Chapter 27 – Magnetic Field and Magnetic Forces

- Magnetism
- Magnetic Field
- Magnetic Field Lines and Magnetic Flux
- Motion of Charged Particles in a Magnetic Field
- Applications of Motion of Charged Particles
- Magnetic Force on a Current-Carrying Conductor
- Force and Torque on a Current Loop

1) A moving charge or collection of moving charges (e.g. electric current) produces a magnetic field. (Chap. 28).

2) A second current or charge responds to the magnetic field and experiences a magnetic force. (Chap. 27).

1. Magnetism

Permanent magnets: exert forces on each other as well as on unmagnetized Fe pieces.

- The needle of a compass is a piece of magnetized Fe.

- If a bar-shaped permanent magnet is free to rotate, one end points north (north pole of magnet).

- An object that contains Fe is not by itself magnetized, it can be attracted by either the north or south pole of permanent magnet.

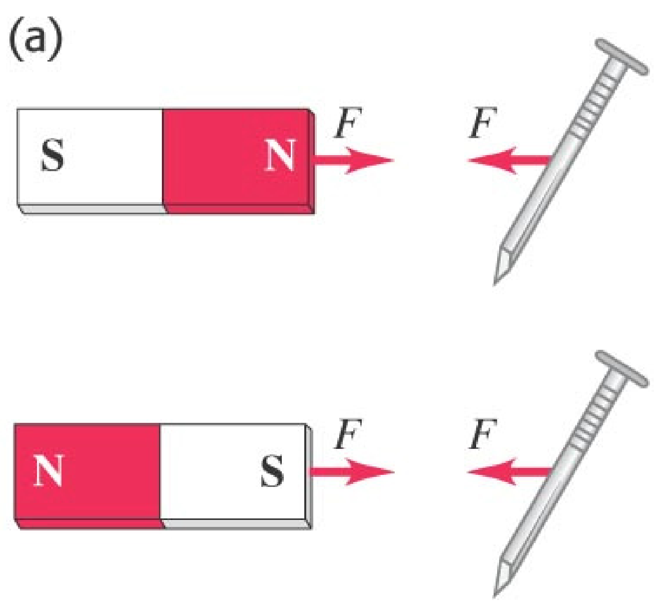
- A bar magnet sets up a magnetic field in the space around it and a second body responds to that field. A compass needle tends to align with the magnetic field at the needle's position.

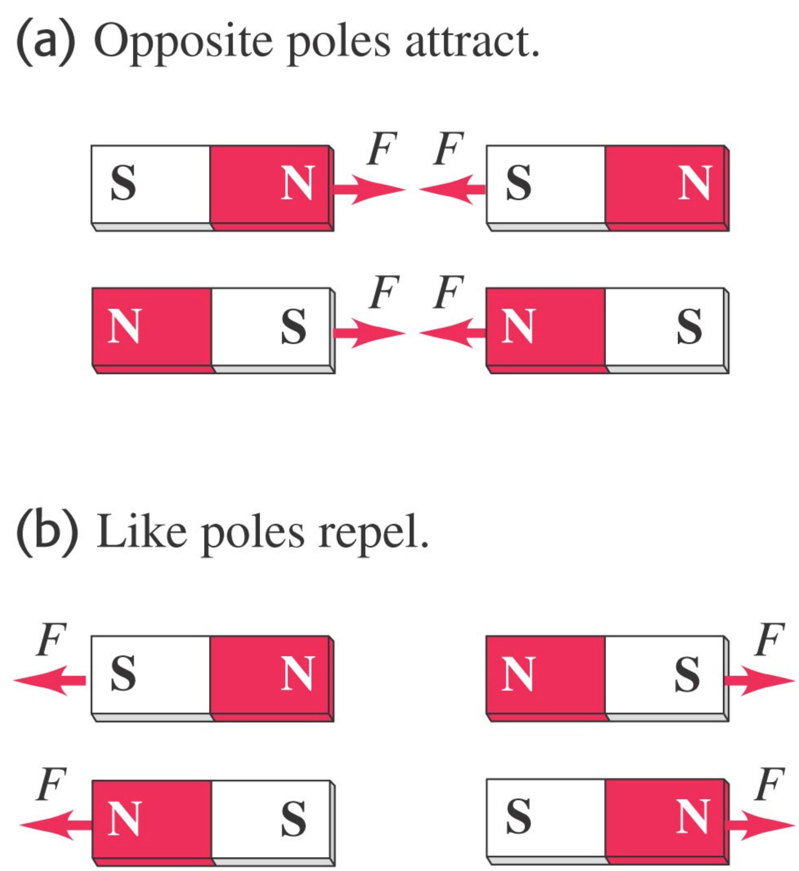
1. Magnetism

- Magnets exert forces on each other just like charges. You can draw magnetic field lines just like you drew electric field lines.

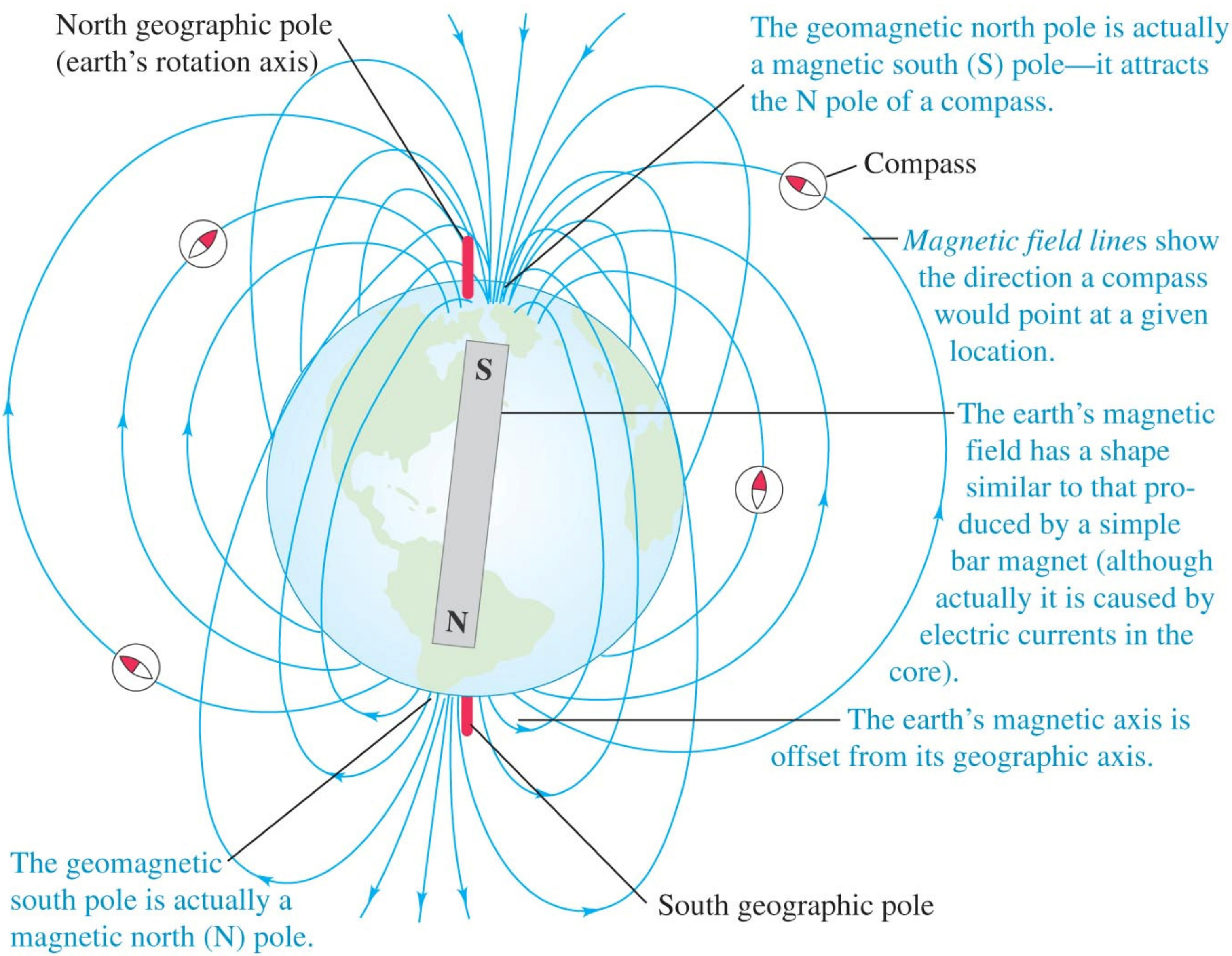
- Magnetic north and south pole's behavior is not unlike electric charges. For magnets, like poles repel and opposite poles attract.

