Chapter 27 – Magnetic Field and Magnetic Forces

- Magnetism
- Magnetic Field
- Magnetic Field Lines and Magnetic Flux
- Motion of Charged Particles in a Magnetic Field
- Applications of Motion of Charged Particles
- Magnetic Force on a Current-Carrying Conductor
- Force and Torque on a Current Loop
- The Direct-Current Motor
- The Hall effect

1) A moving charge or collection of moving charges (e.g. electric current) produces a magnetic field. (Chap. 28).

2) A second current or charge responds to the magnetic field and experiences a magnetic force. (Chap. 27).

1. Magnetism

Permanent magnets: exert forces on each other as well as on unmagnetized Fe pieces.

- The needle of a compass is a piece of magnetized Fe.

- If a bar-shaped permanent magnet is free to rotate, one end points north (north pole of magnet).

- An object that contains Fe is not by itself magnetized, it can be attracted by either the north or south pole of permanent magnet.

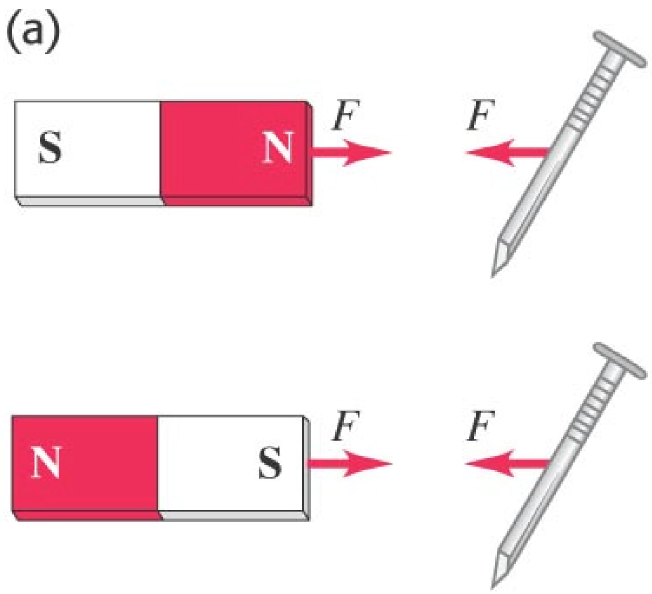
- A bar magnet sets up a magnetic field in the space around it and a second body responds to that field. A compass needle tends to align with the magnetic field at the needle's position.

1. Magnetism

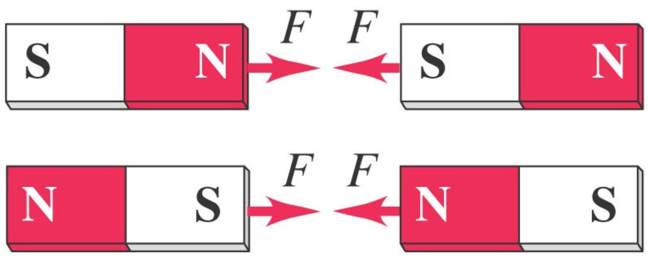
- Magnets exert forces on each other just like charges. You can draw magnetic field lines just like you drew electric field lines.

- Magnetic north and south pole's behavior is not unlike electric charges. For magnets, like poles repel and opposite poles attract.

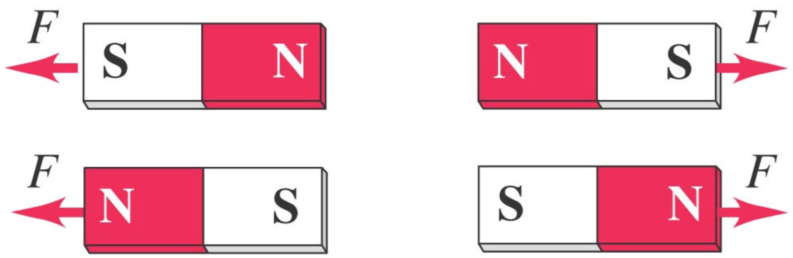
- A permanent magnet will attract a metal like iron with either the north or south pole.



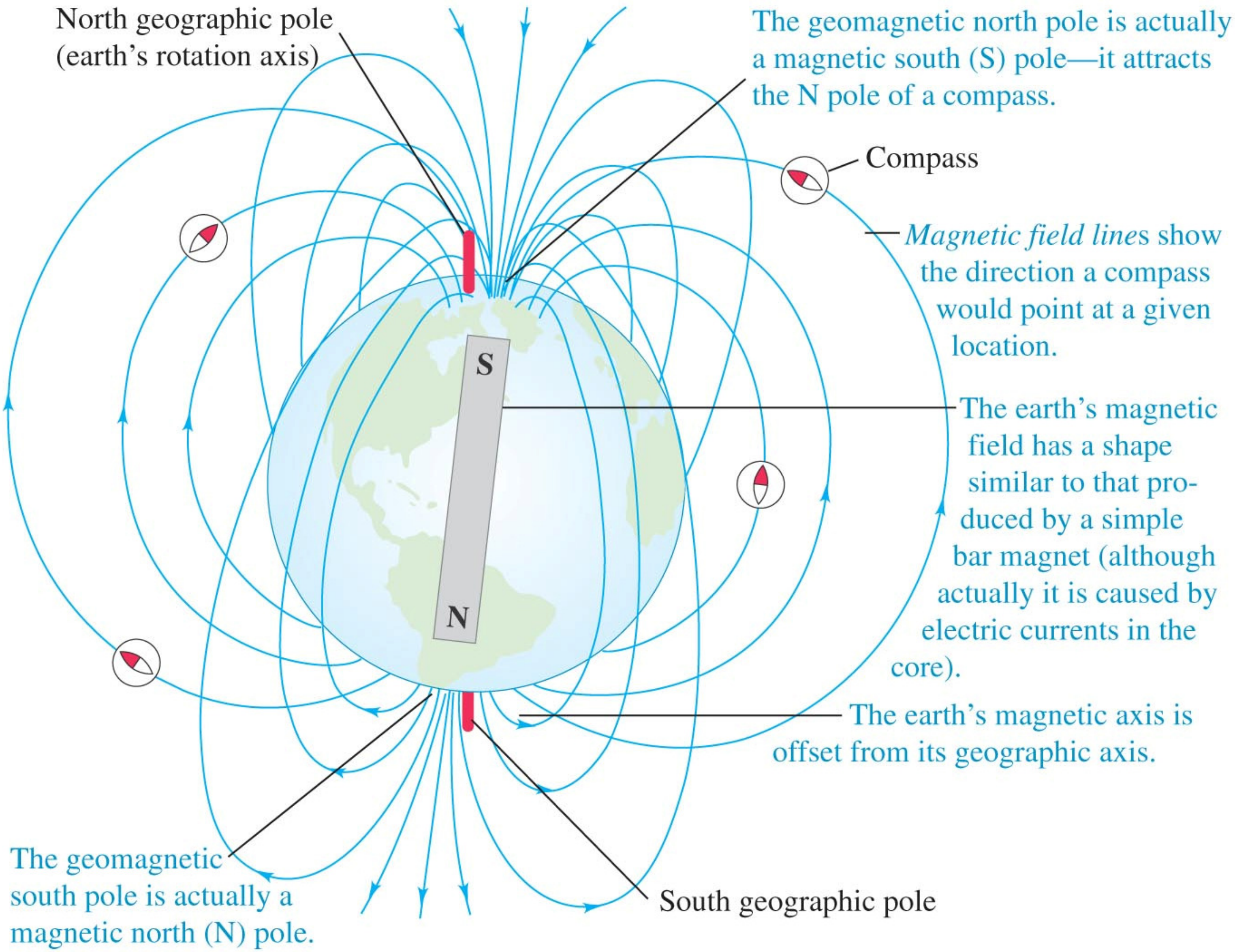
(a) Opposite poles attract.



(b) Like poles repel.



Magnetic poles about our planet



The geomagnetic north pole is actually a magnetic south (S) pole—it attracts the N pole of a compass.

Magnetic field lines show the direction a compass would point at a given location.

The earth's magnetic field has a shape similar to that produced by a simple bar magnet (although actually it is caused by electric currents in the core).

The earth's magnetic axis is offset from its geographic axis.

The geomagnetic south pole is actually a magnetic north (N) pole.

Magnetic declination / magnetic variation: the Earth’s magnetic axis is not parallel to its geographic axis (axis of rotation) → a compass reading deviates from geographic north.

Magnetic inclination: the magnetic field is not horizontal at most of earth’s surface, its angle up or down. The magnetic field is vertical at magnetic poles.

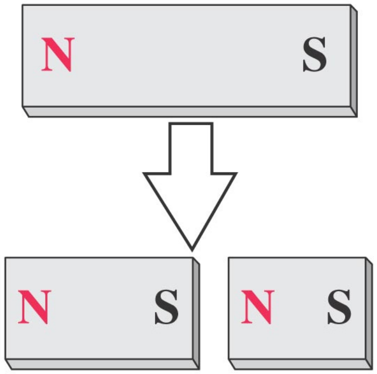
Magnetic Poles versus Electric Charge

- We observed monopoles in electricity. A (+) or (-) alone was stable, and field lines could be drawn around it.

