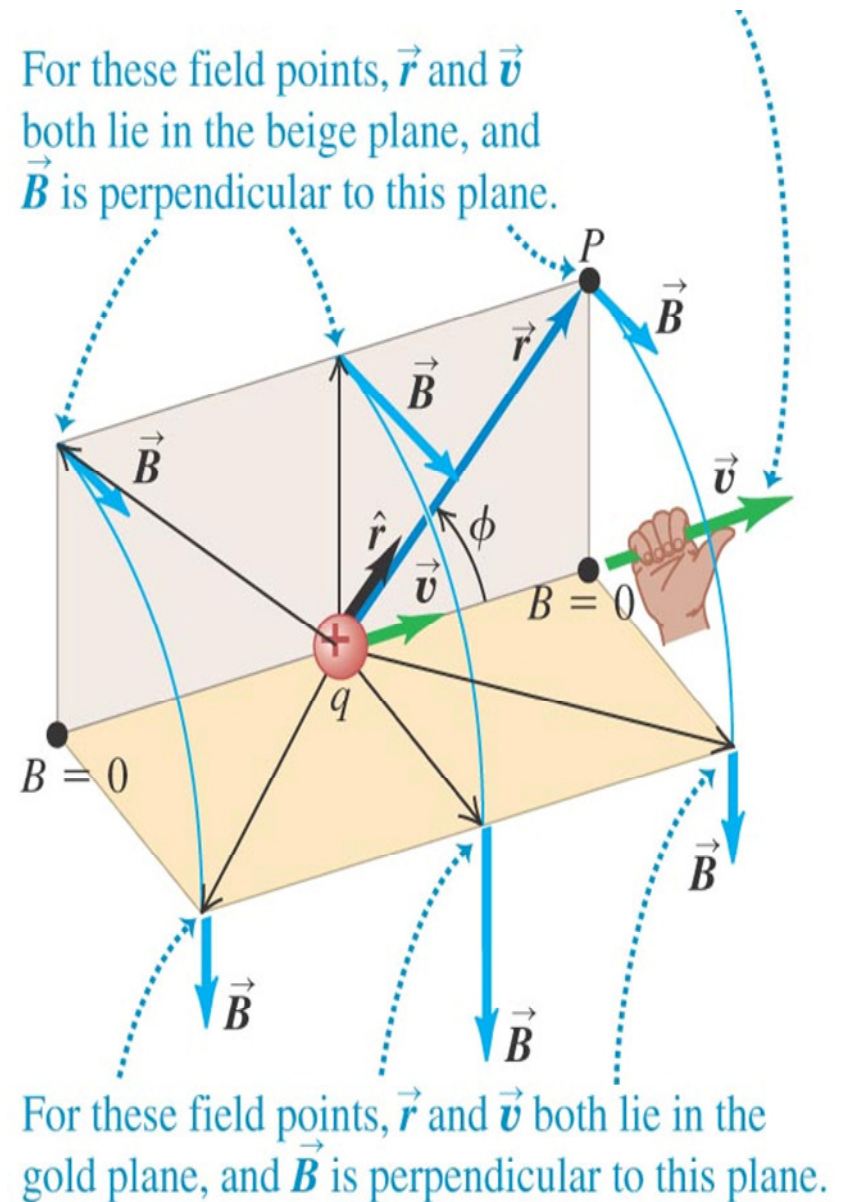
Field point: point P where we want to find the field.

Magnetic field from a point charge moving with constant speed

$$B = \frac{\mu_0}{4\pi} \frac{|q|v \sin \phi}{r^2}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Wb/A}\cdot\text{m} = \text{N s}^2/\text{C}^2 = \text{N/A}^2 \\ = \text{T m/A} \text{ (permeability of vacuum)}$$