- A permanent magnet will attract a metal like iron with either the north or south pole.





Magnetic poles about our planet



Magnetic declination / magnetic variation: the Earth's magnetic axis is not parallel to its geographic axis (axis of rotation) → a compass reading deviates from geographic north.

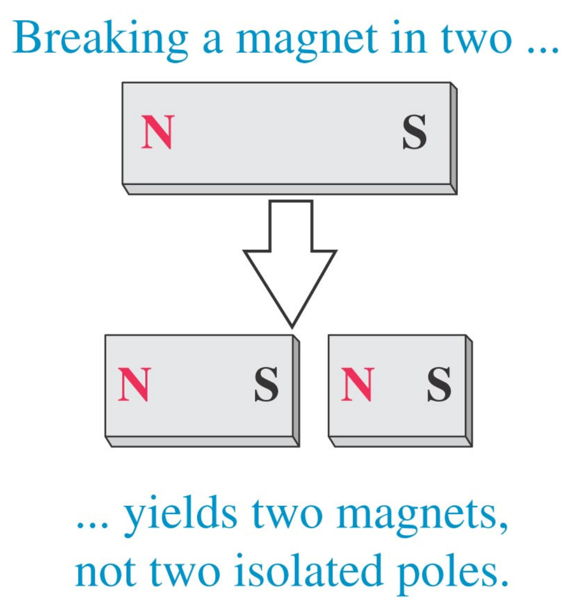
Magnetic inclination: the magnetic field is not horizontal at most of earth's surface, its angle up or down. The magnetic field is vertical at magnetic poles.

Magnetic Poles versus Electric Charge

- We observed monopoles in electricity. A (+) or (-) alone was stable, and field lines could be drawn around it.

- Magnets cannot exist as monopoles. If you break a bar magnet between N and S poles, you get two smaller magnets, each with its own N and S pole.

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

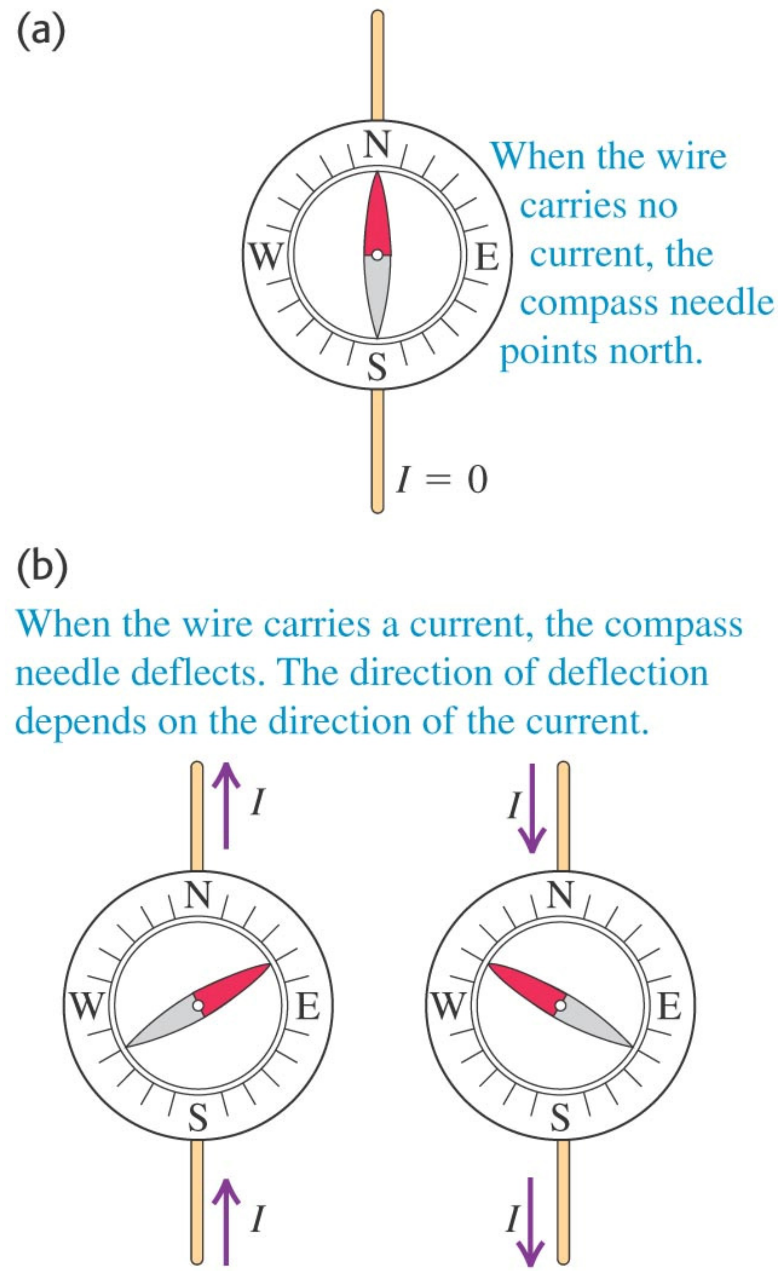


-In 1820, Oersted ran experiments with conducting wires run near a sensitive compass. The orientation of the wire and the direction of the flow both moved the compass needle.

- Ampere / Faraday / Henry → moving a magnet near a conducting loop can induce a current.

- The magnetic forces between two bodies are due to the interaction between moving electrons in the atoms.

- Inside a magnetized body (permanent magnet) there is a coordinated motion of certain atomic electrons. Not true for unmagnetized objects.



2. Magnetic Field

Electric field:

1) A distribution of electric charge at rest creates an electric field E in the surrounding space.

2) The electric field exerts a force $\vec{F}_E = q\,\vec{E}$ on any other charges in presence of that field.

Magnetic field:

1) A moving charge or current creates a magnetic field in the surrounding space (in addition to $\vec{E}$).

2) The magnetic field exerts a force $\vec{F}_m$ on any other moving charge or current present in that field.

