Chapter 28 – Sources of Magnetic Field

- Magnetic Field of a Moving Charge
- Magnetic Field of a Current Element
- Magnetic Field of a Straight Current-Carrying Conductor
- Force Between Parallel Conductors
- Magnetic Field of a Circular Current Loop
- Ampere's Law
- Applications of Ampere's Law
- Magnetic Materials

1. Magnetic Field of a Moving Charge

- A charge creates a magnetic field only when the charge is moving.

Source point: location of the moving charge.

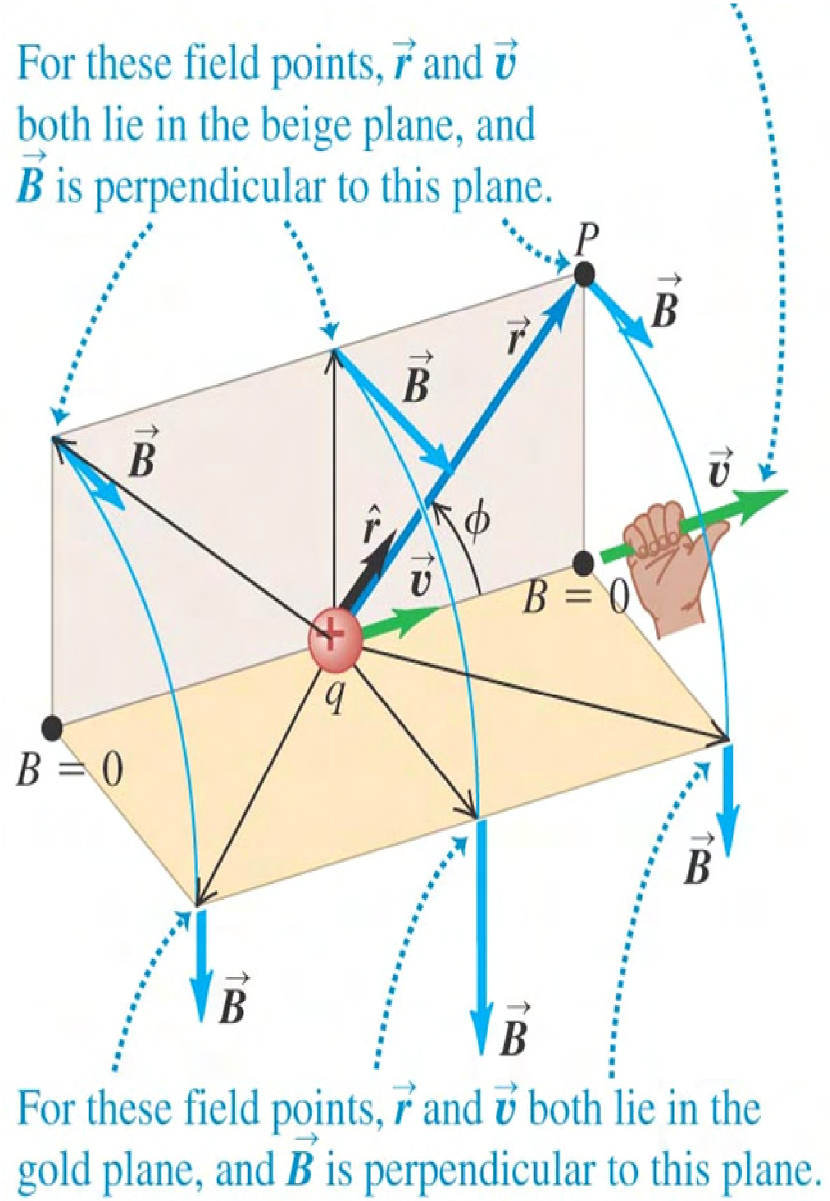
Field point: point P where we want to find the field.

Magnetic field from a point charge moving with constant speed

$$B = \frac{\mu_0}{4\pi}\frac{|q| v \sin\varphi}{r^2}$$

$\mu_0 = 4\pi \cdot 10^{-7}$ Wb/A·m = N s^2/C^2 = N/A^2 = T m/A (permeability of vacuum)

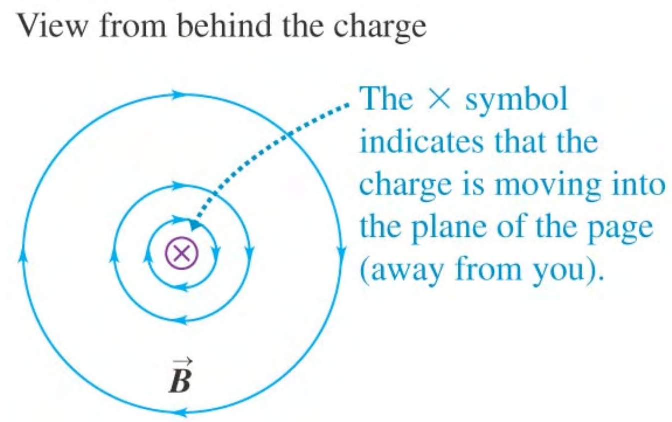
$c = (1/\mu_0\varepsilon_0)^{1/2}$ → speed of light



Magnetic field of a point charge moving with constant velocity

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$\hat{r} = \vec{r} / r$ = vector from source to field point



Moving Charge: Magnetic Field Lines

- The magnetic field lines are circles centered on the line of $\vec{v}$ and lying in planes perpendicular to that line.

- Direction of field line: right hand rule for + charge → point right thumb in direction of $\vec{v}$. Your fingers curl around the charge in direction of magnetic field lines.

2. Magnetic Field of a Current Element

- The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

$$dQ = nqAdl$$ (total moving charge in volume element dl A)

Moving charges in current element are equivalent to dQ moving with drift velocity.

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d \sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d Adl \sin\varphi}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl \sin\varphi}{r^2}$$

($I = n\,q\,v_d\,A$)

Current Element: Vector Magnetic Field

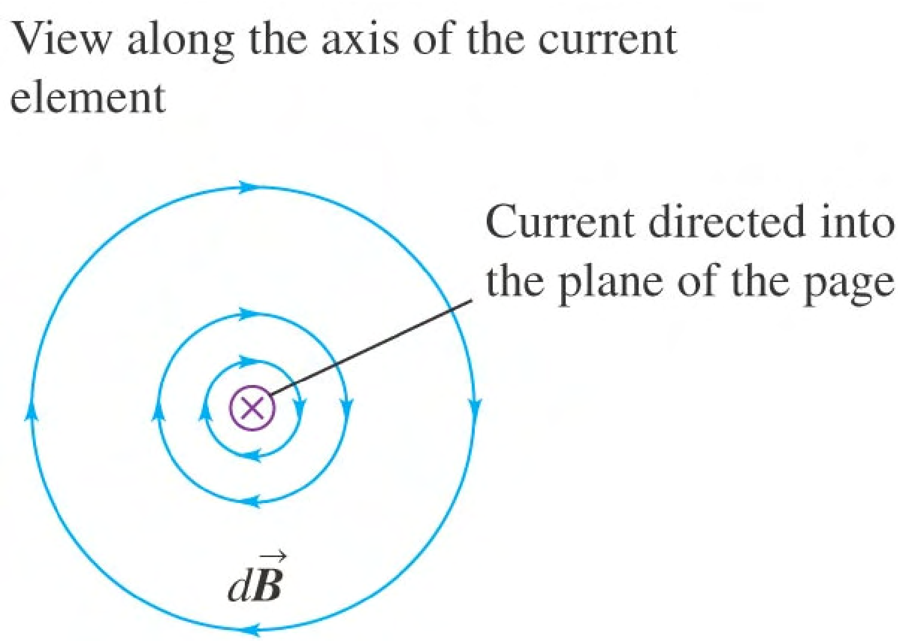
$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

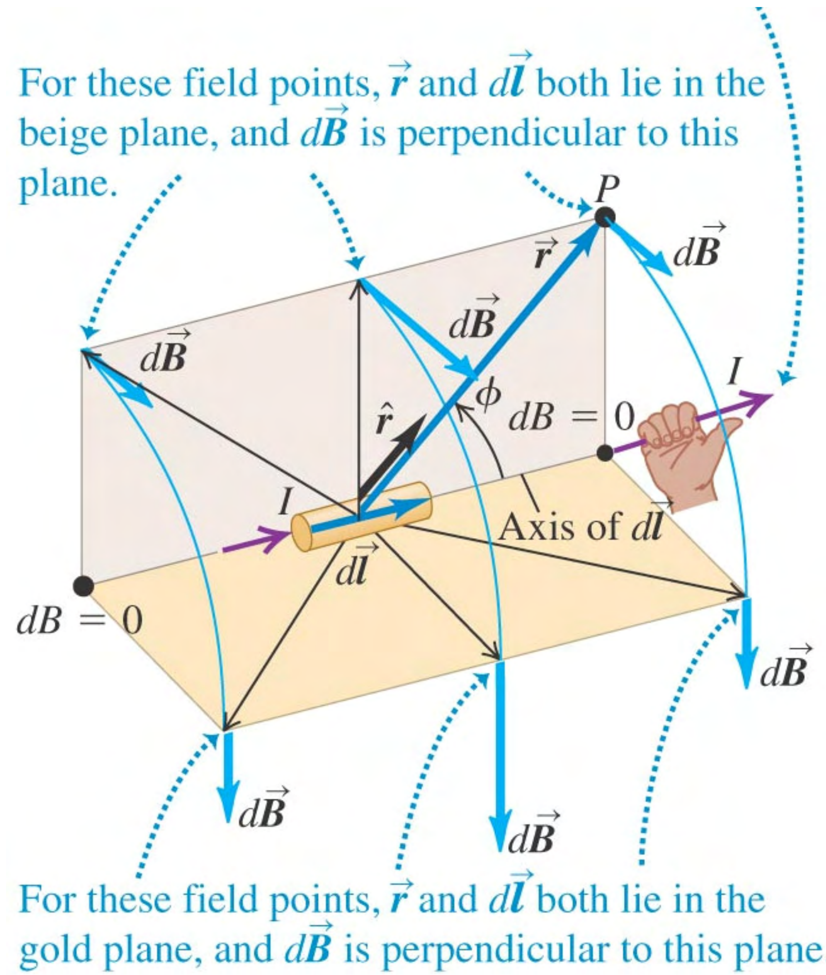
Law of Biot and Savart

Current Element: Magnetic Field Lines

- Field vectors ($d\vec{B}$) and magnetic field lines of a current element ($d\vec{l}$) are like those generated by a + charge dQ moving in direction of v_{drift}.