- Magnets cannot exist as monopoles. If you break a bar magnet between N and S poles, you get two smaller magnets, each with its own N and S pole.

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Breaking a magnet in two ...



... yields two magnets, not two isolated poles.

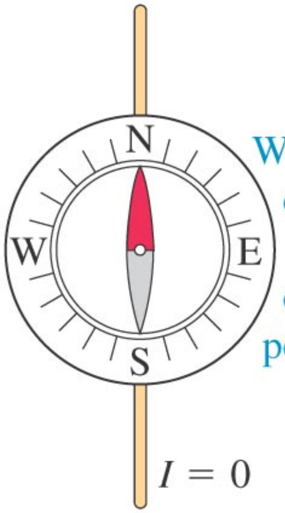
-In 1820, Oersted ran experiments with conducting wires run near a sensitive compass. The orientation of the wire and the direction of the flow both moved the compass needle.

- Ampere / Faraday / Henry → moving a magnet near a conducting loop can induce a current.

- The magnetic forces between two bodies are due to the interaction between moving electrons in the atoms.

- Inside a magnetized body (permanent magnet) there is a coordinated motion of certain atomic electrons. Not true for unmagnetized objects.

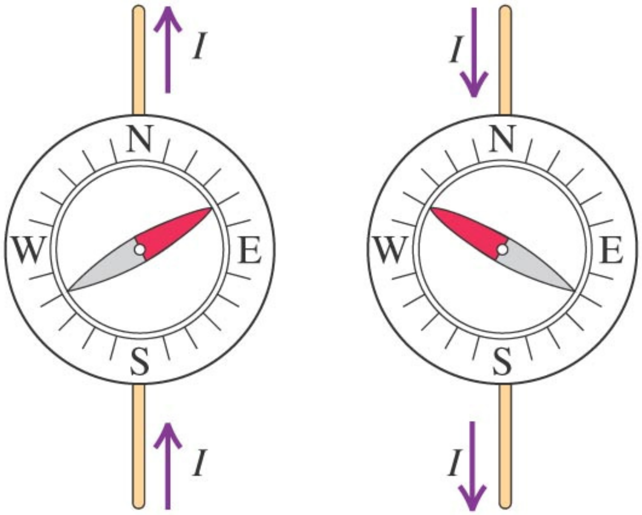
(a)



When the wire carries no current, the compass needle points north.

(b)

When the wire carries a current, the compass needle deflects. The direction of deflection depends on the direction of the current.



2. Magnetic Field

Electric field:

1) A distribution of electric charge at rest creates an electric field E in the surrounding space.
2) The electric field exerts a force $\vec{F}_E = q\,\vec{E}$ on any other charges in presence of that field.

Magnetic field:

1) A moving charge or current creates a magnetic field in the surrounding space (in addition to $\vec{E}$).
2) The magnetic field exerts a force $\vec{F}_m$ on any other moving charge or current present in that field.

- The magnetic field is a vector field → vector quantity associated with each point in space.

$$F_m = |q| v_{\perp} B = |q| v \; B \sin\varphi$$

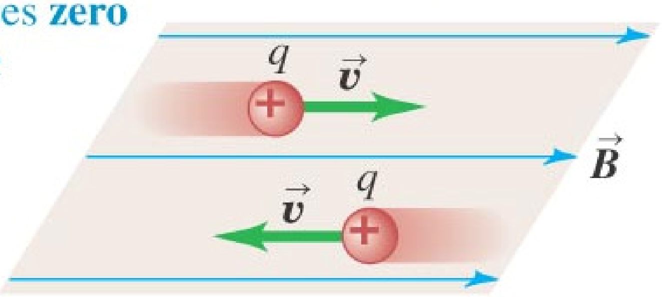
$$\boxed{\vec{F}_m = q\vec{v} \times \vec{B}}$$

- $\vec{F}_m$ is always perpendicular to $\vec{B}$ and $\vec{v}$.

2. Magnetic Field

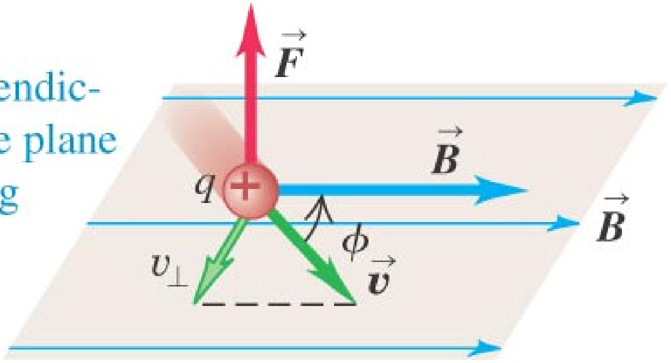
- The moving charge interacts with the fixed magnet. The force between them is at a maximum when the velocity of the charge is perpendicular to the magnetic field.

A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**



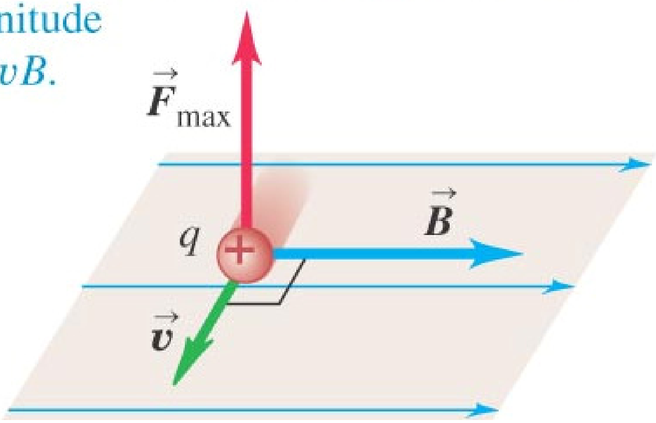
Interaction of magnetic force and charge

A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB\sin\phi$.



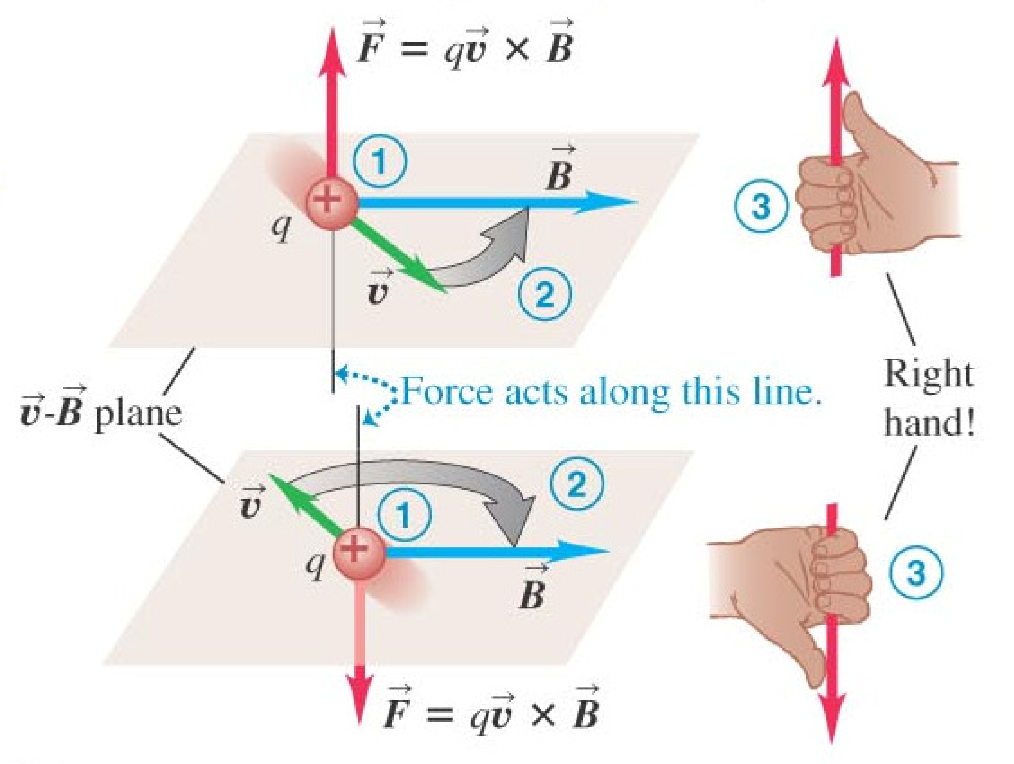
$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{\max} = qvB$.