- The magnetic field is a vector field → vector quantity associated with each point in space.

$$F_m = |q| v_\perp B = |q| v\ B \sin\varphi$$

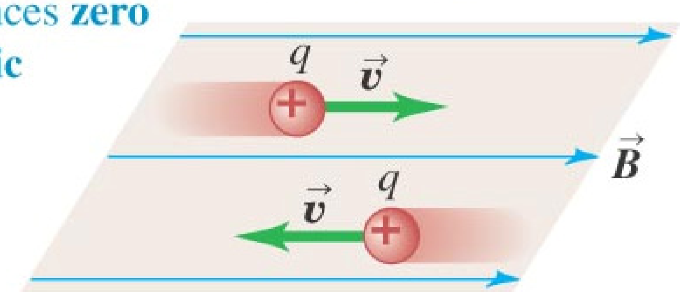
$$\vec{F}_m = q\vec{v} \times \vec{B}$$

- $\vec{F}_m$ is always perpendicular to $\vec{B}$ and $\vec{v}$.

2. Magnetic Field

- The moving charge interacts with the fixed magnet. The force between them is at a maximum when the velocity of the charge is perpendicular to the magnetic field.

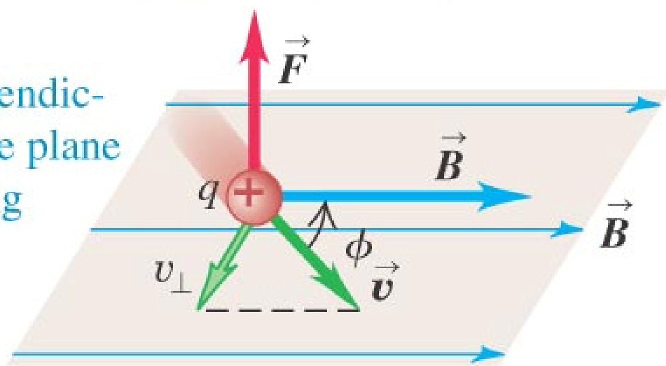
A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**



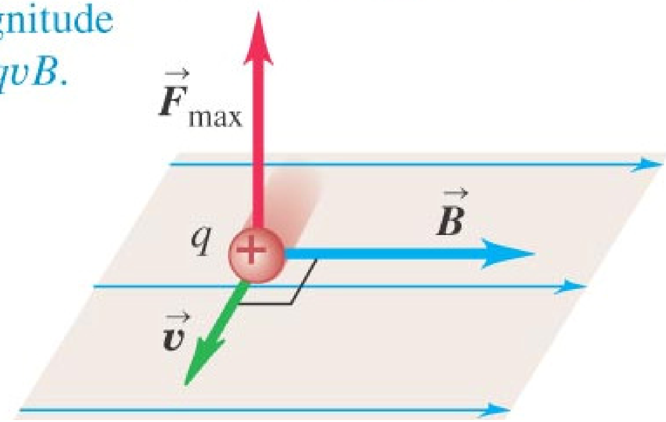
Interaction of magnetic force and charge

A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB \sin\phi$.

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.



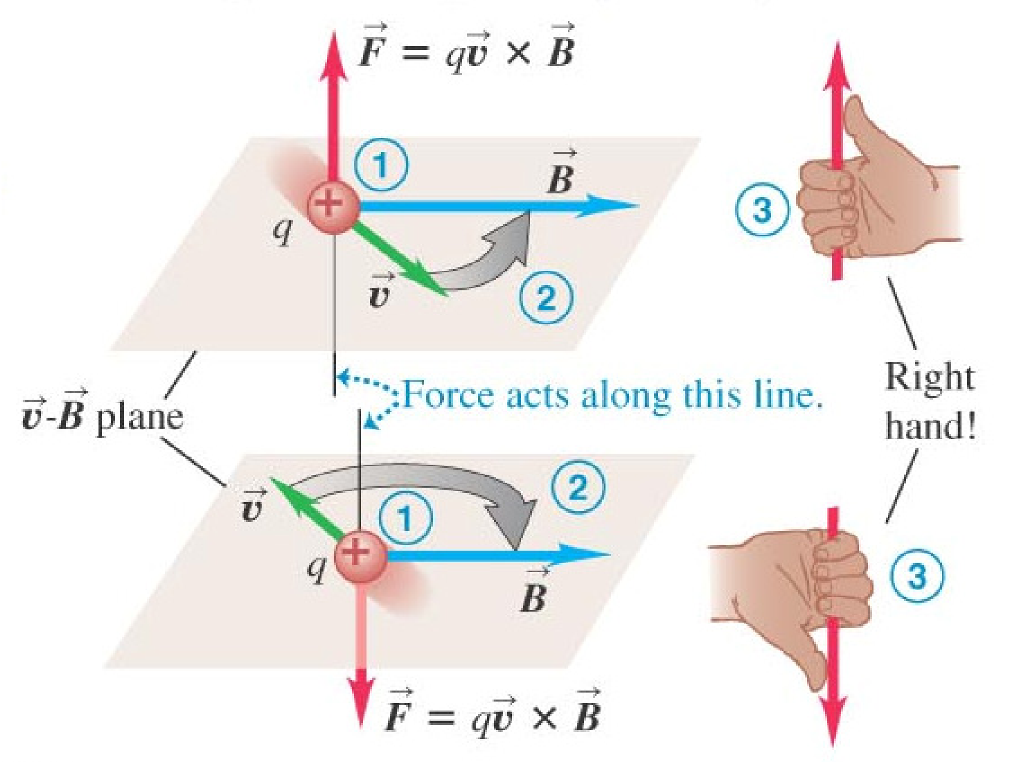
A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{max} = qvB$.

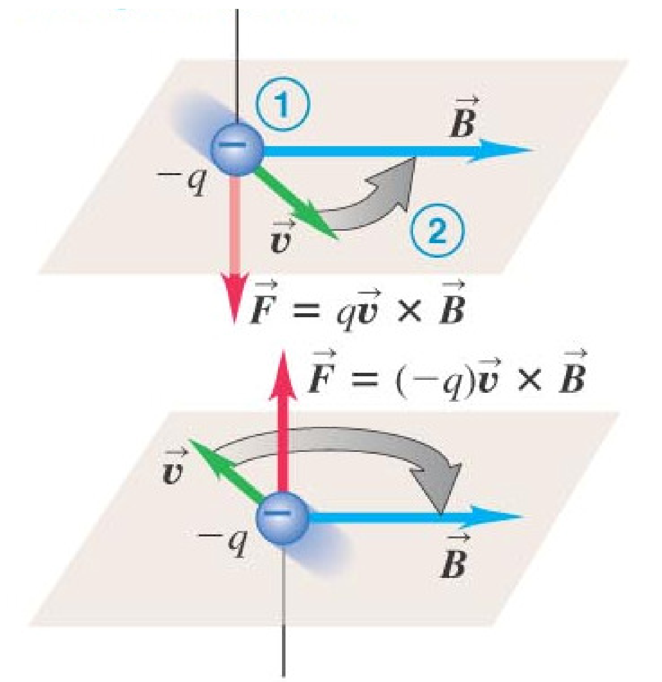


Right Hand Rule

Positive charge moving in magnetic field →direction of force follows right hand rule

Negative charge → F direction contrary to right hand rule.