- Field lines are circles in planes ⊥ to $d\vec{l}$ and centered on line of $d\vec{l}$.



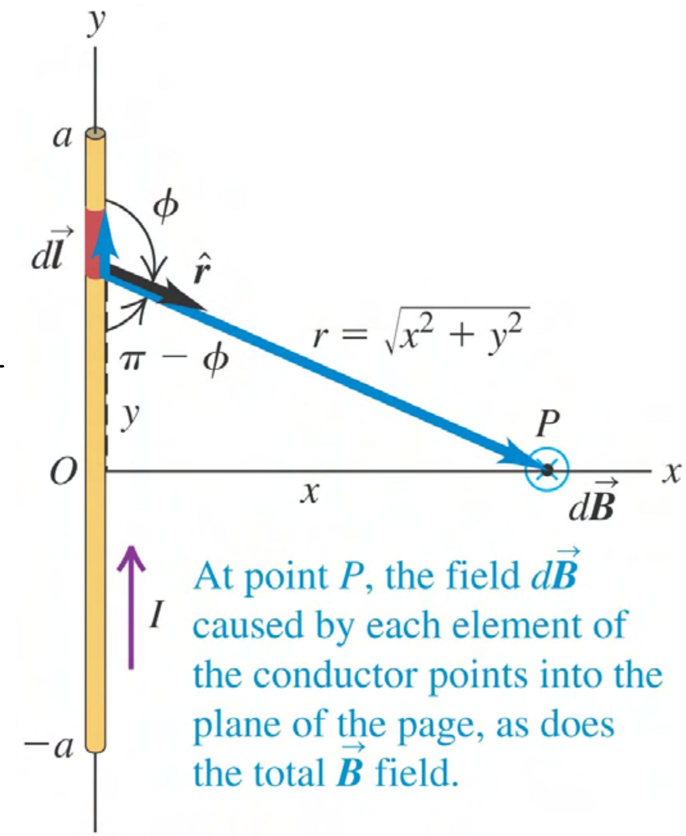


3. Magnetic Field of a Straight Current-Carrying Conductor

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

$$\left|d\vec{l} \times \hat{r}\right| = dl \cdot 1 \cdot \sin\varphi = dl \cdot \sin(\pi - \varphi) = \frac{dl \cdot x}{\sqrt{x^2 + y^2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x \cdot dy}{(x^2 + y^2)^{3/2}} = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2 + a^2}}$$



If conductors length 2a >> x

$$B = \frac{\mu_0 I(2a)}{4\pi \cdot x \cdot a} = \frac{\mu_0 I}{2\pi \cdot x}$$

B direction: into the plane of the figure, perpendicular to x-y plane

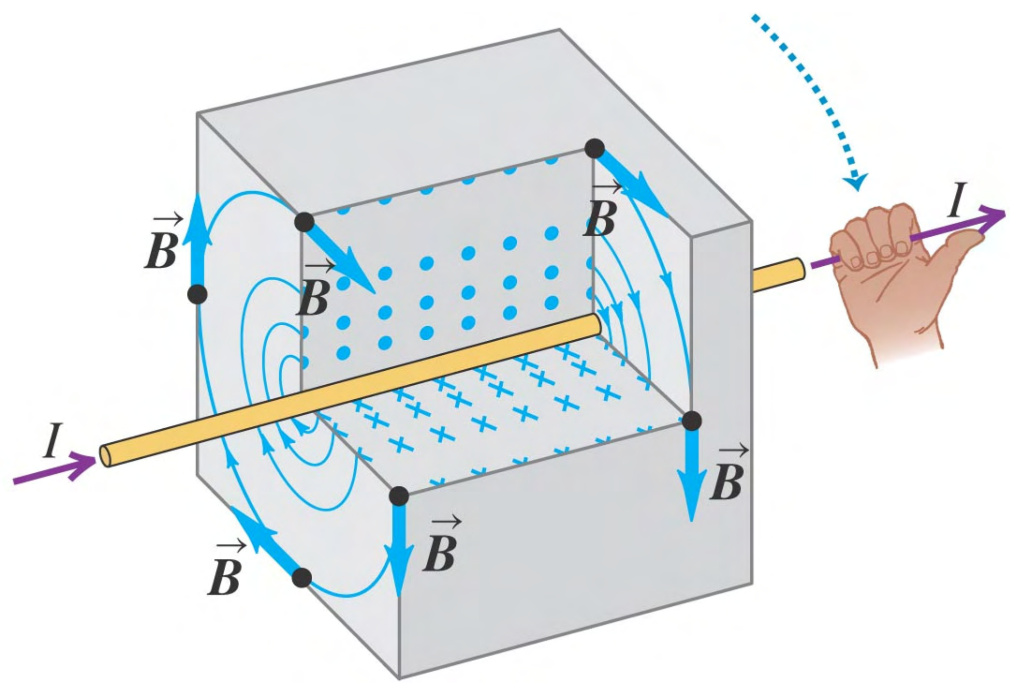
$$B = \frac{\mu_0 I}{2\pi \cdot r}$$

Field near a long, straight current-carrying conductor

- Electric field lines radiate outward from + line charge distribution. They begin and end at electric charges.

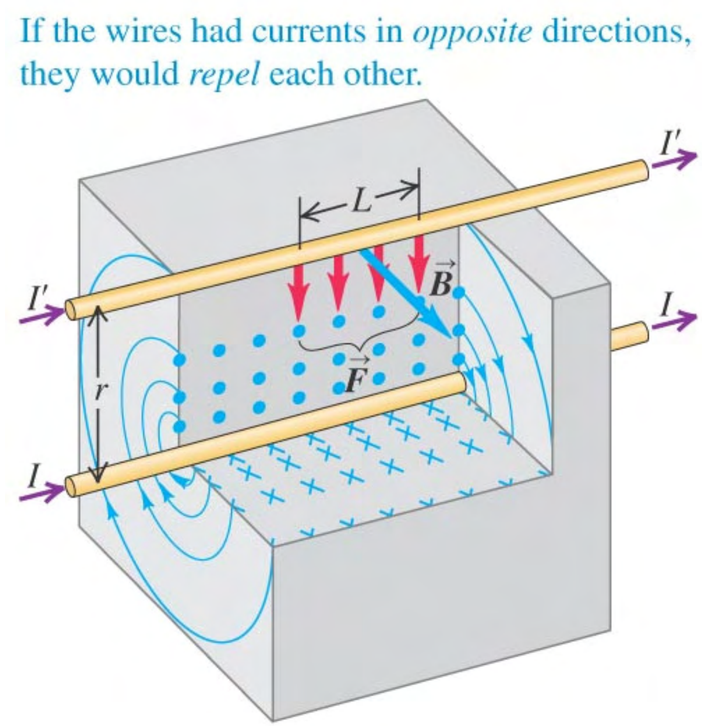
- Magnetic field lines encircle the current that acts as their source. They form closed loops and never have end points.

-The total magnetic flux through any closed surface is zero → there are no isolated magnetic charges (or magnetic monopoles) → any magnetic field line that enters a closed surface must also emerge from that surface.



4. Force Between Parallel Conductors

- Two conductors with current in same direction. Each conductor lies in B set-up by the other conductor.



$\vec{B}$ generated by lower conductor at the position of upper conductor:

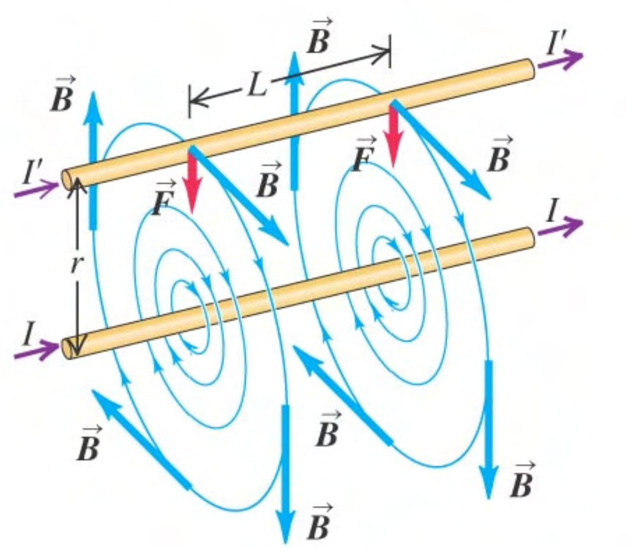
$$B = \frac{\mu_0 I}{2\pi \cdot r}$$

$$\vec{F} = I' \vec{L} \times \vec{B}$$

$$F = I' L B = I' L \frac{\mu_0 I}{2\pi \cdot r}$$

$$\boxed{\frac{F}{L} = \frac{\mu_0 I \cdot I'}{2\pi \cdot r}}$$

Two long parallel current-carrying conductors



Force on upper conductor is downward.