$$c = (1/\mu_0\epsilon_0)^{1/2} \rightarrow \text{speed of light}$$



Magnetic field of a point charge moving with constant velocity

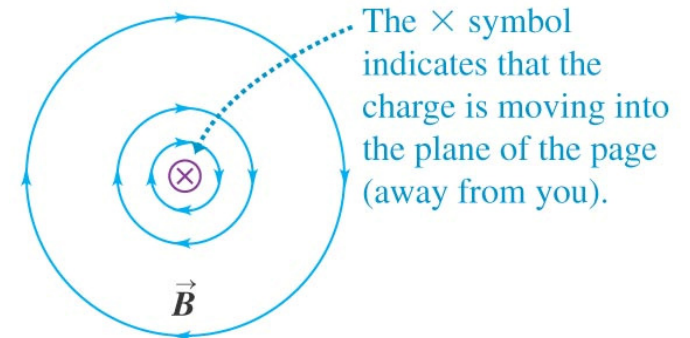
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$\hat{r} = \vec{r} / r =$ vector from source to field point

Moving Charge: Magnetic Field Lines

- The magnetic field lines are circles centered on the line of \vec{v} and lying in planes perpendicular to that line.

View from behind the charge



- **Direction of field line:** right hand rule for + charge → point right thumb in direction of \vec{v} . Your fingers curl around the charge in direction of magnetic field lines.

2. Magnetic Field of a Current Element

- The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

$$dQ = nqAdl \quad (\text{total moving charge in volume element } dl \text{ A})$$

Moving charges in current element are equivalent to dQ moving with drift velocity.

$$dB = \frac{\mu_0}{4\pi} \frac{|dQ|v_d \sin \varphi}{r^2} = \frac{\mu_0}{4\pi} \frac{n|q|v_d Adl \sin \varphi}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl \sin \varphi}{r^2}$$

$$(I = n q v_d A)$$

Current Element: Vector Magnetic Field

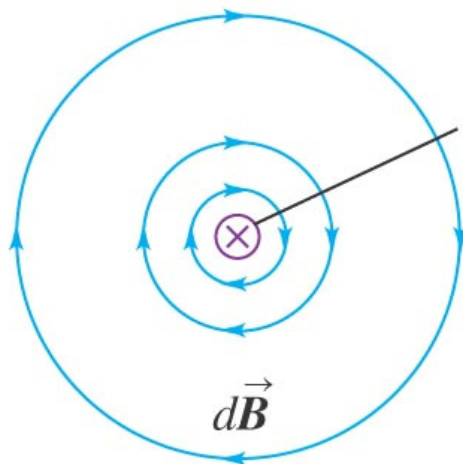
$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