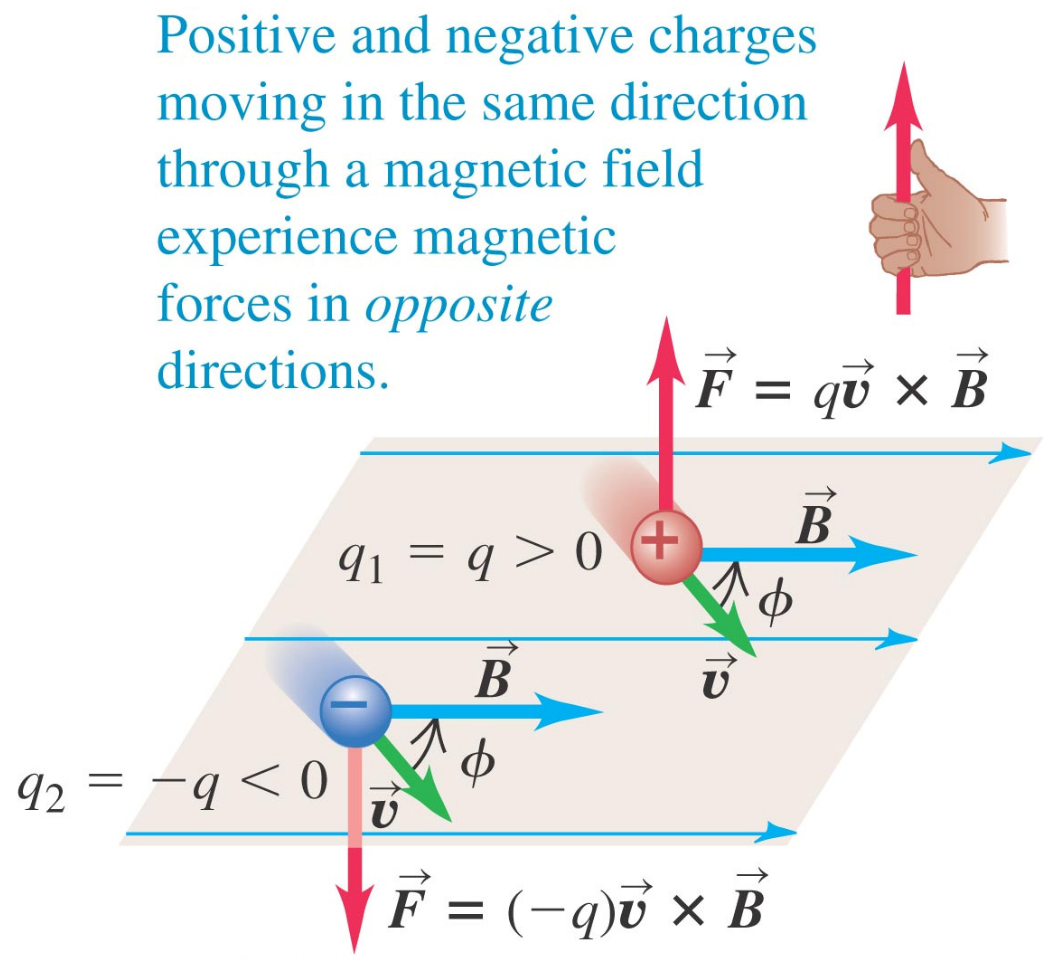


$$F = |q| v B_{\perp}$$

Units: 1 Tesla = 1 N s / C m = 1 N/A m

1 Gauss = 10^{-4} T

Right Hand Rule



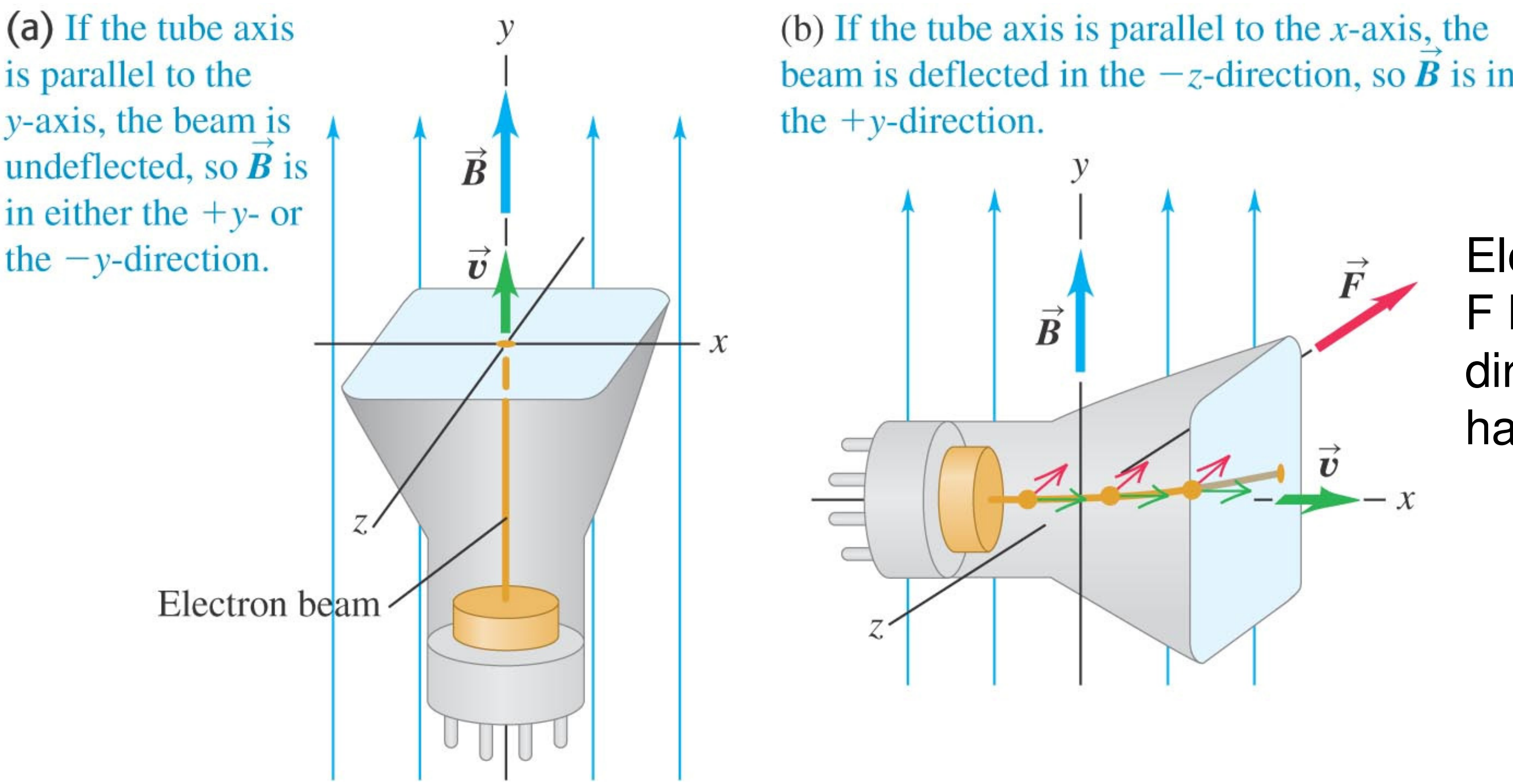
If charged particle moves in region where both, E and B are present:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Measuring Magnetic Fields with Test Charges

Ex: electron beam in a cathode X-ray tube.