- Parallel conductors carrying currents in same direction attract each other. If I has contrary direction they repel each other.

Magnetic Forces and Defining the Ampere

- One Ampere is the unvarying current that, if present in each of the two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2 x 10^{-7} N per meter of length.

5. Magnetic Field of a Circular Current Loop

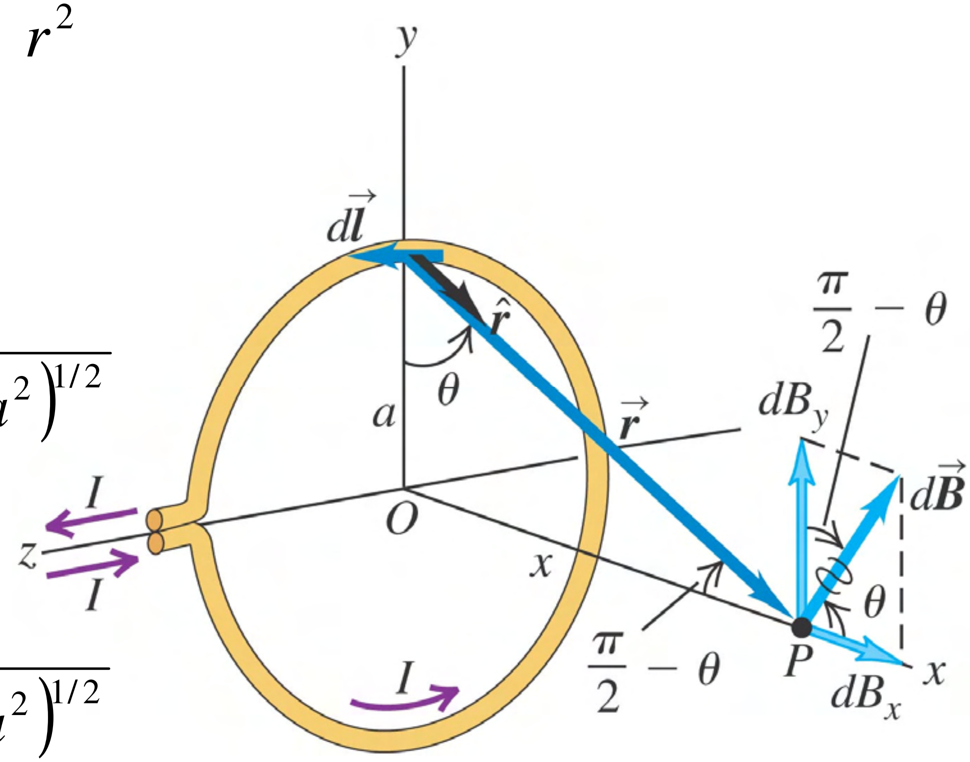
$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{I dl}{r^2}$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2 + a^2)}$$

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2 + a^2)}\frac{a}{(x^2 + a^2)^{1/2}}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2 + a^2)}\frac{x}{(x^2 + a^2)^{1/2}}$$

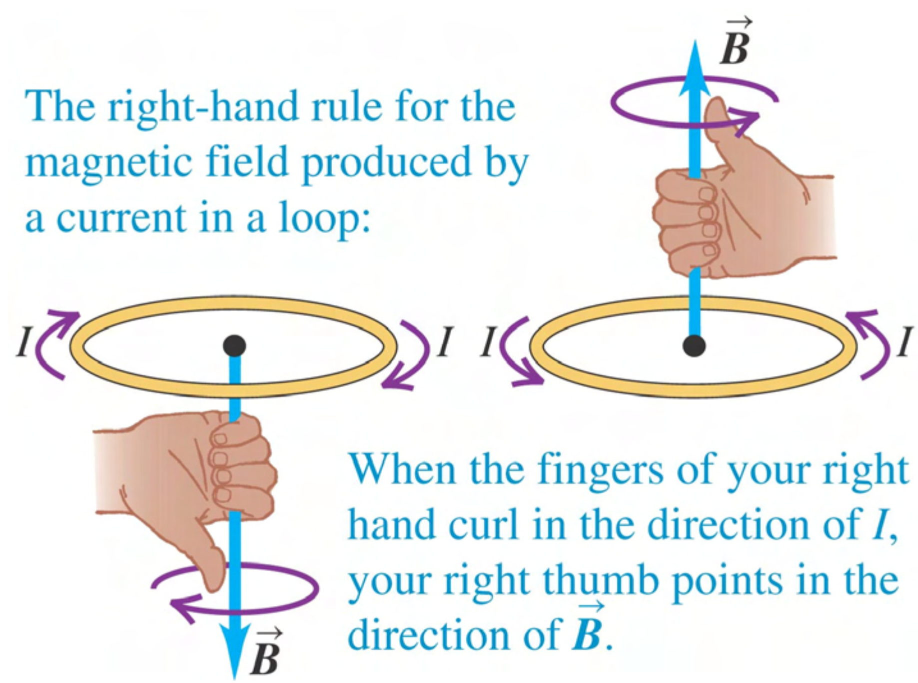


- Rotational symmetry about x axis → no $\vec{B}$ component perpendicular to x. For dl on opposite sides of loop, dB_x are equal in magnitude and in same direction, dB_y have same magnitude but opposite direction (cancel).

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a\,dl}{\left(x^2 + a^2\right)^{3/2}} = \frac{\mu_0 I}{4\pi} \frac{a}{\left(x^2 + a^2\right)^{3/2}} \int dl = \frac{\mu_0 I}{4\pi} \frac{a}{\left(x^2 + a^2\right)^{3/2}} (2\pi a)$$

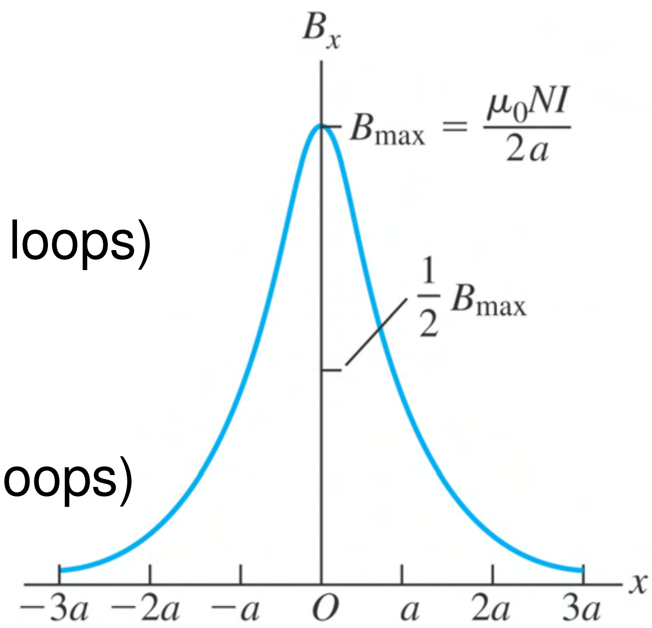
$$B_x = \frac{\mu_0 I a^2}{2\left(x^2 + a^2\right)^{3/2}}$$

(on the axis of a circular loop)



Magnetic Field on the Axis of Coil

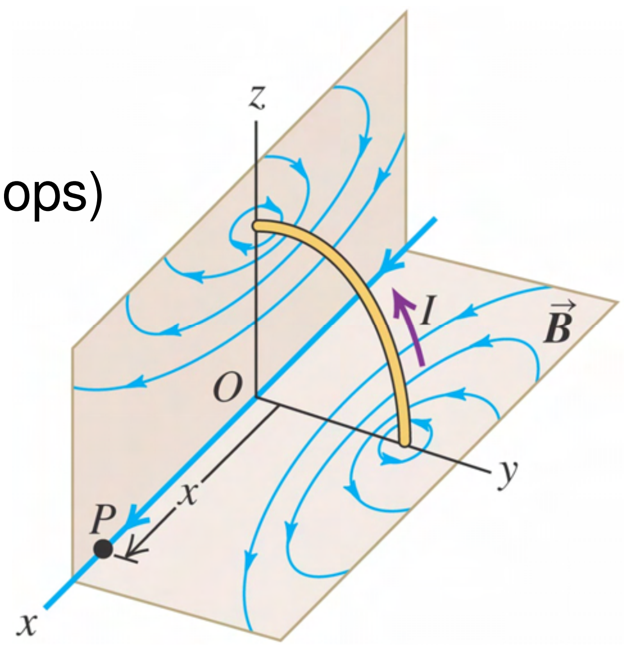
$$B_x = \frac{\mu_0 N I a^2}{2\left(x^2 + a^2\right)^{3/2}}$$ (on the axis of N circular loops)