Law of Biot and Savart

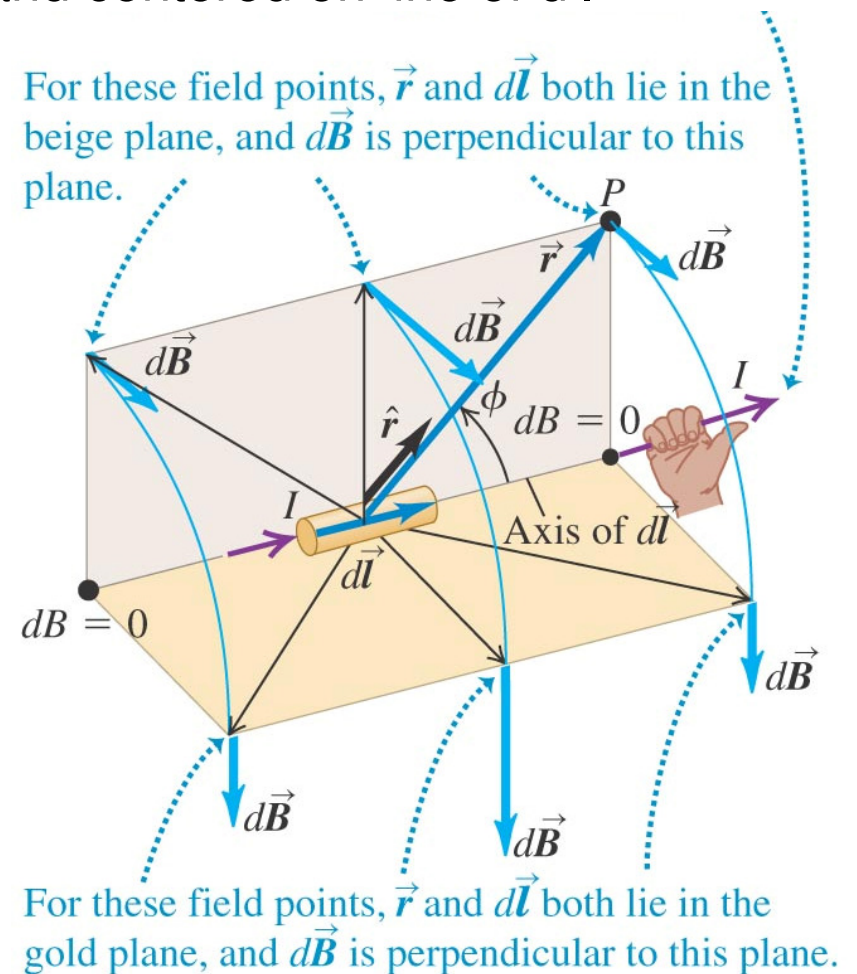
Current Element: Magnetic Field Lines

- Field vectors ($d\vec{B}$) and magnetic field lines of a current element ($d\vec{l}$) are like those generated by a + charge dQ moving in direction of v_{drift} .
- Field lines are circles in planes \perp to $d\vec{l}$ and centered on line of $d\vec{l}$.

View along the axis of the current element



Current directed into the plane of the page



3. Magnetic Field of a Straight Current-Carrying Conductor

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$|d\vec{l} \times \hat{r}| = dl \cdot 1 \cdot \sin \varphi = dl \cdot \sin(\pi - \varphi) = \frac{dl \cdot x}{\sqrt{x^2 + y^2}}$$

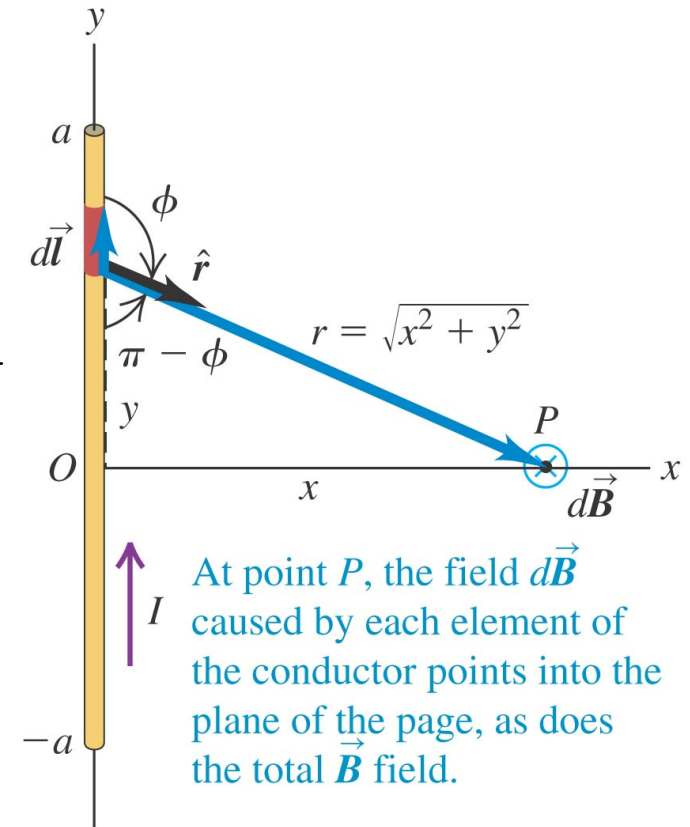
$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x \cdot dy}{(x^2 + y^2)^{3/2}} = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$$