- In general, if a magnetic field (B) is present, the electron beam is deflected. However this is not true if the beam is // to B ($\varphi = 0, \pi$ → F=0 → no deflection).



Electron q< 0 → F has contrary direction to right hand rule

No deflection → $\vec{F} = 0$ → $\vec{v}$ // $\vec{B}$

Deflection → $\vec{F} \neq 0$ → $\vec{F} \perp \vec{v}, \vec{B}$

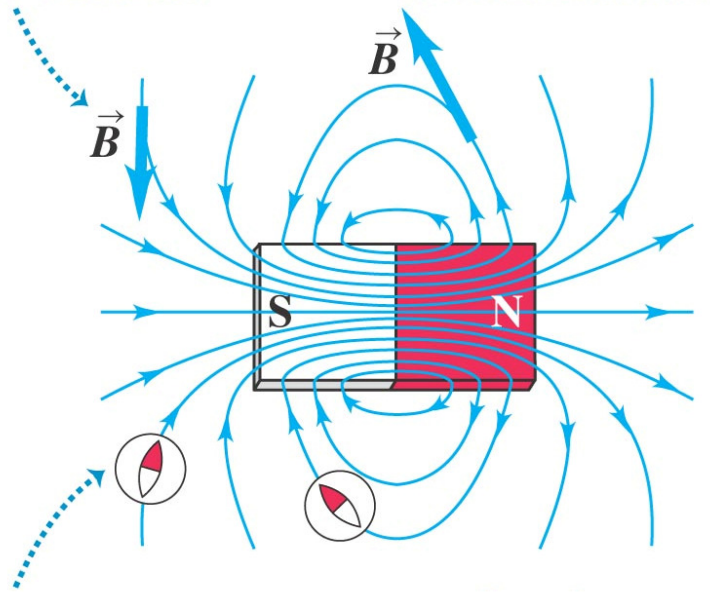
3. Magnetic Field Lines and Magnetic Flux

- Magnetic field lines may be traced from N toward S (analogous to the electric field lines).

- At each point they are tangent to magnetic field vector.

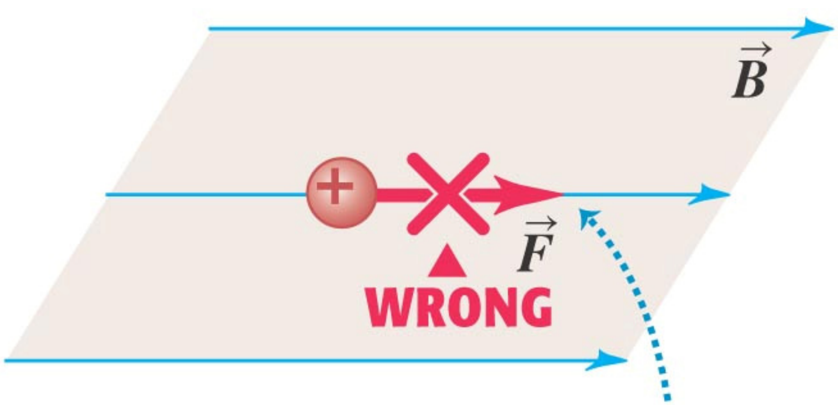
- The more densely packed the field lines, the stronger the field at a point.

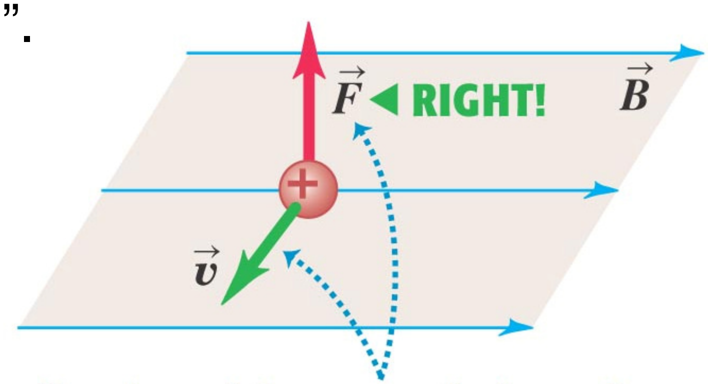
- Field lines never intersect.



- The field lines point in the same direction as a compass (from N toward S).

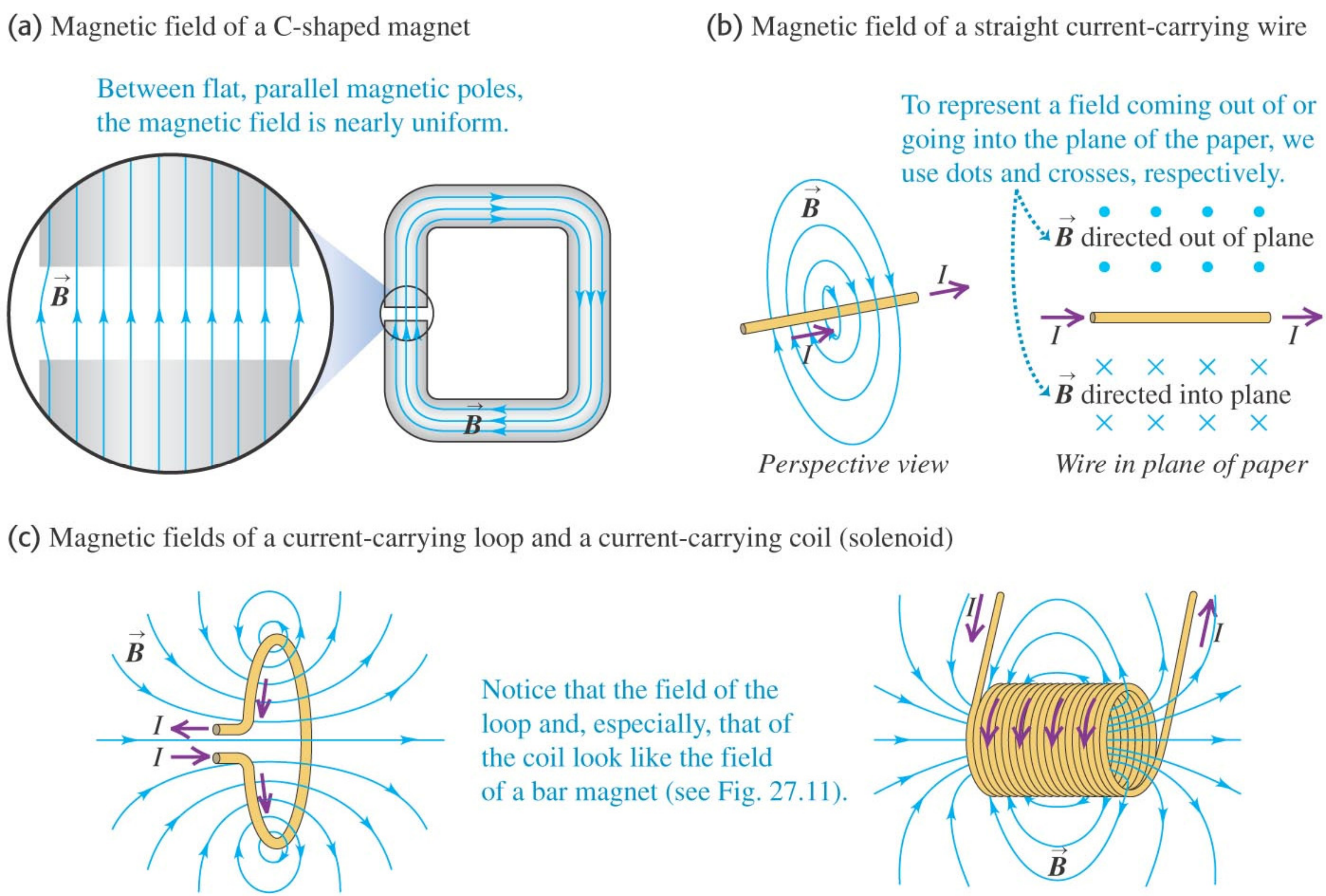
- Magnetic field lines are not "lines of force".