$$\boxed{B_x = \frac{\mu_0 N I}{2a}}$$ (at the center, x=0, of N circular loops)

$$\mu = N \cdot I \cdot A = N \cdot I \cdot (\pi a^2)$$

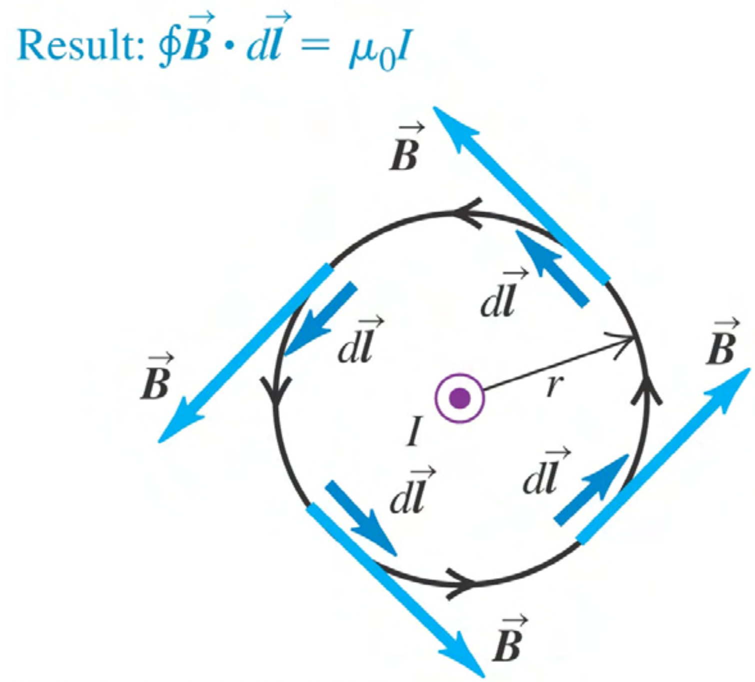
$$\boxed{B_x = \frac{\mu_0 \mu}{2\pi (x^2 + a^2)^{3/2}}}$$ (on the axis of any number of circular loops)



6. Ampere's Law

- Law that allows us to obtain the magnetic fields caused by highly symmetric current distributions.

$$\oint \vec{B} \cdot d\vec{l} = \oint B_{//} dl = \mu_0 I$$



Ampere's Law for a Long Straight Conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \vec{B} \cdot d\vec{l} = B \oint dl = \frac{\mu_0 I}{2\pi \cdot r}(2\pi \cdot r) = \mu_0 I$$

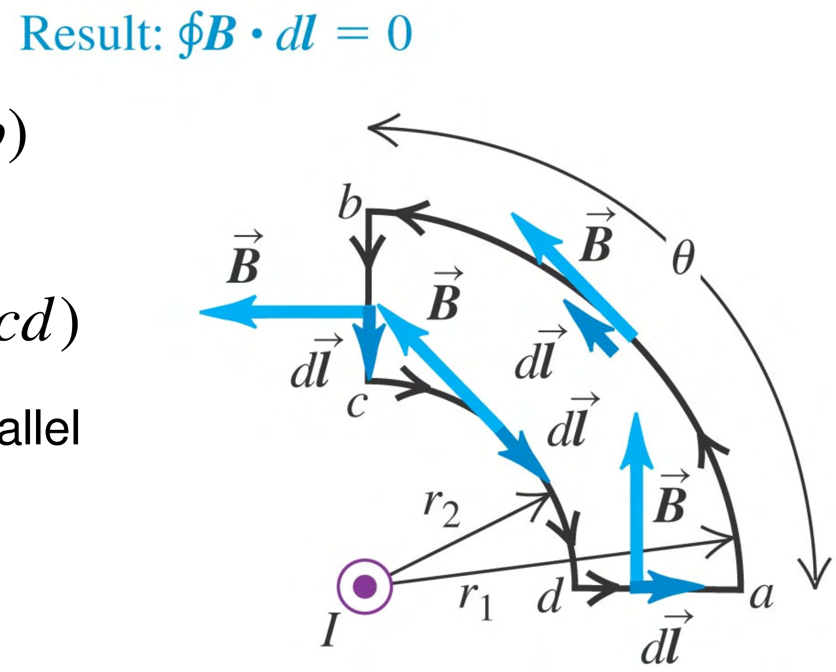
- Direction of current: right hand rule → curl fingers of right hand around the integration path , the thumb indicates positive current direction.

For an <u>integration path that does not enclose the conductor</u>:

$$B_{//} = B_1 = \frac{\mu_0 I}{2\pi \cdot r_1} \qquad (circular\ arc\ ab)$$

$$B_{//} = -B_2 = \frac{-\mu_0 I}{2\pi \cdot r_2} \qquad (circular\ arc\ cd)$$

$\vec{B}$ and $\vec{dl}$ antiparallel



arc = (angle) x (radius) = θ r

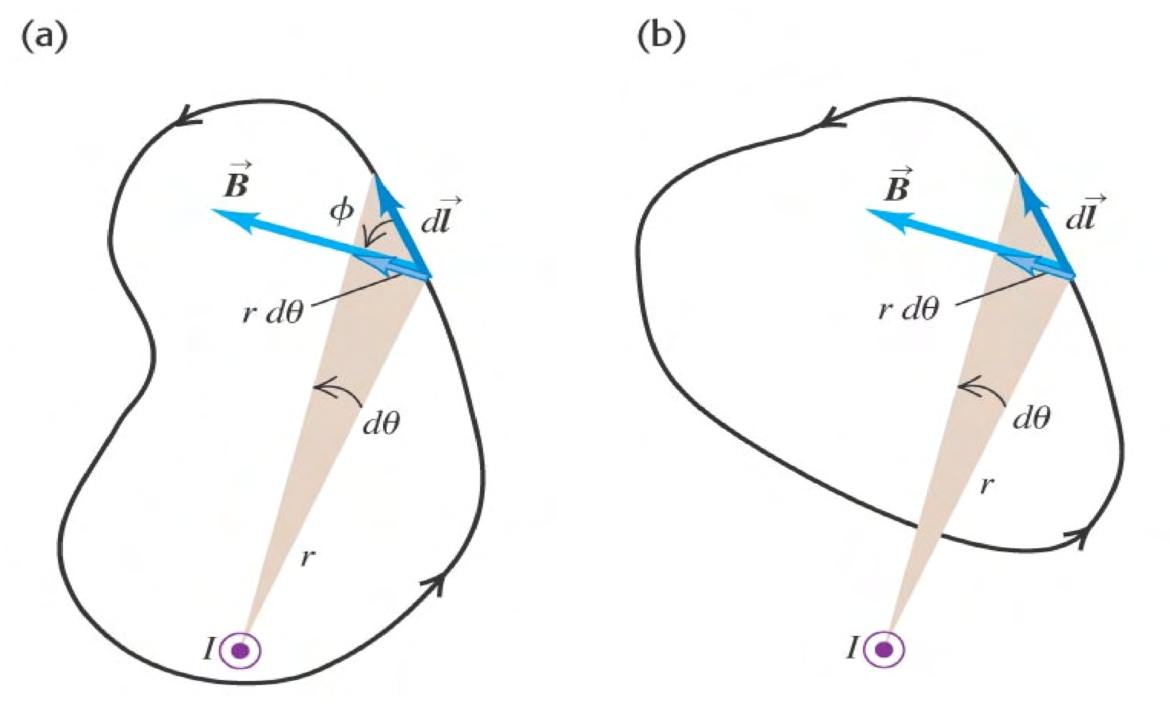
$$\oint \vec{B}\cdot d\vec{l} = \oint B_{//} dl = B_1 \int_a^b dl + (0)\int_b^c dl + (-B_2)\int_c^d dl + (0)\int_d^a dl =$$

$$= \frac{\mu_0 I}{2\pi \cdot r_1}(r_1\theta) + 0 - \frac{\mu_0 I}{2\pi \cdot r_2}(r_2\theta) + 0 = 0$$

$$\vec{B}\cdot d\vec{l} = B\cdot dl\cdot\cos\varphi = B\cdot r\cdot d\theta$$

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi\cdot r}(r d\theta) = \frac{\mu_0 I}{2\pi}\oint d\theta = \mu_0 I$$

- This result does not depend on the shape of the path or on position of the wire inside it.
- If the path does not enclose the wire → $\oint d\theta = 0$ around integration path.



Ampere's Law: General Statement

- The total magnetic field at any point in the path is the vector sum of all fields produced by the individual conductors.