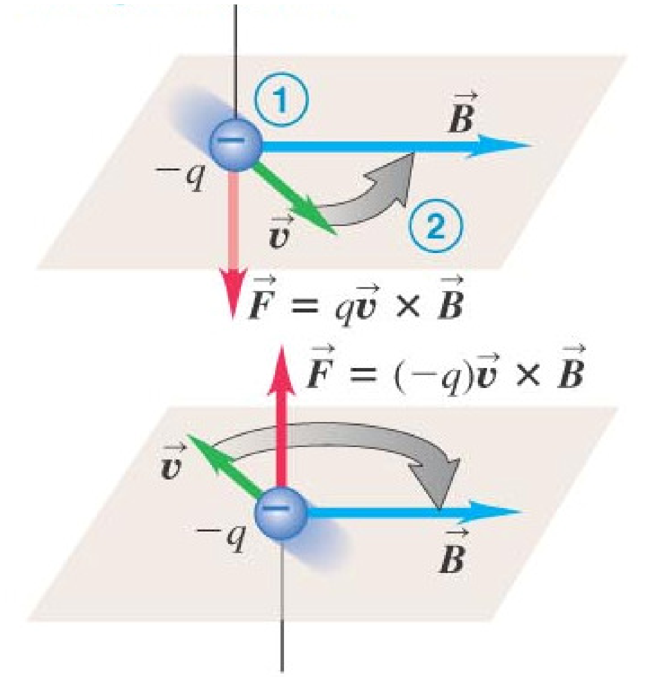


Right Hand Rule

Positive charge moving in magnetic field
→direction of force follows right hand rule



Negative charge → F direction
contrary to right hand rule.

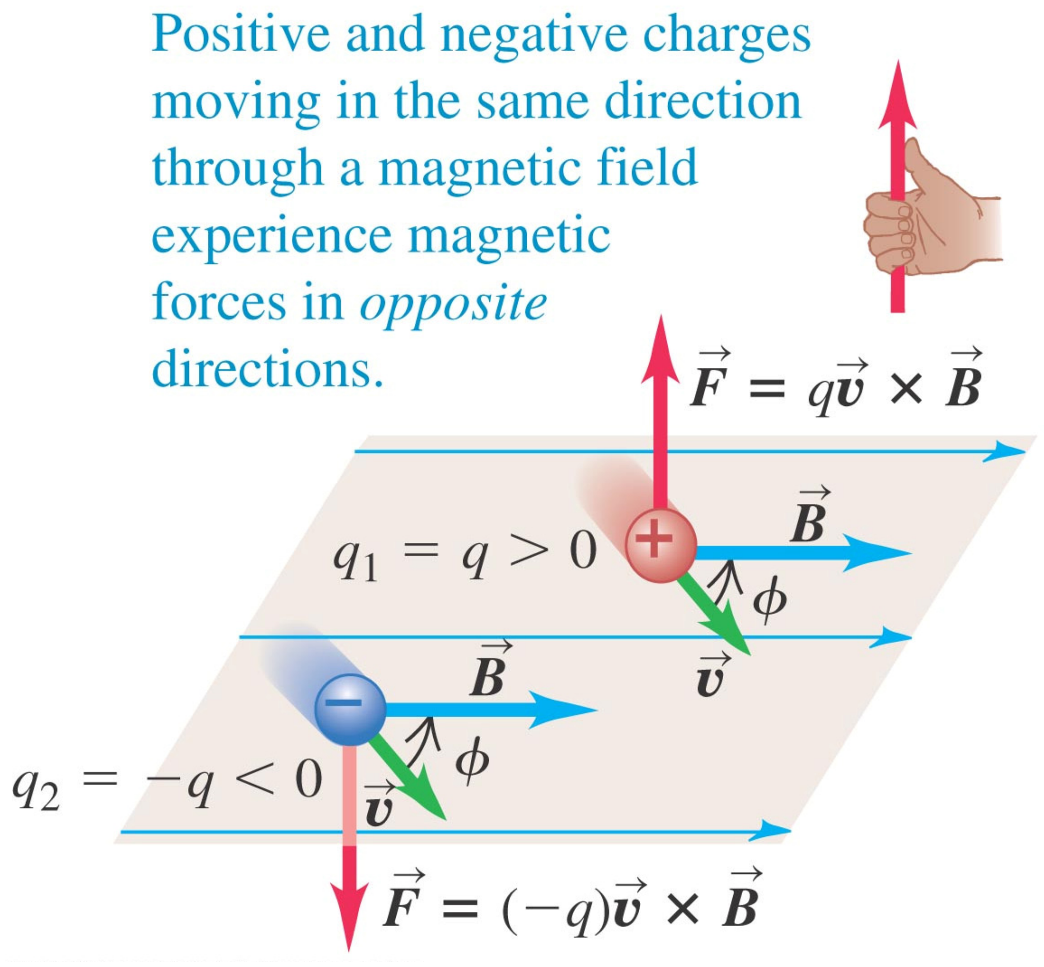


$$F = |q| v B_\perp$$

Units: 1 Tesla = 1 N s / C m = 1 N/A m

1 Gauss = 10^{-4} T

Right Hand Rule



Positive and negative charges moving in the same direction through a magnetic field experience magnetic forces in *opposite* directions.

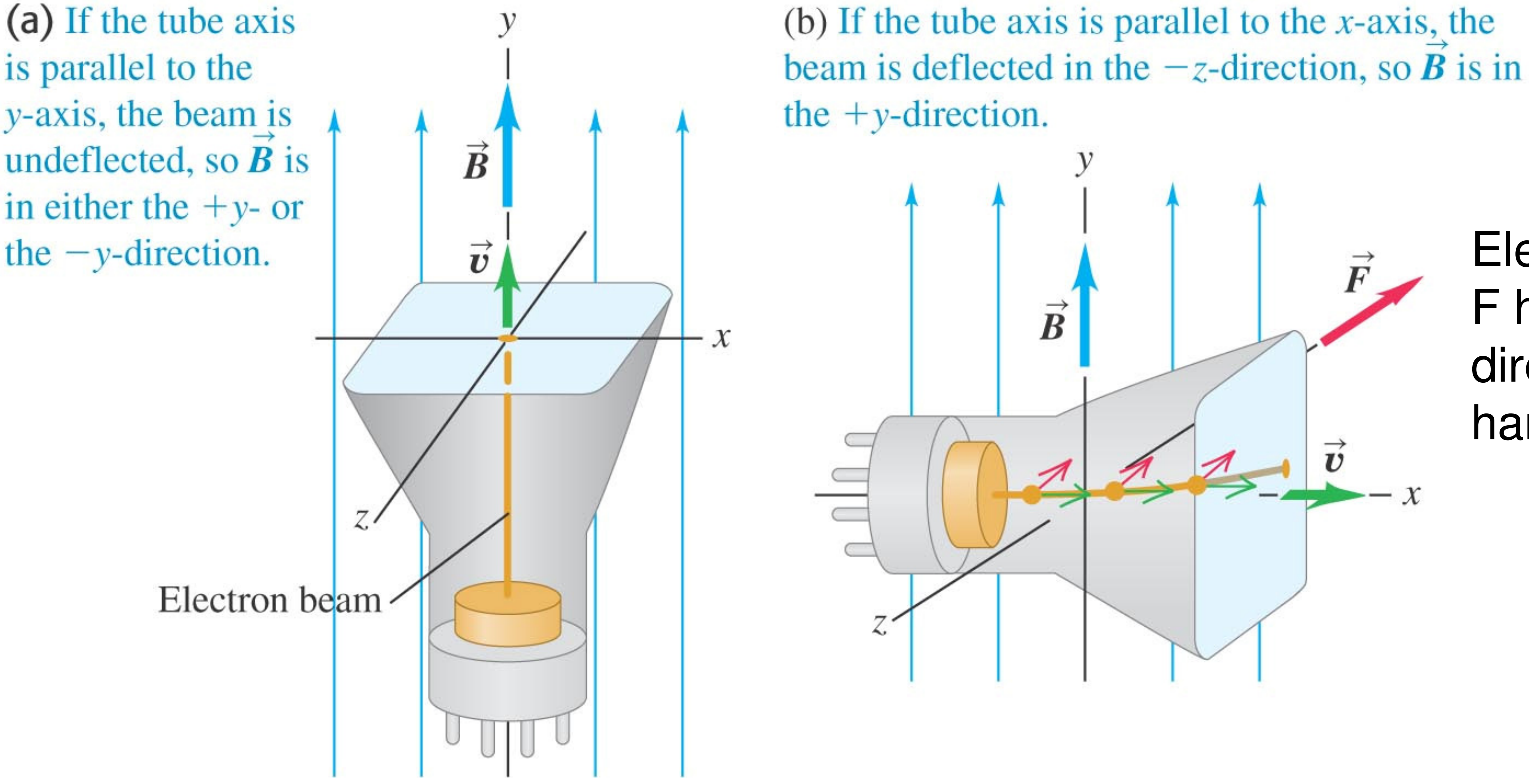
If charged particle moves in region where both, E and B are present:

$$\boxed{\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})}$$

Measuring Magnetic Fields with Test Charges

Ex: electron beam in a cathode X-ray tube.

- In general, if a magnetic field (B) is present, the electron beam is deflected. However this is not true if the beam is // to B ($\varphi = 0, \pi$ → F=0 → no deflection).