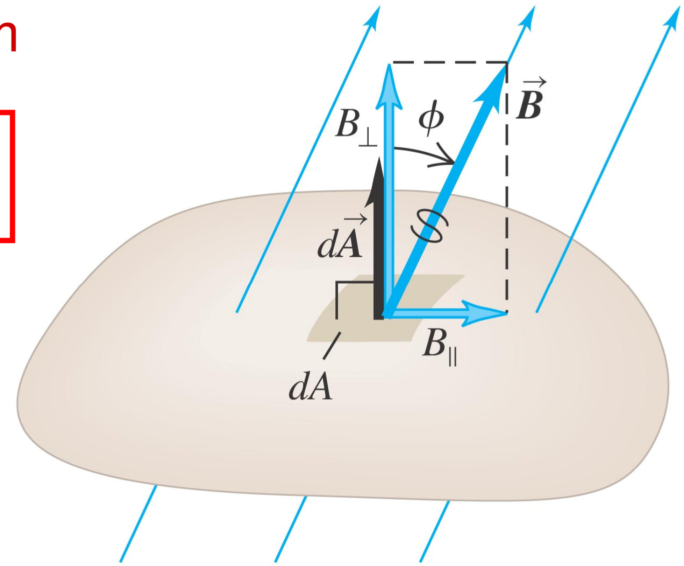
The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

- Magnetic field lines have no ends → they continue through the interior of the magnet.



Magnetic Flux and Gauss's Law for Magnetism

$$\Phi_B = \int B_\perp dA = \int B\cos\varphi \cdot dA = \int \vec{B}\cdot d\vec{A}$$



- Magnetic flux is a scalar quantity.

- If $\vec{B}$ is uniform: $$\Phi_B = B_\perp A = BA\cos\varphi$$

Units: 1 Weber (1 Wb = 1 T m^2 = 1 N m / A)

- Difference with respect to electric flux → the total magnetic flux through a closed surface is always zero. This is because there is no isolated magnetic charge ("monopole") that can be enclosed by the Gaussian surface.

$$\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0$$

$$B = \frac{d\Phi_B}{dA_\perp}$$

- The magnetic field is equal to the flux per unit area across an area at right angles to the magnetic field = magnetic flux density.

4. Motion of Charged Particles in a Magnetic Field

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

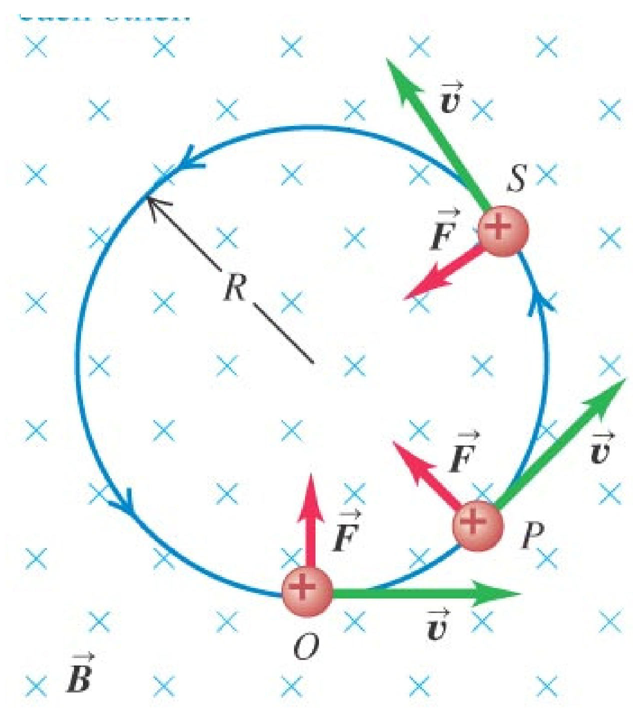
- Magnetic force perpendicular to $\vec{v}$ → it cannot change the magnitude of the velocity, only its direction.
- $\vec{F}$ does not have a component parallel to particle's motion → cannot do work.
- Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.
- Magnitudes of F and v are constant (v perp. B) → uniform circular motion.

$$F = |q| \cdot v \cdot B = m\frac{v^2}{R}$$

Radius of circular orbit in magnetic field:

$$R = \frac{mv}{|q|B}$$

+ particle → counter-clockwise rotation.
- particle → clockwise rotation.

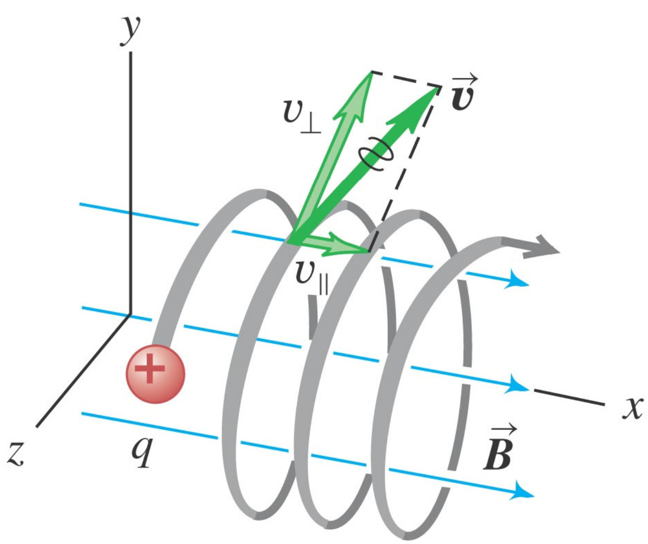


Angular speed: ω = v/R → $\omega = v\frac{|q|B}{mv} = \frac{|q|B}{m}$

Cyclotron frequency: f = ω/2π

- If v is not perpendicular to B → $v_{//}$ (parallel to B) constant because $F_{//} = 0$ → particle moves in a helix. (R same as before, with $v = v_{\perp}$).

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



A charged particle will move in a plane perpendicular to the magnetic field.