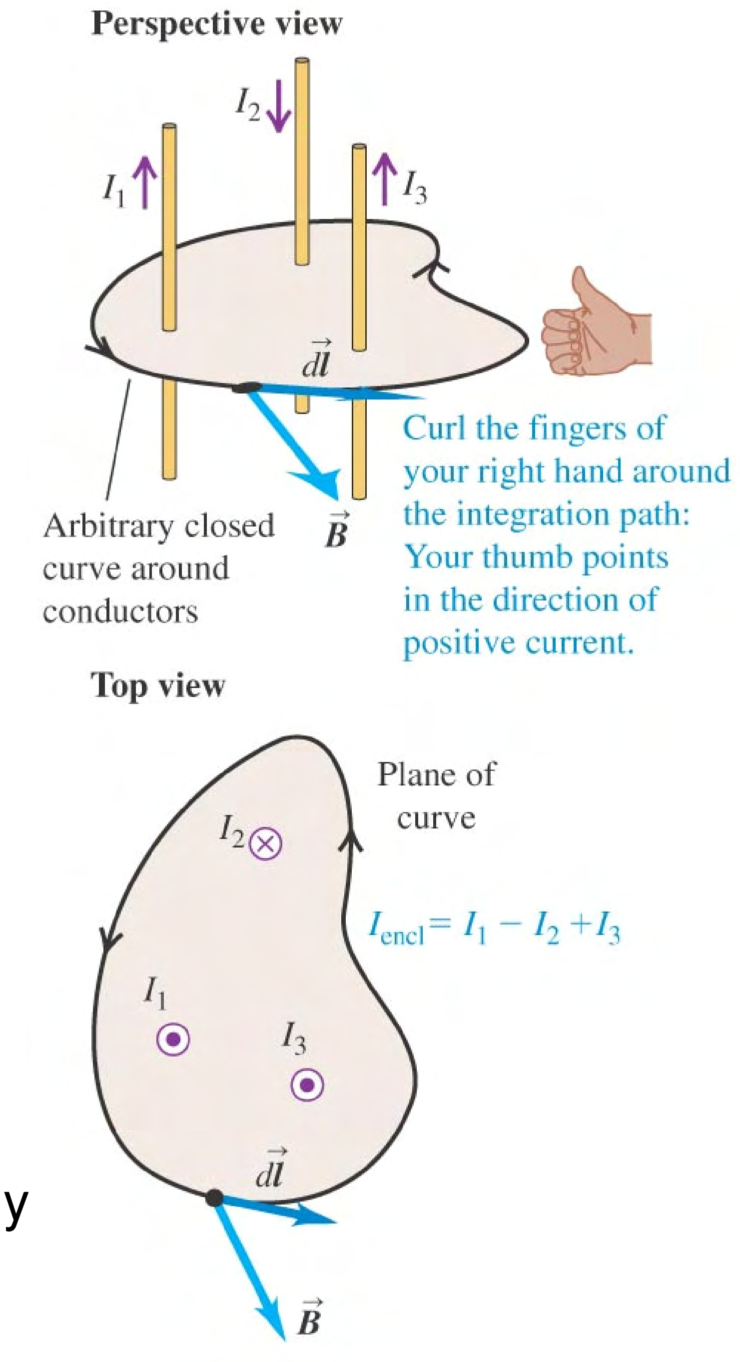
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

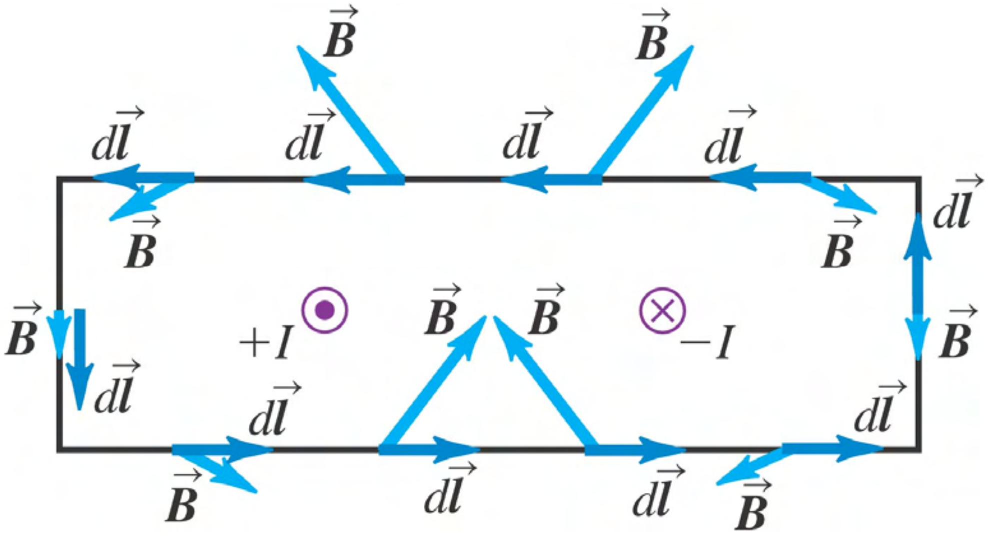
-If the integration path does not enclose a wire →

$$\oint \vec{B} \cdot d\vec{l} = 0$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$

$\oint \vec{B} \cdot d\vec{l} = 0$ → does not mean that B = 0 everywhere along the path, only that $I_{encl} = 0$.