(a) If the tube axis is parallel to the y-axis, the beam is undeflected, so $\vec{B}$ is in either the $+y$- or the $-y$-direction.

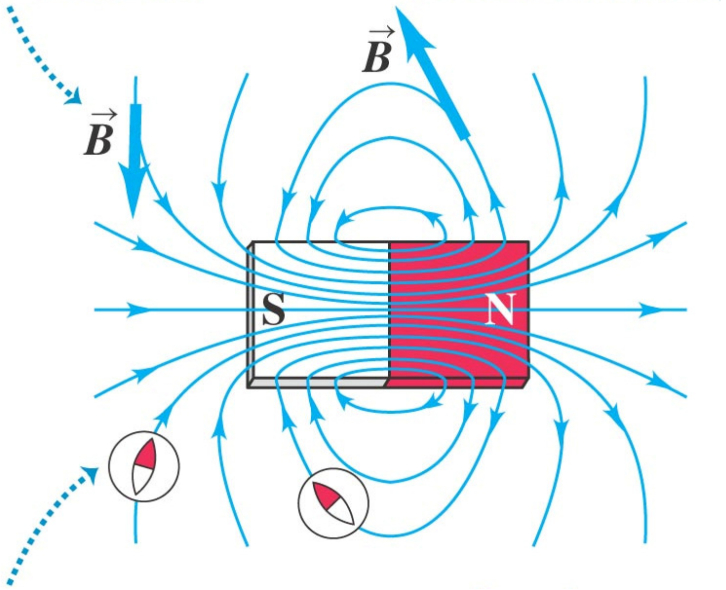
(b) If the tube axis is parallel to the x-axis, the beam is deflected in the $-z$-direction, so $\vec{B}$ is in the $+y$-direction.

Electron q< 0 → F has contrary direction to right hand rule

No deflection → $\vec{F} = 0$ → $\vec{v} \,//\, \vec{B}$

Deflection → $\vec{F} \neq 0$ → $\vec{F} \perp \vec{v}, \vec{B}$

3. Magnetic Field Lines and Magnetic Flux



- Magnetic field lines may be traced from N toward S (analogous to the electric field lines).

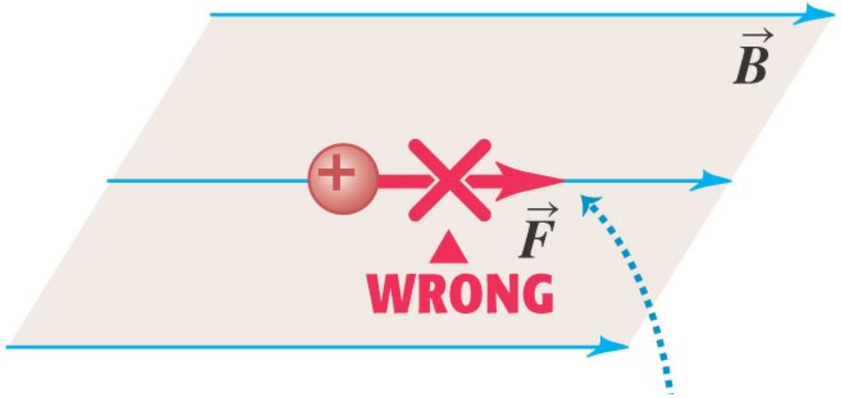
- At each point they are tangent to magnetic field vector.

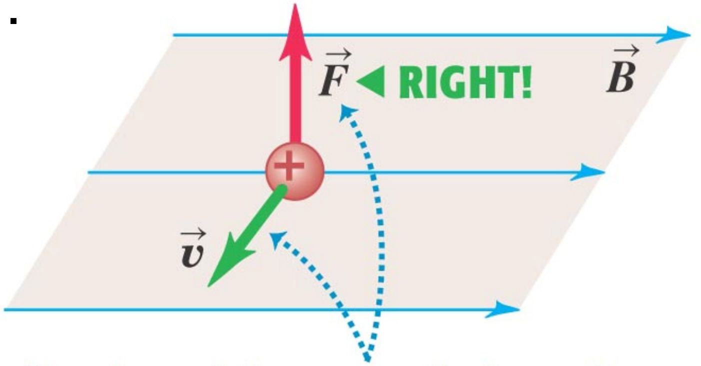
- The more densely packed the field lines, the stronger the field at a point.

- Field lines never intersect.

- The field lines point in the same direction as a compass (from N toward S).

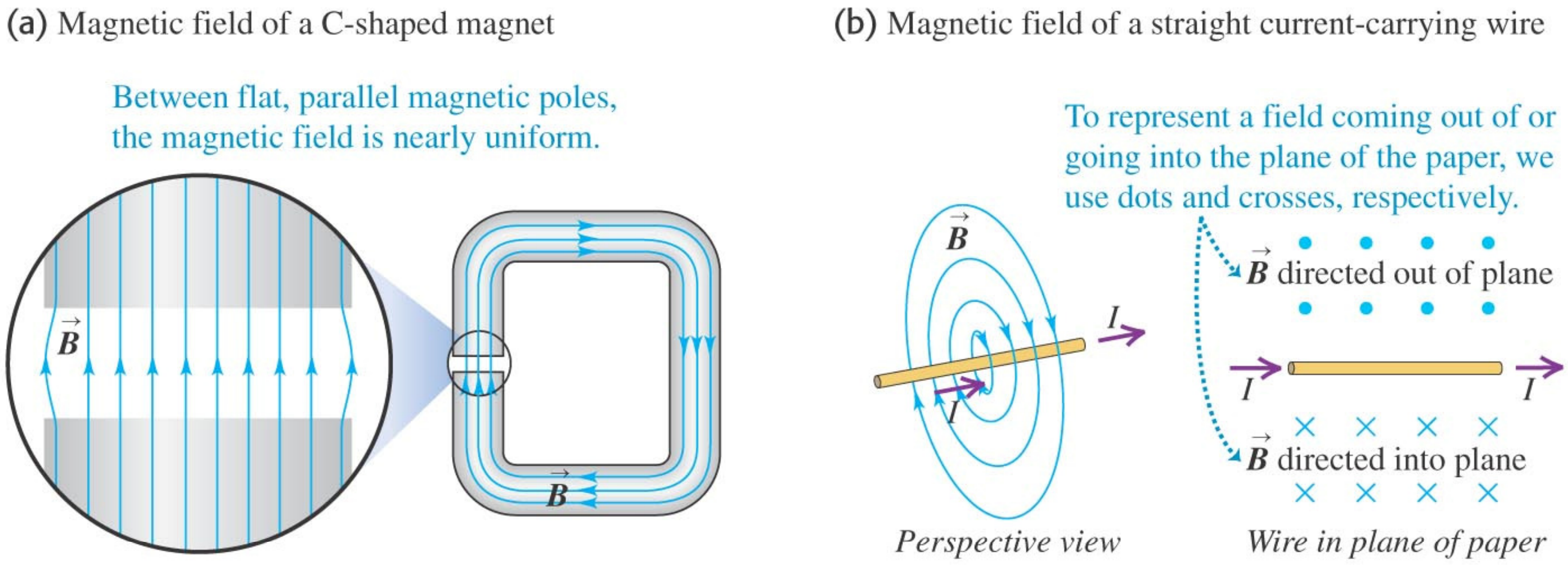
- Magnetic field lines are not "lines of force".





The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

- Magnetic field lines have no ends → they continue through the interior of the magnet.

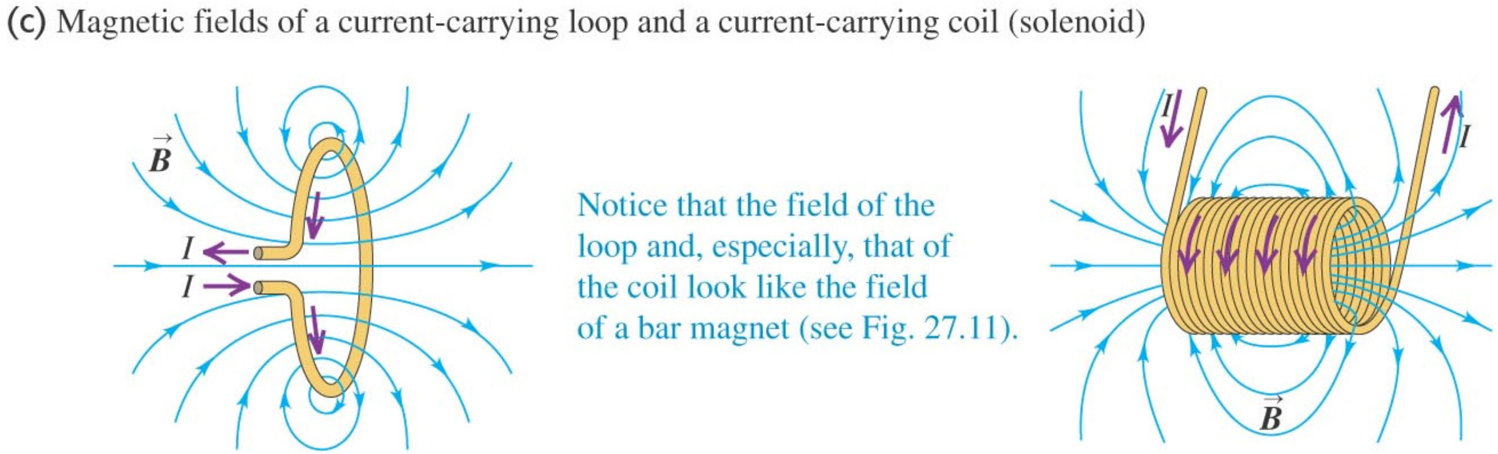


(a) Magnetic field of a C-shaped magnet

Between flat, parallel magnetic poles, the magnetic field is nearly uniform.