5. Applications of Motion of Charged Particles

Velocity selector

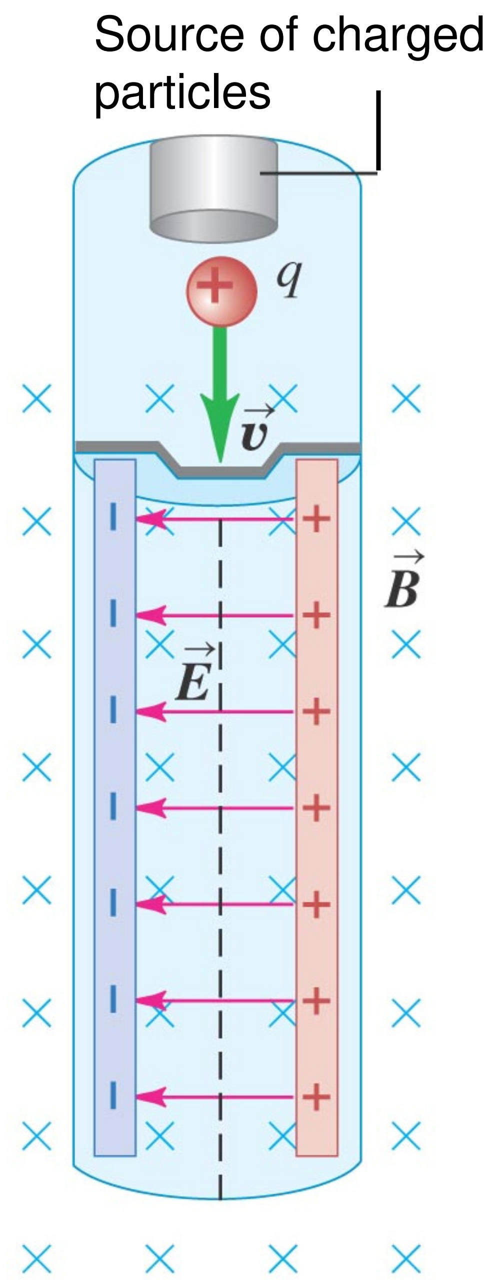
- Particles of a specific speed can be selected from the beam using an arrangement of E and B fields.

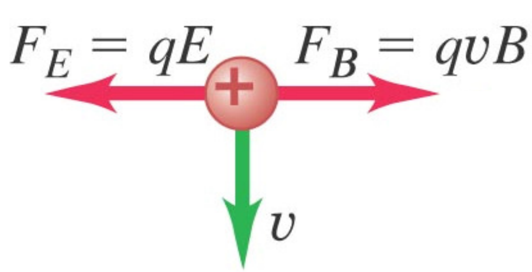
- F_m (magnetic) for + charge towards right (q v B).

- F_E (electric) for + charge to left (q E).

- $F_{net} = 0$ if $F_m = F_E$ → $-qE + q v B = 0$ → **$v = E/B$**

- Only particles with speed E/B can pass through without being deflected by the fields.





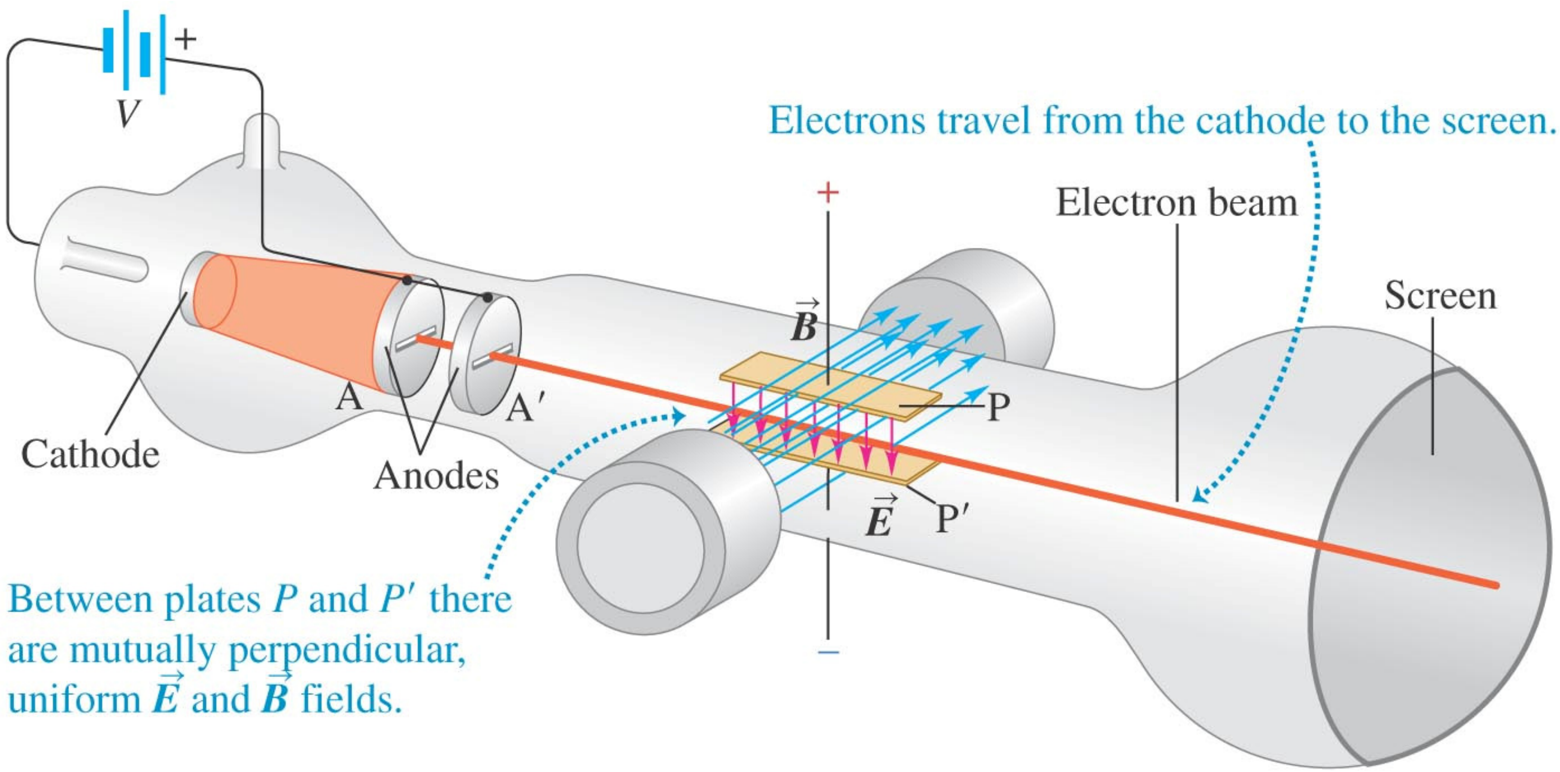
Thomson's *e/m* Experiment

$\Delta E = \Delta K + \Delta U = 0 \rightarrow 0.5\ m\ v^2 = U = e\ V$

$$v = \frac{E}{B} = \sqrt{\frac{2eV}{m}}$$

$$\frac{e}{m} = \frac{E^2}{2VB^2}$$

e/m does not depend on the cathode material or residual gas on tube → particles in the beam (electrons) are a common constituent of all matter.