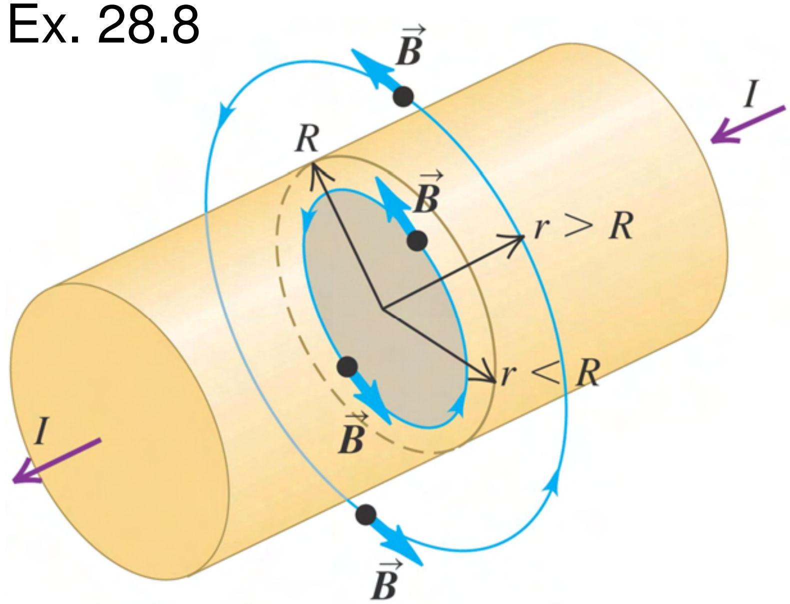


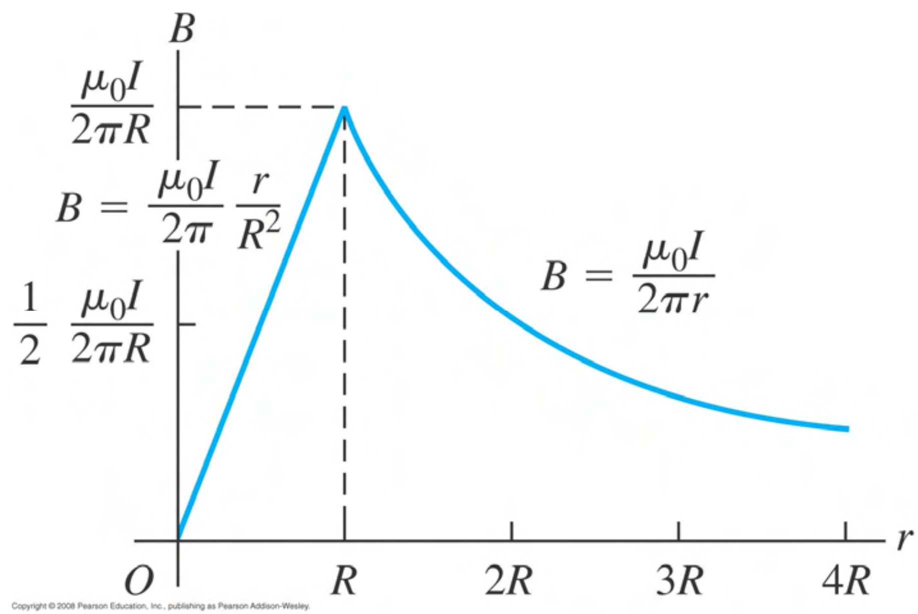


$$\oint \vec{B}\cdot d\vec{l} = 0$$

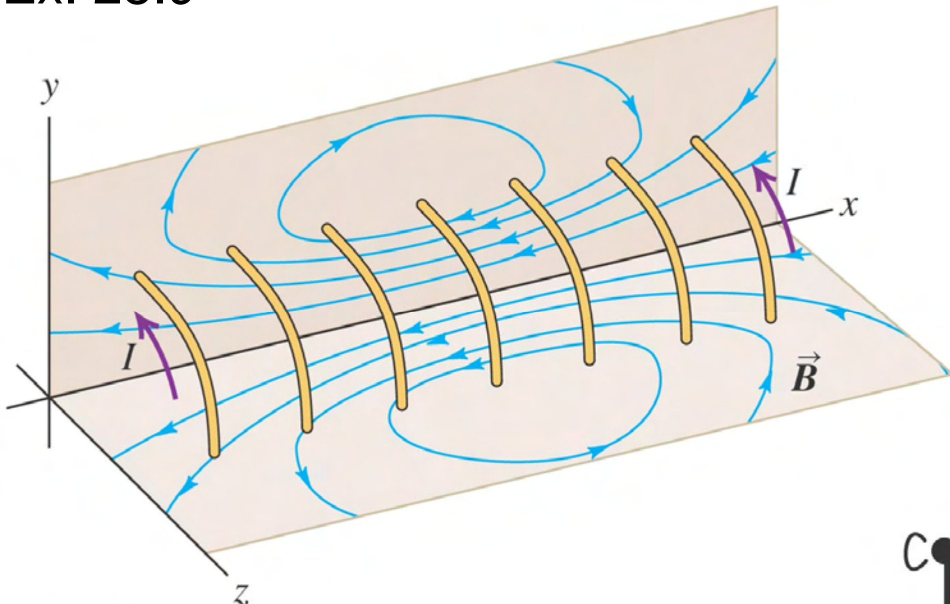
7. Applications of Ampere's Law

Ex. 28.8

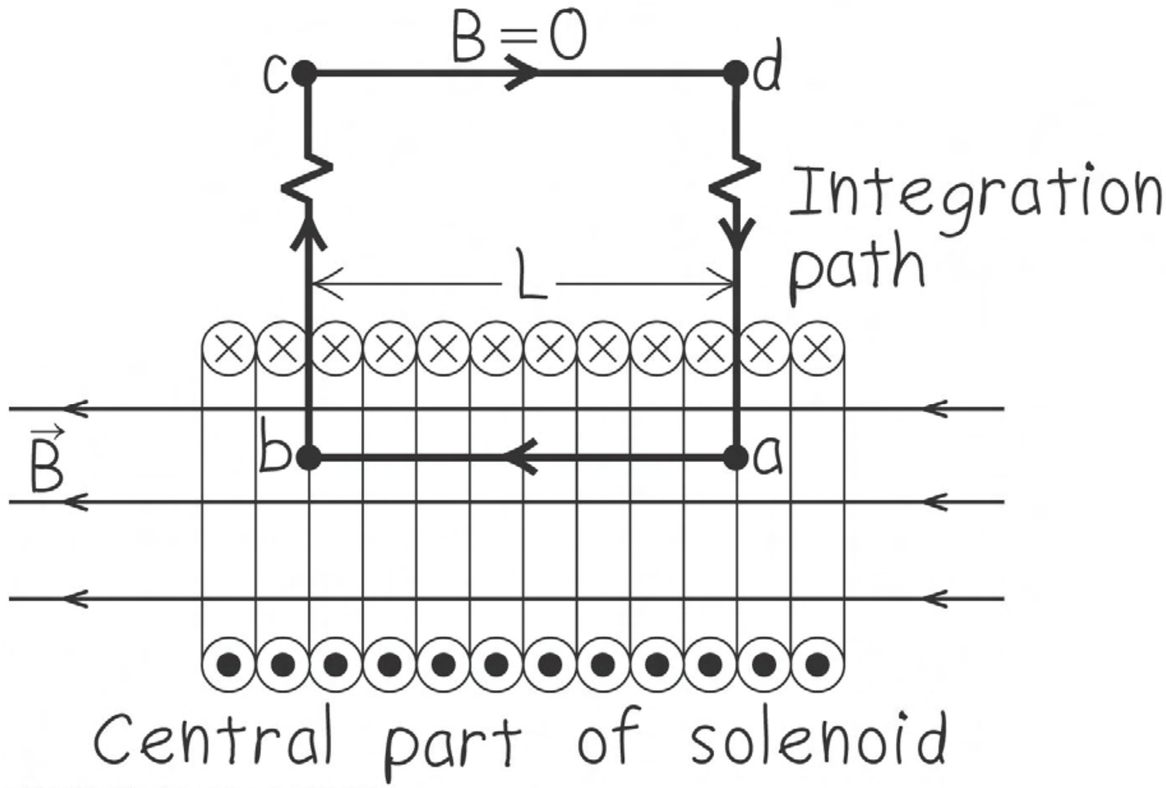




Ex. 28.9

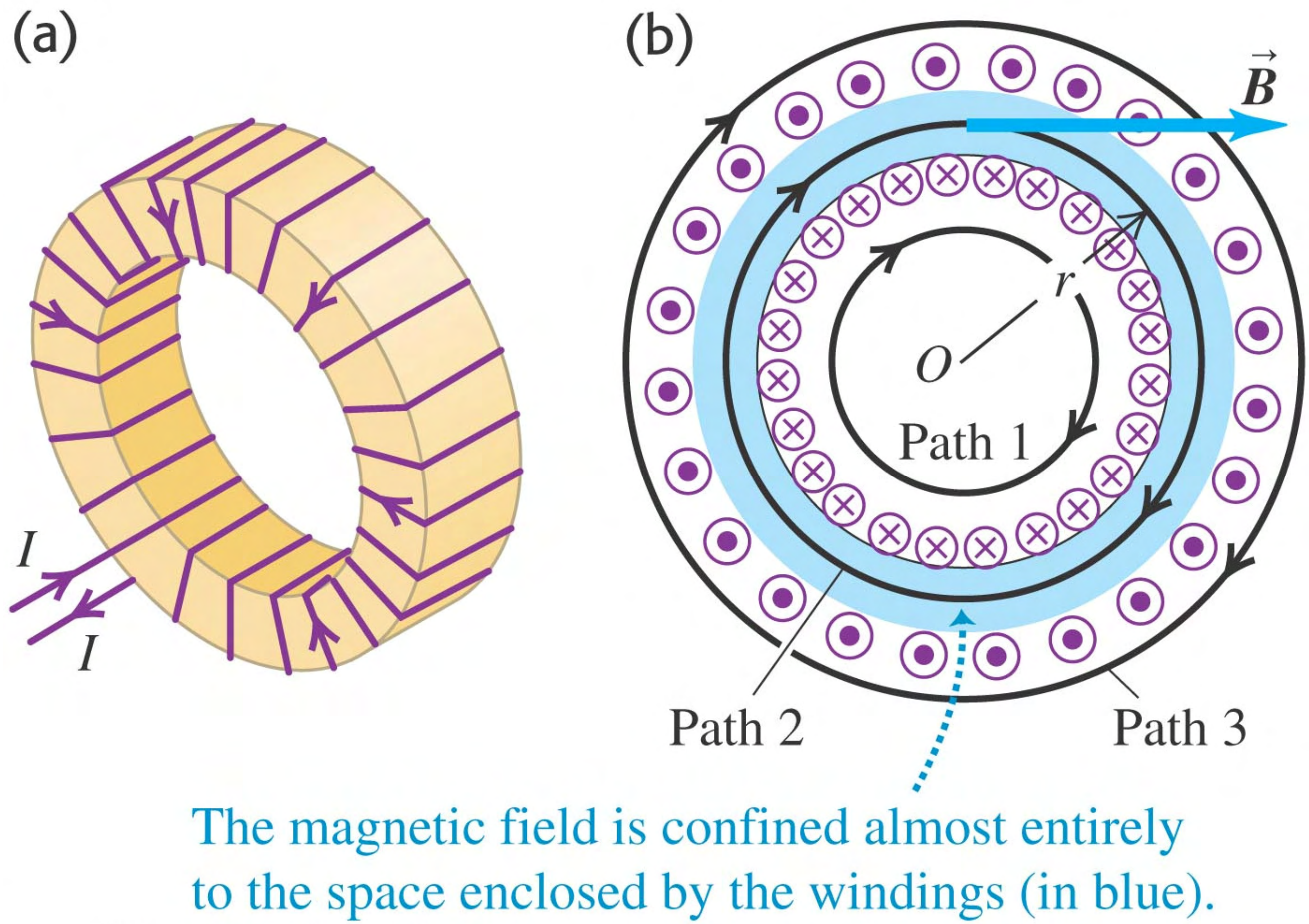


y
I
x
I
B
z



B = 0
c
d
Integration path
L
b
a
B
Central part of solenoid

Ex. 28.10

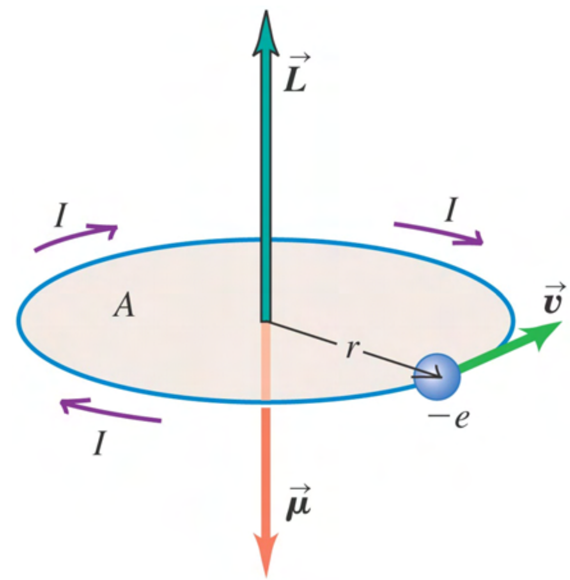


(a)
I
I
(b)
B
r
O
Path 1
Path 2
Path 3
The magnetic field is confined almost entirely
to the space enclosed by the windings (in blue).

8. Magnetic Materials

- Atoms contain moving electrons, e- form microscopy current loops that produce magnetic fields (randomly oriented, no net B_{int}). In some materials, external B_{ext} causes these loops to orient with field, adding to the B_{ext} → magnetized material.

- An electron moving with speed v in a circular orbit of radius r has an angular momentum $\vec{L}$ and oppositely directed orbital magnetic dipole moment $\vec{\mu}$. It also has a spin angular momentum and oppositely directed spin magnetic dipole moment.