(b) Magnetic field of a straight current-carrying wire

To represent a field coming out of or going into the plane of the paper, we use dots and crosses, respectively.

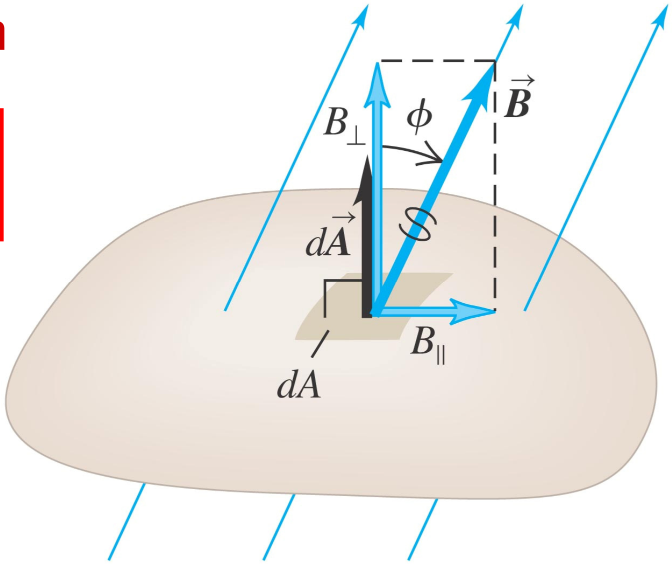


(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)

Notice that the field of the loop and, especially, that of the coil look like the field of a bar magnet (see Fig. 27.11).

Magnetic Flux and Gauss's Law for Magnetism

$$\boxed{\Phi_B = \int B_\perp dA = \int B\cos\varphi \cdot dA = \int \vec{B}\cdot d\vec{A}}$$



- Magnetic flux is a scalar quantity.

- If $\vec{B}$ is uniform: $\boxed{\Phi_B = B_\perp A = BA\cos\varphi}$

Units: 1 Weber (1 Wb = 1 T m^2 = 1 N m / A)

- Difference with respect to electric flux → the total magnetic flux through a closed surface is always zero. This is because there is no isolated magnetic charge ("monopole") that can be enclosed by the Gaussian surface.

$$\boxed{\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0}$$

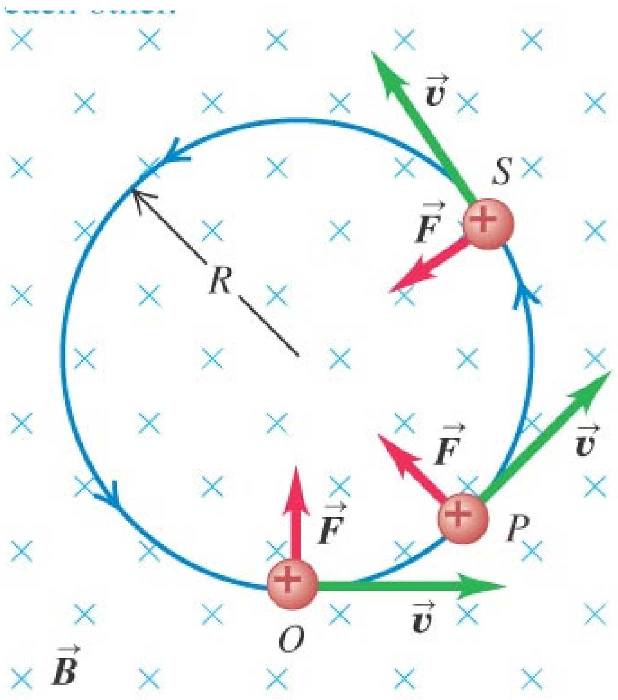
$$\boxed{B = \frac{d\Phi_B}{dA_\perp}}$$

- The magnetic field is equal to the flux per unit area across an area at right angles to the magnetic field = magnetic flux density.

4. Motion of Charged Particles in a Magnetic Field

$$\boxed{\vec{F}_m = q\vec{v}\times\vec{B}}$$

- Magnetic force perpendicular to $\vec{v}$ → it cannot change the magnitude of the velocity, only its direction.
- $\vec{F}$ does not have a component parallel to particle's motion → cannot do work.
- Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.
- Magnitudes of F and v are constant (v perp. B) → uniform circular motion.



$$F = |q|\cdot v\cdot B = m\frac{v^2}{R}$$

Radius of circular orbit in magnetic field:

$$\boxed{R = \frac{mv}{|q|B}}$$

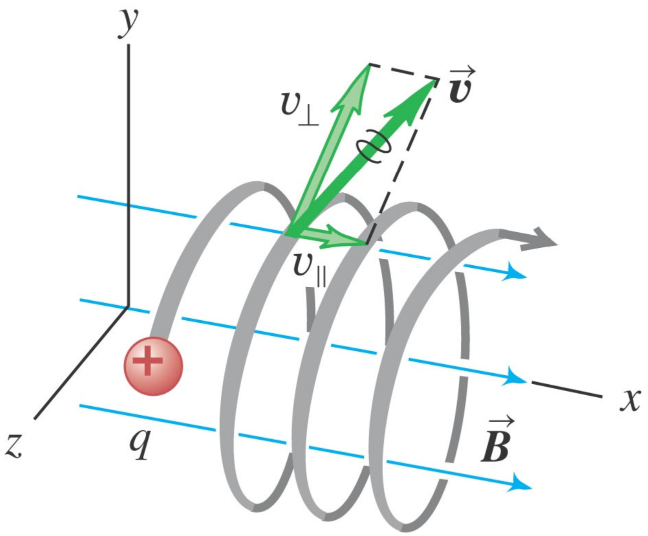
+ particle → counter-clockwise rotation.
- particle → clockwise rotation.

Angular speed: ω = v/R → $\boxed{\omega = v\frac{|q|B}{mv} = \frac{|q|B}{m}}$

Cyclotron frequency: $f = \omega/2\pi$

- If v is not perpendicular to B → $v_{//}$ (parallel to B) constant because $F_{//} = 0$ → particle moves in a helix. (R same as before, with $v = v_\perp$).

This particle's motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.



A charged particle will move in a plane perpendicular to the magnetic field.

5. Applications of Motion of Charged Particles

Velocity selector

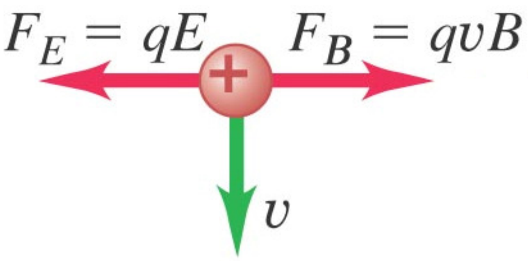
- Particles of a specific speed can be selected from the beam using an arrangement of E and B fields.

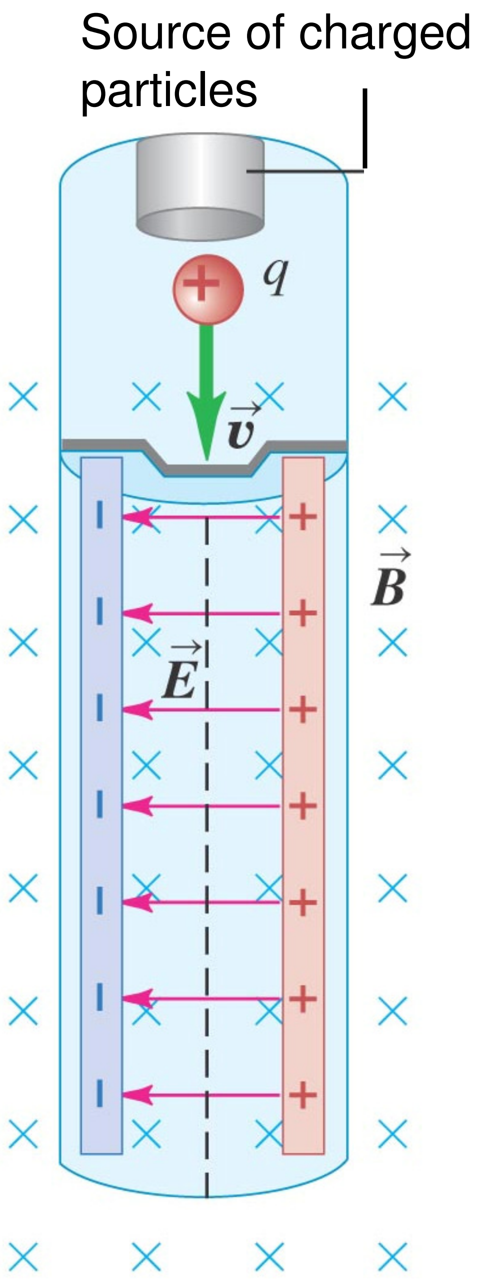
- F_m (magnetic) for + charge towards right ($q\,v\,B$).