If conductor length $2a \gg x$

$$B = \frac{\mu_0 I (2a)}{4\pi \cdot x \cdot a} = \frac{\mu_0 I}{2\pi \cdot x}$$

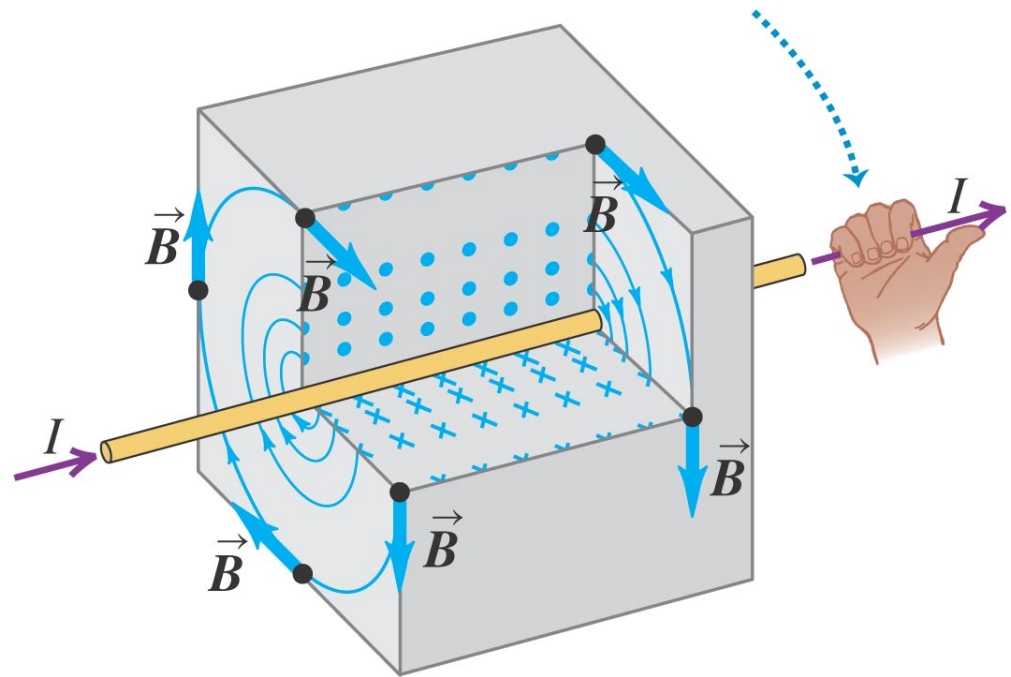
$$B = \frac{\mu_0 I}{2\pi \cdot r}$$

Field near a long, straight current-carrying conductor



B direction: into the plane of the figure, perpendicular to x - y plane

- **Electric field lines** radiate outward from + line charge distribution. They begin and end at electric charges.
 - **Magnetic field lines** encircle the current that acts as their source. They form closed loops and never have end points.
- The total magnetic flux through any closed surface is zero \rightarrow there are no isolated magnetic charges (or magnetic monopoles) \rightarrow any magnetic field line that enters a closed surface must also emerge from that surface.



4. Force Between Parallel Conductors

- Two conductors with current in same direction. Each conductor lies in B set-up by the other conductor.

\vec{B} generated by lower conductor at the position of upper conductor:

$$B = \frac{\mu_0 I}{2\pi \cdot r}$$

$$\vec{F} = I' \vec{L} \times \vec{B}$$

$$F = I' L B = I' L \frac{\mu_0 I}{2\pi \cdot r}$$

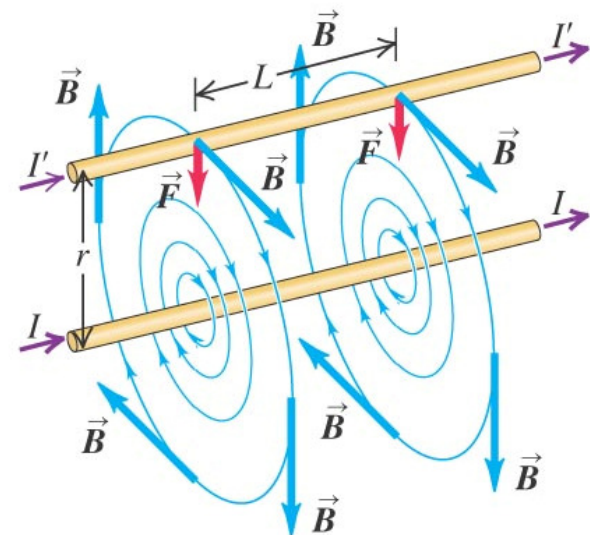
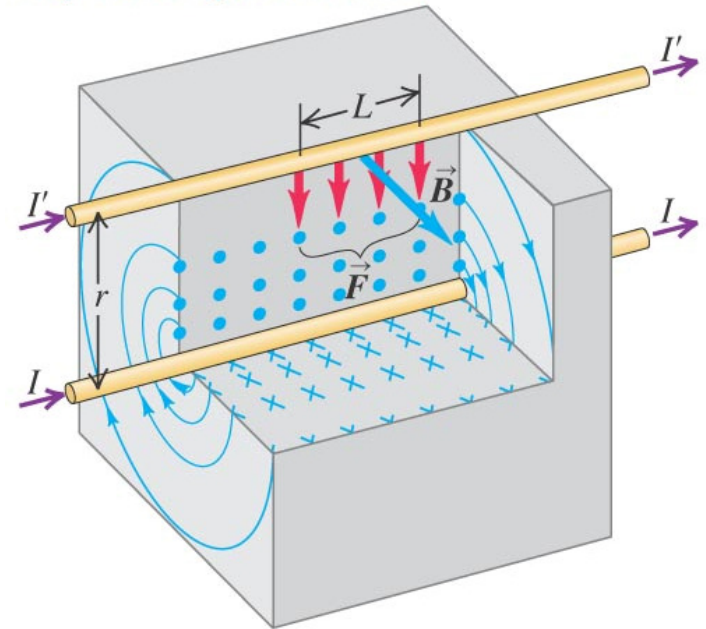
$$\frac{F}{L} = \frac{\mu_0 I \cdot I'}{2\pi \cdot r}$$

Two long parallel current-carrying conductors

Force on upper conductor is downward.

- Parallel conductors carrying currents in same direction attract each other. If I has contrary direction they repel each other.

If the wires had currents in *opposite* directions, they would *repel* each other.



Magnetic Forces and Defining the Ampere

- One Ampere is the unvarying current that, if present in each of the two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} N per meter of length.

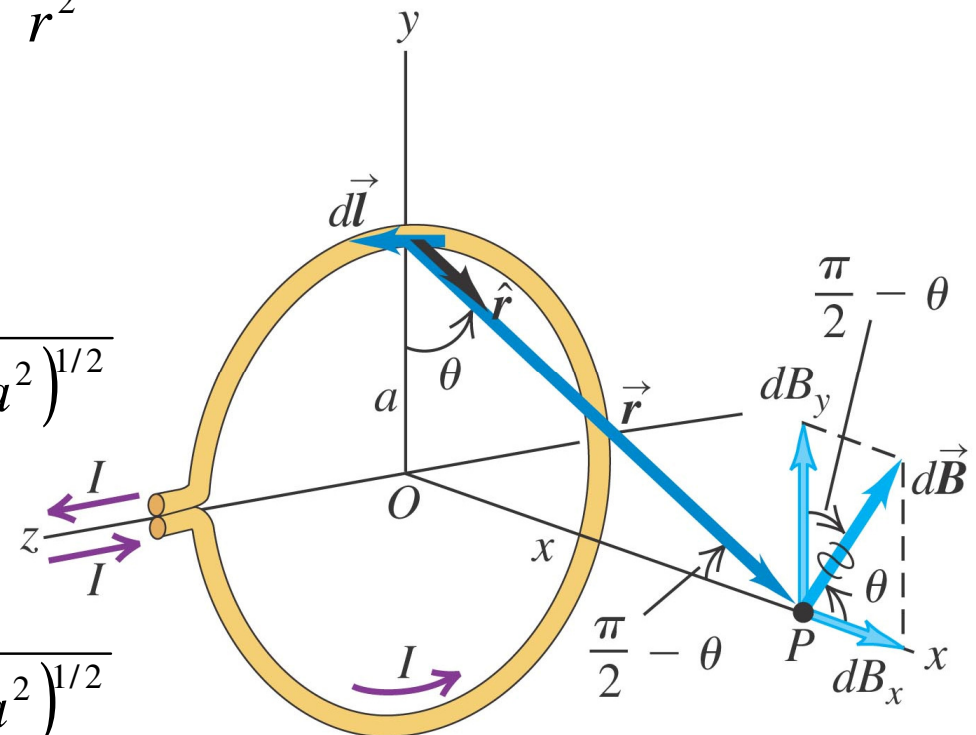
5. Magnetic Field of a Circular Current Loop