Model of electron in an atom

$$I = \frac{e}{T} = \frac{e}{2\pi \cdot r / v} = \frac{ev}{2\pi \cdot r}$$

$$\mu = I \cdot A = \frac{ev}{2\pi \cdot r}(\pi \cdot r^2) = \frac{evr}{2} \quad \rightarrow \quad v = \frac{2\mu}{e \cdot r}$$

Angular momentum of e-:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = r \cdot p = r \cdot mv = r \cdot m \frac{2\mu}{e \cdot r} = \frac{2\mu m}{e}$$

$$\mu = \frac{e}{2m} L$$

- Atomic angular momentum is quantized: L ~ h/2π (its component along a direction is always an integer multiple of this value).

(h = Planck constant = 6.626 x 10^{-34} J s)

- Associated with the quantization of L is an uncertainty in direction of $\vec{L}$ and of $\vec{\mu}$ (since they are related).

$$\mu_B = \frac{e}{2m} L = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi \cdot m} = 9.274 \cdot 10^{-24}\, Am^2 \;\; or\; J/T$$ Bohr Magneton

- Electrons have intrinsic angular momentum (Spin) that is not related to orbital motion, but can be seen as spinning on an axis. The angular momentum has an associated magnetic moment with magnitude ≈ μ_B.

Magnetic Materials

- When magnetic materials are present, the magnetization of the material causes an additional contribution to B.

Paramagnetism

- In an atom, most of the orbital and spin magnetic moments add to zero. However, in case cases the atom has magnetic moment μ_B. If such atom is placed on $\vec{B}$, the field will exert a torque:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

this torque aligns the magnetic moment with magnetic field (position of minimum potential energy). In that position, the current loops add to the externally applied B.

- $\vec{B}$ produced by a current loop is proportional to loop's magnetic dipole moment $\vec{\mu}$ → additional B produced by electron current loops proportional to $\vec{\mu}_{total}$ per unit volume of material (V) = Magnetization.

Magnetization:

$$\vec{M} = \frac{\vec{\mu}_{total}}{V}$$

Units: (A m^2)/m^3 = A/m

- Additional magnetic field due to $\vec{M}$ of material is: $\mu_0 \vec{M}$

- When a magnetized material surrounds a current-carrying conductor, the total $\vec{B}$ is:

$$\vec{B} = \vec{B}_0 + \vec{\mu}_0 \vec{M}$$

B_0 = field caused by the current conductor

behavior typical of a paramagnetic material.

- The magnetic field at any point in a paramagnetic material is greater by the factor K_m (relative permeability of the material) than it will be if the material were replaced by vacuum.

- All equations from this chapter can be adapted to the situation in which the current-carrying conductor is embedded in a paramagnetic material by replacing $\mu_0 \rightarrow K_m \mu_0$.

Permeability:

$$\mu = K_m \mu_0$$

Careful ! → We have used the same symbol "μ" to represent two different physical quantities, the magnetic dipole moment (a vector), and the permeability (an scalar).

Paramagnetism (χ_m small but >0)

Magnetic Susceptibility:

$$\chi_m = K_m - 1$$

K_m and χ_m are dimensionless.

- The tendency of atomic magnetic moments to align themselves parallel to $\vec{B}$ is opposed by random thermal motion → χ_m decreases with increasing T.

$$M = C\frac{B}{T}$$

Curie's Law

C = constant
T = temperature

Table 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20°C$

Material	$\chi_m = K_m - 1\ (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

- An object with magnetic dipoles is attracted to magnet poles. Weak attraction in paramagnetic materials due to thermal randomization of magnetic moments. At low T, M increases → stronger attractive forces.

Diamagnetism

- Total magnetic moment of all atomic current loops = 0 in absence of $\vec{B}$.
- These materials can still show magnetic effects when external $\vec{B}_{ext}$ alters e^- motion in atoms → induced magnetic moment. The additional induced B has opposite direction to B_{ext} (see Chap. 29).
- An induced current always tends to cancel the field change that caused it.
- Susceptibility $\chi < 0$ and small.
- Relative permeability K_m slightly less than 1.

Ferromagnetism (FM)

- Strong interactions between atomic magnetic moments cause them to align parallel to each other in regions (magnetic domains) even when no B_{ext} is present. Ex: Fe, Co.

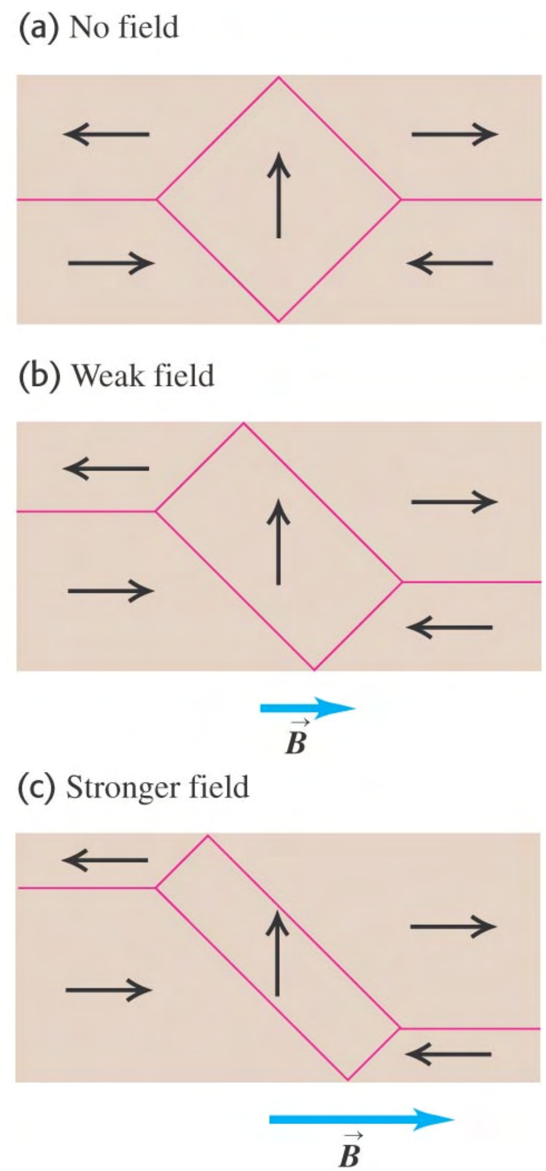
- If no B_{ext} → domain magnetizations are randomly oriented.

- If B_{ext} → domain M tend to align parallel to field, domain boundaries shift, domains with M parallel to B grow.

- $K_m >> 1$

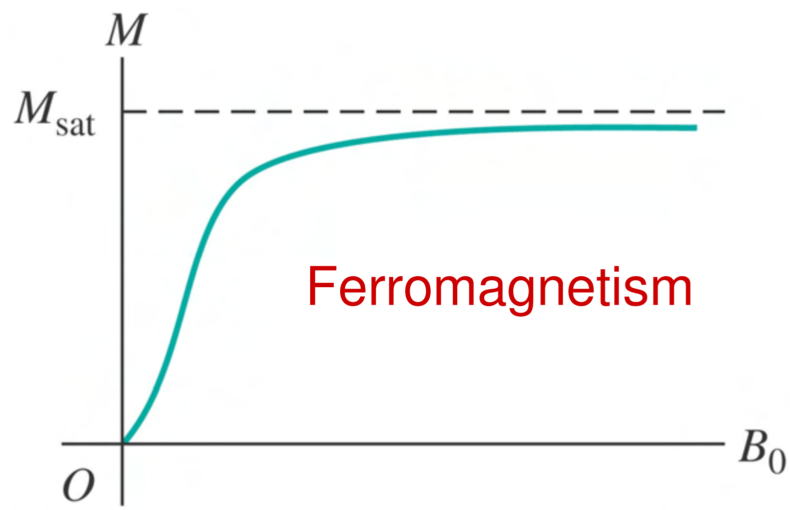
- Ferromagnets are strongly magnetized by B_{ext} and attracted to magnet.