- F_E (electric) for + charge to left ($q\,E$).

- $F_{net} = 0$ if $F_m = F_E \rightarrow -qE + q\,v\,B = 0 \rightarrow \boxed{v = E/B}$

- Only particles with speed E/B can pass through without being deflected by the fields.





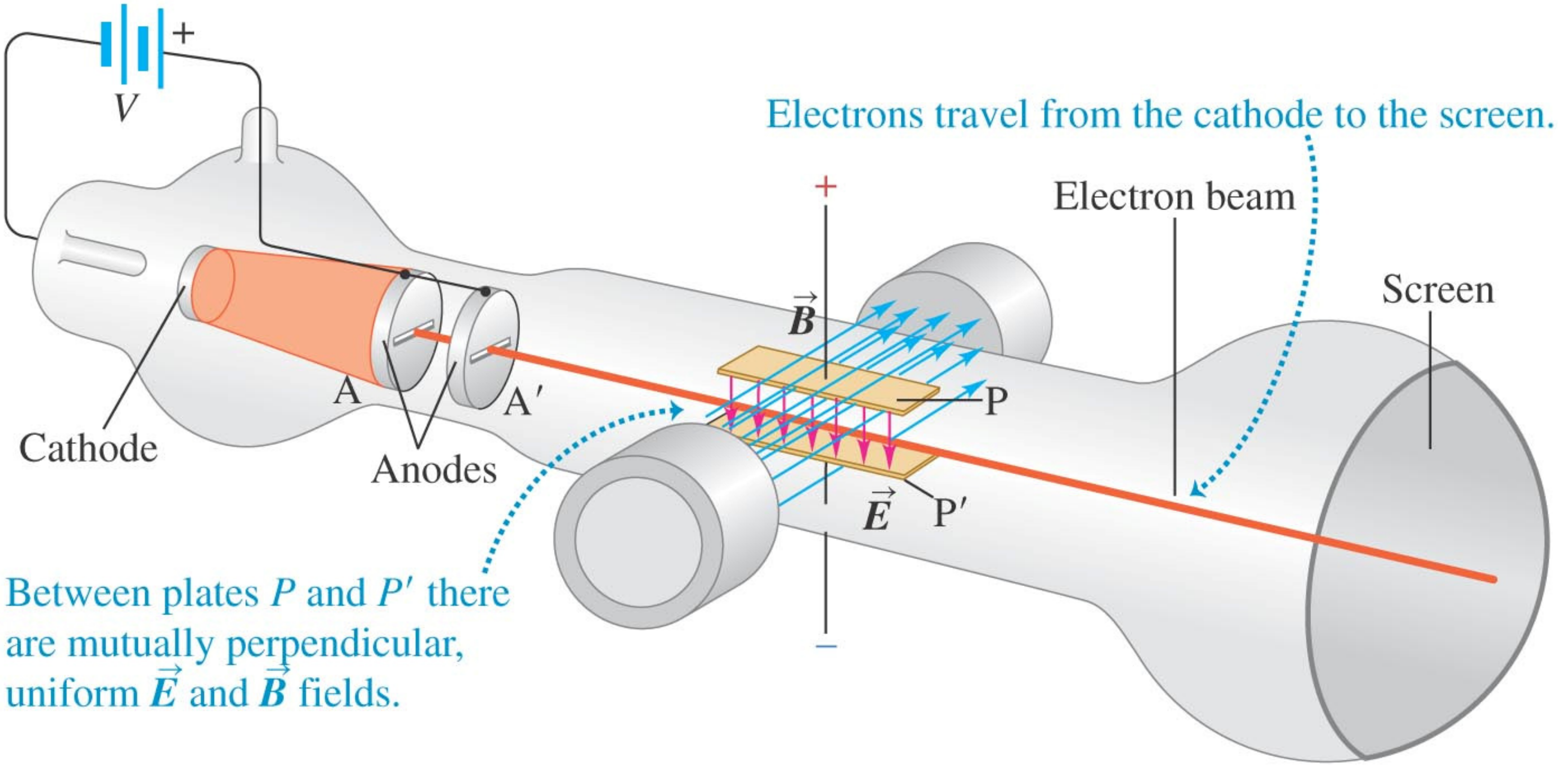
Thomson's *e/m* Experiment

$\Delta E = \Delta K + \Delta U = 0 \rightarrow 0.5\ m\ v^2 = U = e\ V$

$$v = \frac{E}{B} = \sqrt{\frac{2eV}{m}}$$

$$\boxed{\frac{e}{m} = \frac{E^2}{2VB^2}}$$

e/m does not depend on the cathode material or residual gas on tube → particles in the beam (electrons) are a common constituent of all matter.



Electrons travel from the cathode to the screen.

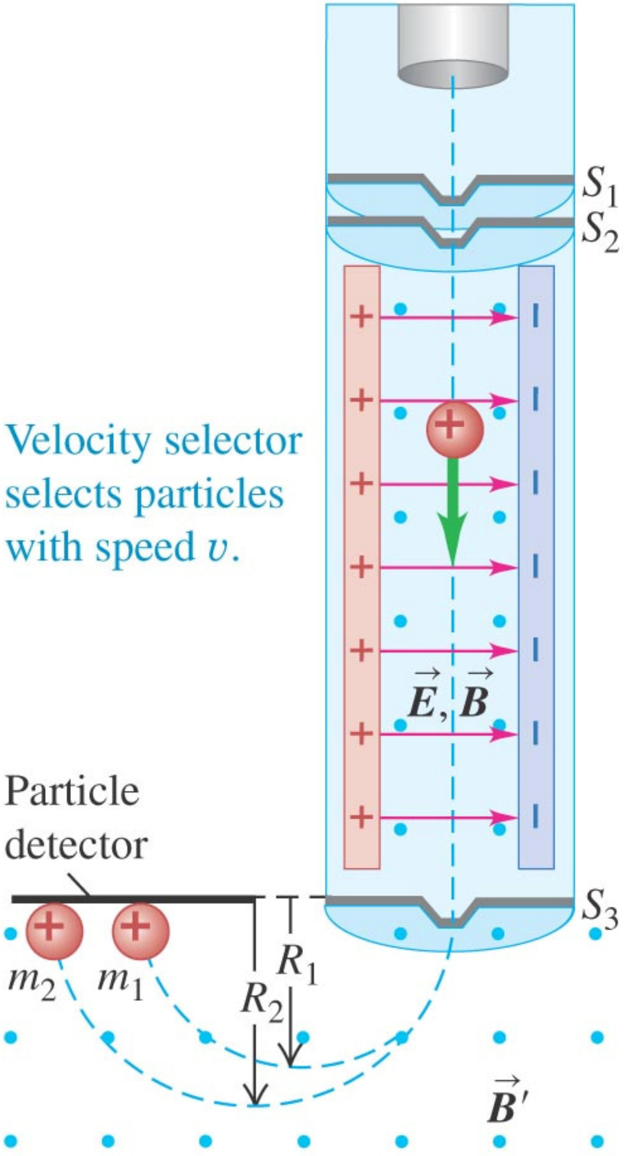
Between plates P and P' there are mutually perpendicular, uniform $\vec{E}$ and $\vec{B}$ fields.

Mass Spectrometer

- Using the same concept as Thompson, Bainbridge was able to construct a device that would only allow one mass in flight to reach the detector.

- Velocity selector filters particles with v = E/B. After this, in the region of B' particles with $m_2 > m_1$ travel with radius ($R_2 > R_1$).

$$R = \frac{mv}{|q|B'}$$



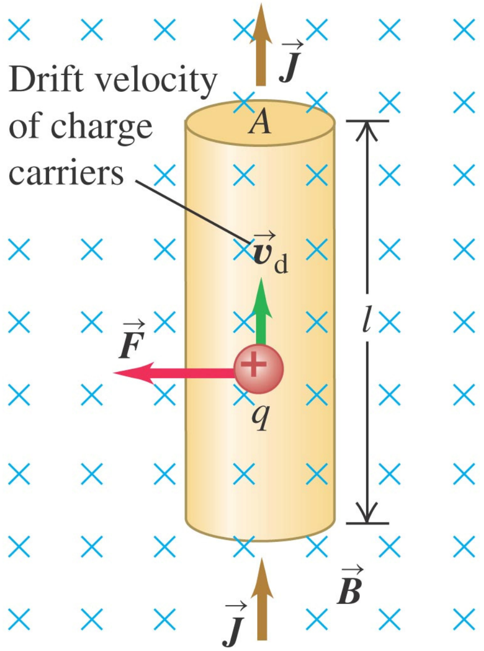
Velocity selector selects particles with speed v.

Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

6. Magnetic Force on a Current-Carrying Conductor

$$\vec{F}_m = q\vec{v}_d \times \vec{B}$$

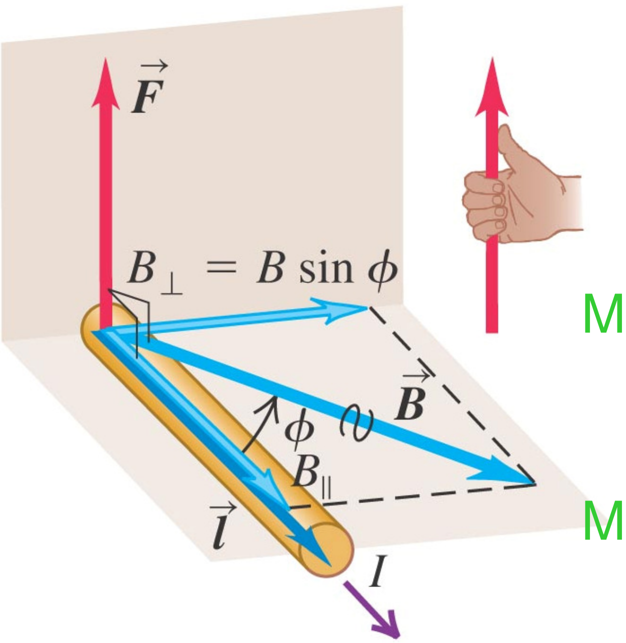
$$F_m = qv_dB$$ Force on one charge



- Total force: $$F_m = (nAl)(qv_dB)$$

n = number of charges per unit volume
A l = volume

$$F_m = (nqv_d)(A)(lB) = (JA)(lB) = IlB$$ (B ⊥ wire)



In general:

$$F = IlB_\perp = IlB\sin\varphi$$

Magnetic force on a straight wire segment:

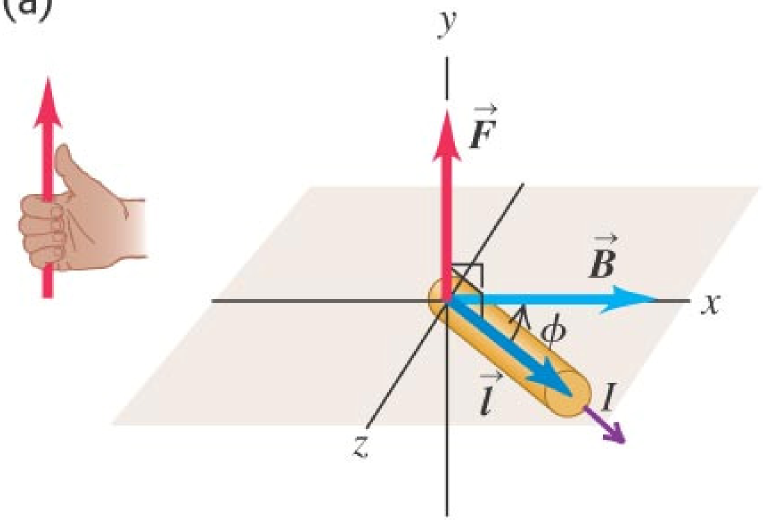
$$\boxed{\vec{F} = I\vec{l} \times \vec{B}}$$

Magnetic force on an infinitesimal wire section:

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

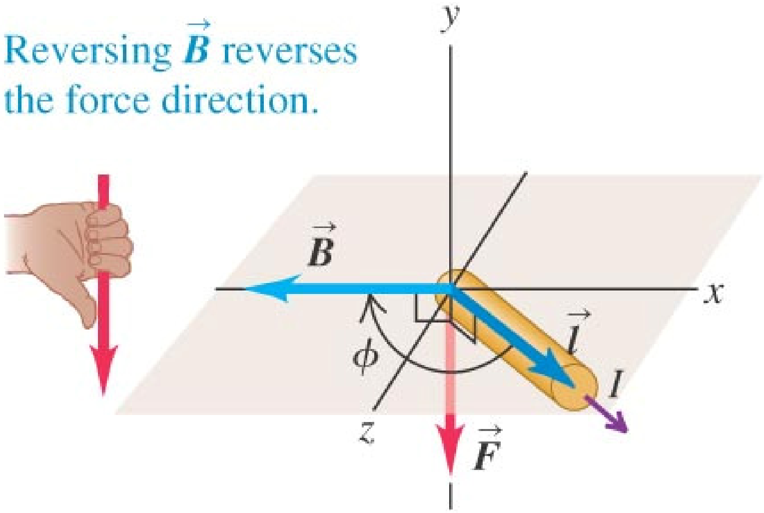
- Current is not a vector. The direction of the current flow is given by $\vec{dl}$, not I. $\vec{dl}$ is tangent to the conductor.

(a)



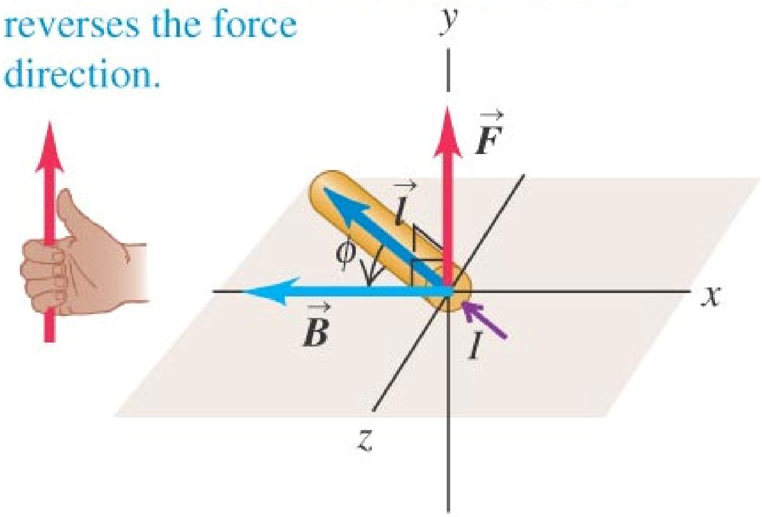
$$\boxed{\vec{F} = I\vec{l} \times \vec{B}}$$

(b)


Reversing $\vec{B}$ reverses the force direction.

(c)

Reversing the current [relative to (b)] reverses the force direction.



7. Force and Torque on a Current Loop

- The net force on a current loop in a uniform magnetic field is zero.

Right wire of length “a” → $F = I\,a\,B$ $(B \perp l)$

Left wire of length “b” → $F' = I\,b\,B\sin(90^\circ - \varphi)$ (B forms $90^\circ - \varphi$ angle with l)

$F' = I\,b\,B\cos\varphi$

$F_{net} = F - F + F' - F' = 0$

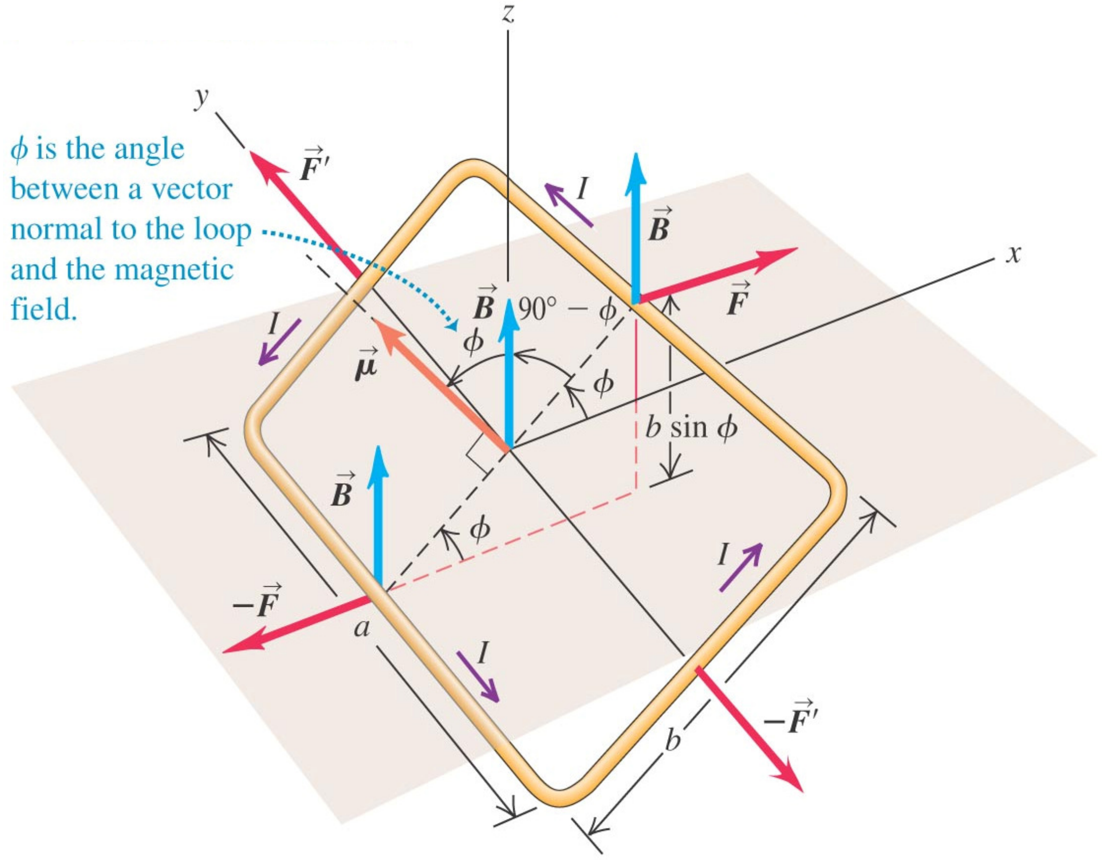
- Net torque ≠ 0 (general).