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2} \quad B = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)}$$

$$dB_x = dB \cos \theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{a}{(x^2 + a^2)^{1/2}}$$

$$dB_y = dB \sin \theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}}$$

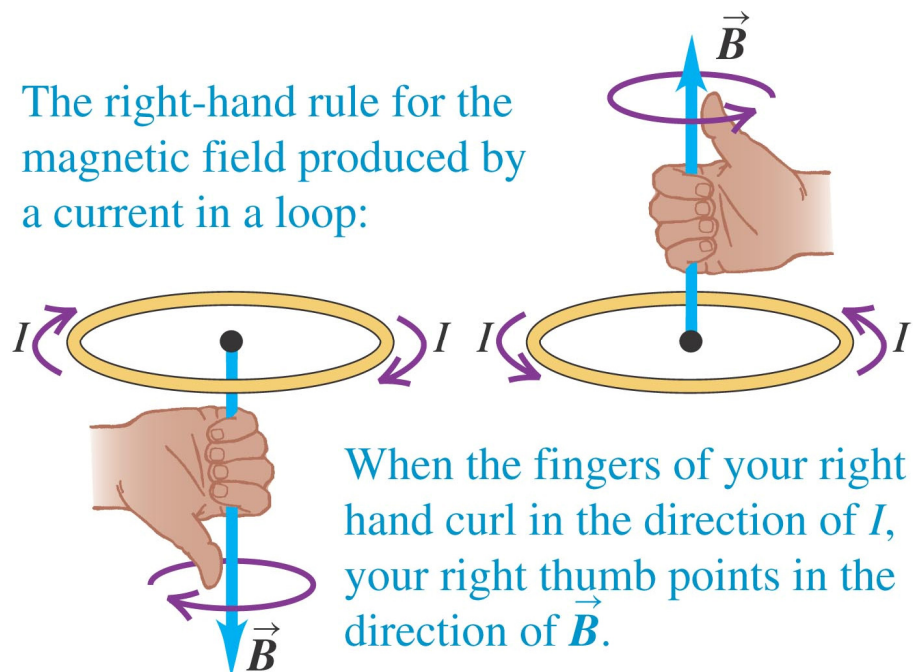


- **Rotational symmetry about x axis** \rightarrow no \vec{B} component perpendicular to x. For dl on opposite sides of loop, dB_x are equal in magnitude and in same direction, dB_y have same magnitude but opposite direction (cancel).

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a dl}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I}{4\pi} \frac{a}{(x^2 + a^2)^{3/2}} \int dl = \frac{\mu_0 I}{4\pi} \frac{a}{(x^2 + a^2)^{3/2}} (2\pi a)$$

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

(on the axis of a circular loop)



Magnetic Field on the Axis of Coil

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} \quad (\text{on the axis of } N \text{ circular loops})$$

$$B_x = \frac{\mu_0 N I}{2a} \quad (\text{at the center, } x=0, \text{ of } N \text{ circular loops})$$

$$\mu = N \cdot I \cdot A = N \cdot I \cdot (\pi a^2)$$

$$B_x = \frac{\mu_0 \mu}{2\pi(x^2 + a^2)^{3/2}}$$

(on the axis of any number of circular loops)