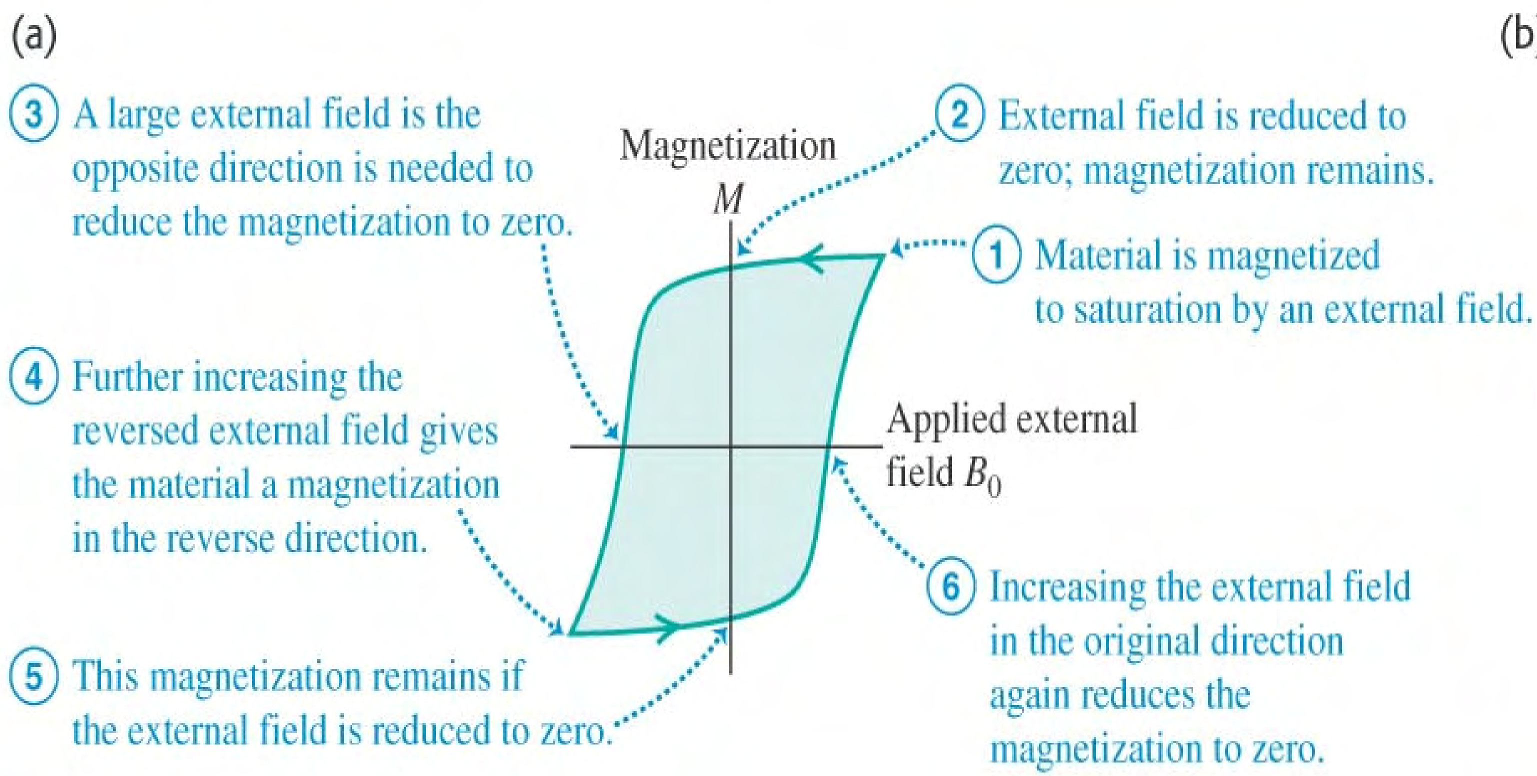
-Saturation magnetization: M reached when all magnetic moments from FM are aligned // B_{ext}. Once M_{sat} is reached, increasing B_{ext} will not change M.



- For many FM materials, the relation between M and $B_{ext} = B_0$ is different when you increase or decrease B_0 → hysteresis loop.

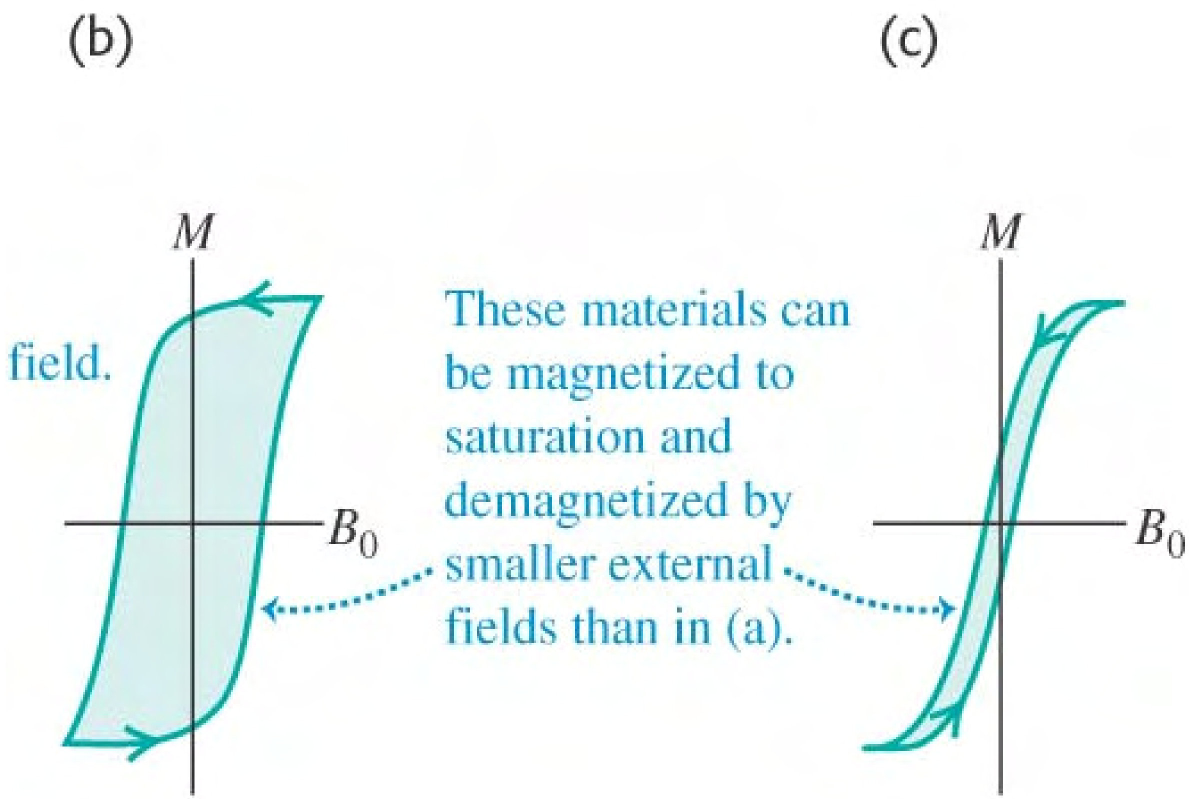
- In permanent magnets, after M_{sat} is reached and B_0 is reduced to zero, some M remains (remnant magnetization).
To reduce M to zero requires B to change direction.

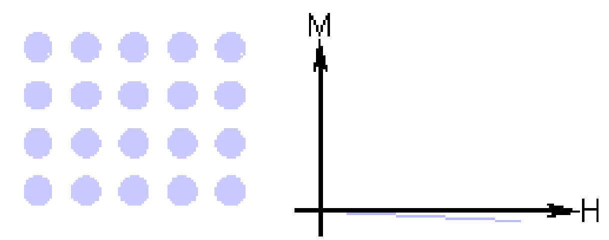
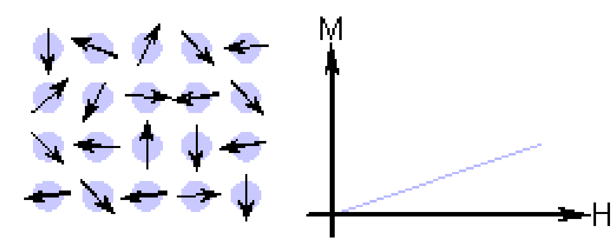
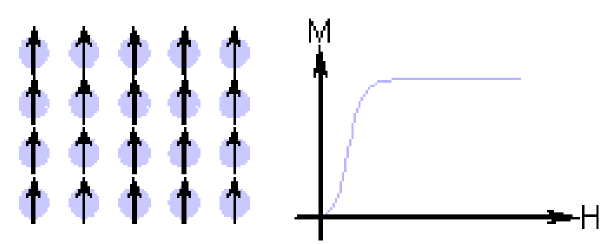
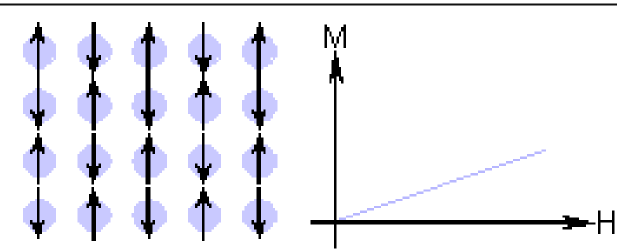
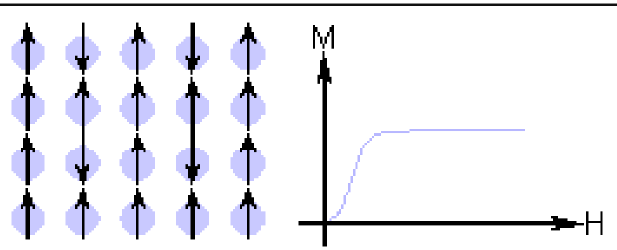




Ferromagnetism

- Magnetizing and de-magnetizing a material that has hysteresis involves dissipation of energy → materials' temperature increases.



Type of Magnetism	Susceptibility	Atomic / Magnetic Behaviour		Example / Susceptibility	
Diamagnetism	Small & negative.	Atoms have no magnetic moment		Au Cu	-2.74×10^{-6} -0.77×10^{-6}
Paramagnetism	Small & positive.	Atoms have randomly oriented magnetic moments		β-Sn Pt Mn	0.19×10^{-6} 21.04×10^{-6} 66.10×10^{-6}
Ferromagnetism	Large & positive, function of applied field, microstructure dependent.	Atoms have parallel aligned magnetic moments		Fe	~100,000
Antiferromagnetism	Small & positive.	Atoms have mixed parallel and anti-parallel aligned magnetic moments		Cr	3.6×10^{-6}
Ferrimagnetism	Large & positive, function of applied field, microstructure dependent	Atoms have anti-parallel aligned magnetic moments		Ba ferrite	~3