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r \cdot F \sin\alpha = r_{\perp} F = r F_{\perp}$$

$$\tau_{F'} = r \cdot F \sin 0^\circ = 0$$

$$\tau_F = F(b/2)\sin\varphi$$



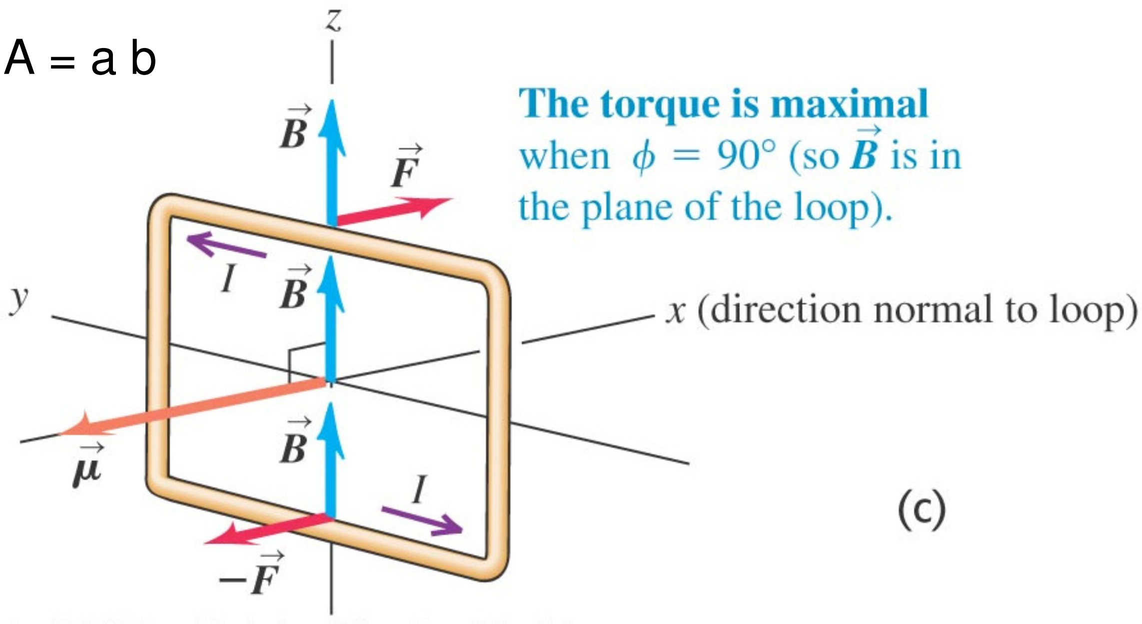
ϕ is the angle between a vector normal to the loop and the magnetic field.

$$\tau_{total} = \tau_{F'} + \tau_{-F'} + \tau_{F} + \tau_{-F} = 0 + 0 + 2(b/2)F\sin\varphi$$

$$\boxed{\tau_{total} = (IBa)(b\sin\varphi) = IBA\sin\varphi}$$

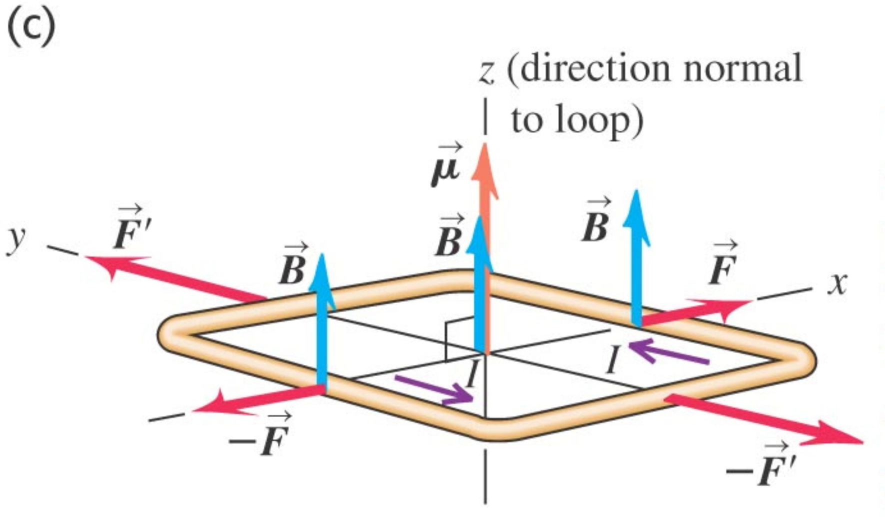
Torque on a current loop

A = a b



The torque is maximal when $\phi = 90°$ (so $\vec{B}$ is in the plane of the loop).

Torque is zero, φ = 0º



φ is angle between a vector perpendicular to loop and $\vec{B}$

$$\tau_{total} = IBA\sin\varphi$$

Magnetic dipole moment: $\boxed{\mu = IA}$

$$\tau_{total} = \mu B\sin\varphi$$

Magnetic torque: $\boxed{\vec{\tau} = \vec{\mu}\times\vec{B}}$

Potential Energy for a Magnetic Dipole:

$$\boxed{U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\varphi}$$

Electric dipole moment: $\boxed{\overline{p} = q\vec{d}}$

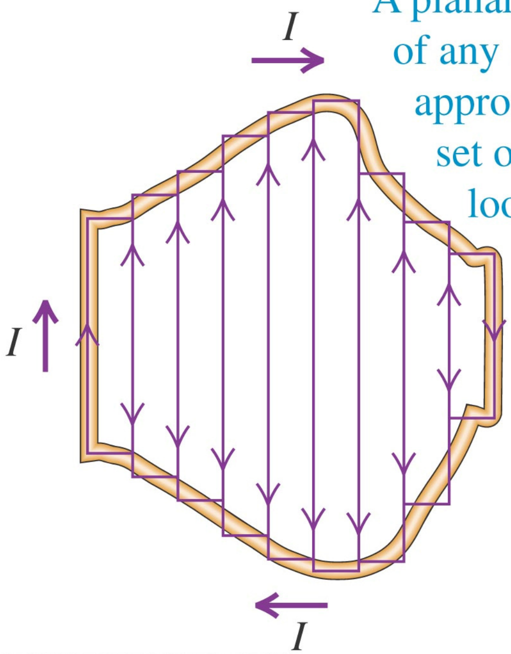
Electric torque: $\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$

Potential Energy for an Electric Dipole:

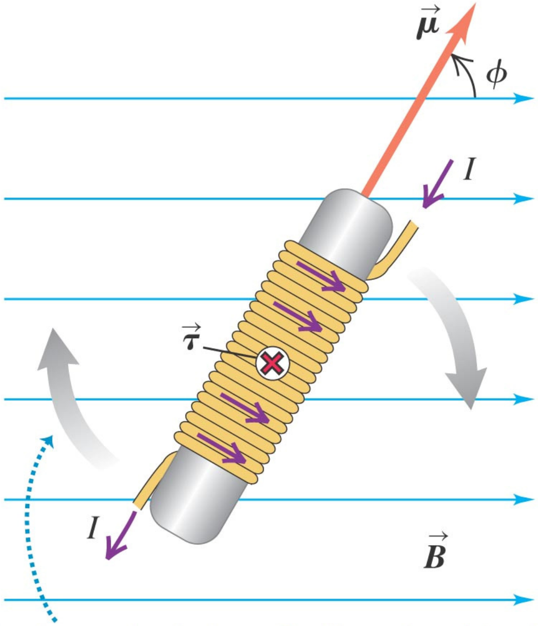
$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

Magnetic Torque: Loops and Coils

If these loops all carry equal current I in same clockwise sense, F and torque on the sides of two adjacent loops cancel, and only forces and torques around boundary ≠ 0.



A planar current loop of any shape can be approximated by a set of rectangular loops.



The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$.

Solenoid

$$\tau = NIBA\sin\varphi$$

N = number of turns

φ is angle between axis of solenoid and B

Max. torque: solenoid axis ⊥ B.

Torque rotates solenoid to position where its axis is parallel to B.