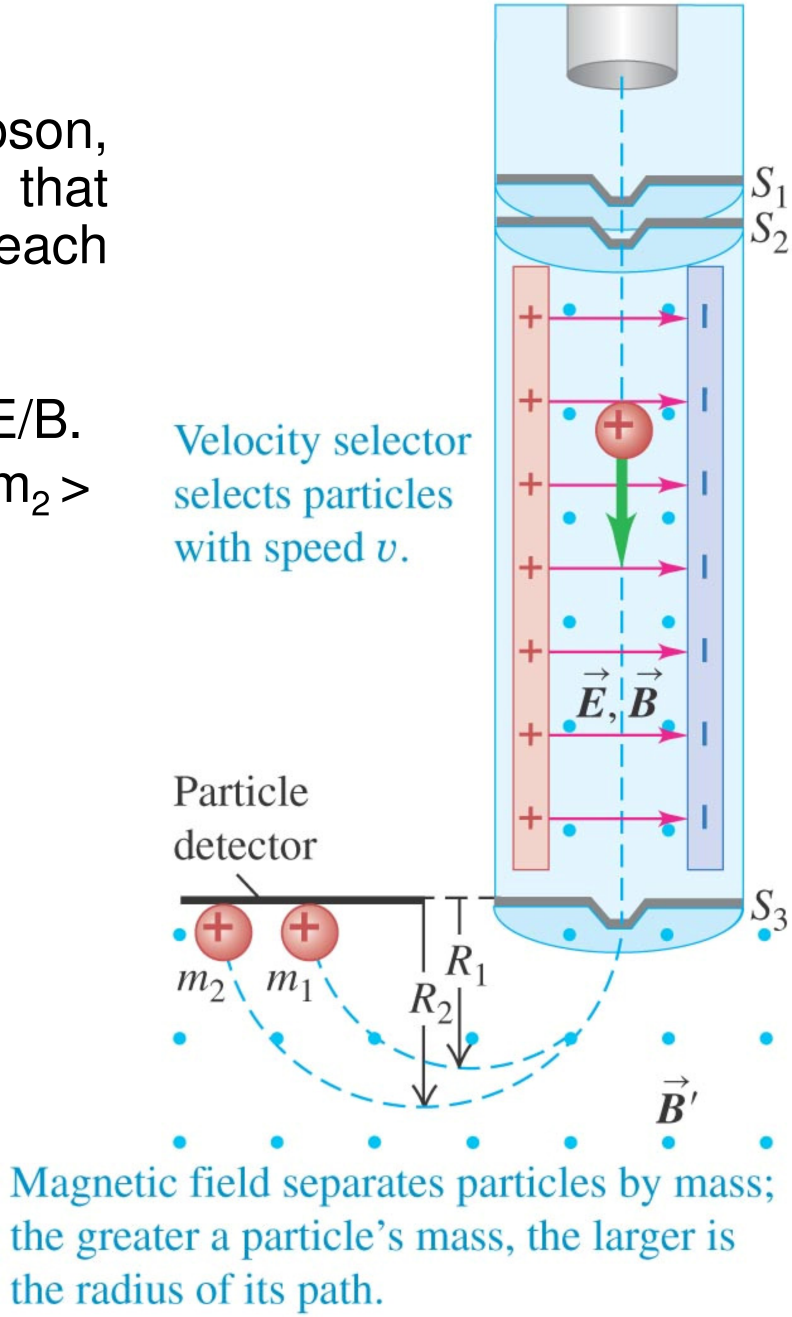


Mass Spectrometer

- Using the same concept as Thompson, Bainbridge was able to construct a device that would only allow one mass in flight to reach the detector.

- Velocity selector filters particles with v = E/B. After this, in the region of B' particles with $m_2 > m_1$ travel with radius ($R_2 > R_1$).

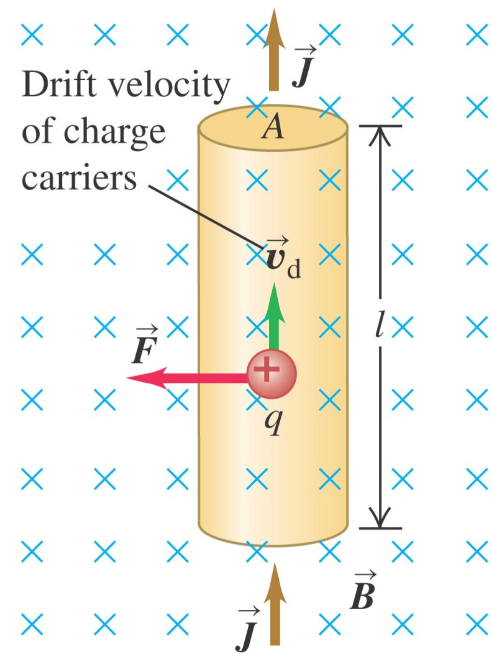
$$R = \frac{mv}{|q|B'}$$



6. Magnetic Force on a Current-Carrying Conductor

$$\vec{F}_m = q\vec{v}_d \times \vec{B}$$

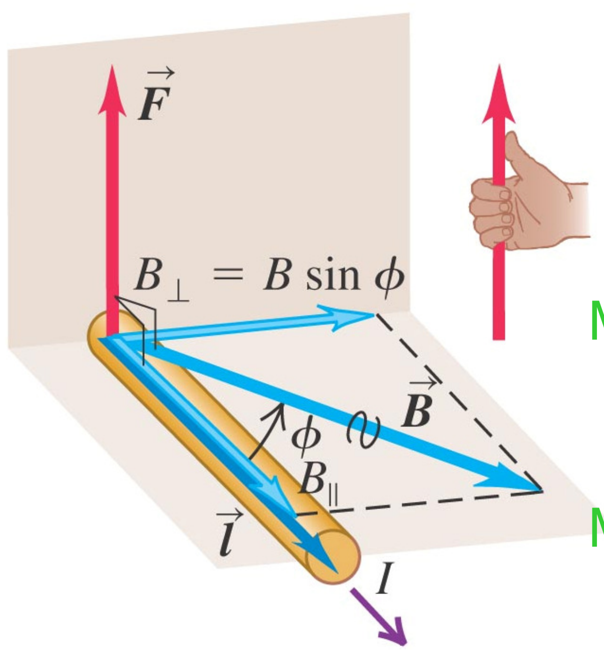
$$F_m = qv_d B$$ Force on one charge



- Total force: $$F_m = (nAl)(qv_d B)$$

n = number of charges per unit volume
A l = volume

$$F_m = (nqv_d)(A)(lB) = (JA)(lB) = IlB$$ (B ⊥ wire)



In general:

$$F = IlB_\perp = IlB \sin \varphi$$

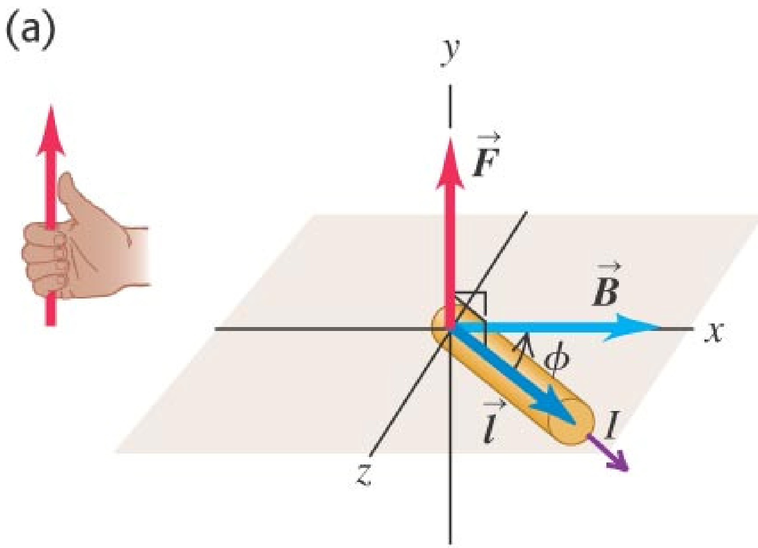
Magnetic force on a straight wire segment:

$$\boxed{\vec{F} = I\vec{l} \times \vec{B}}$$

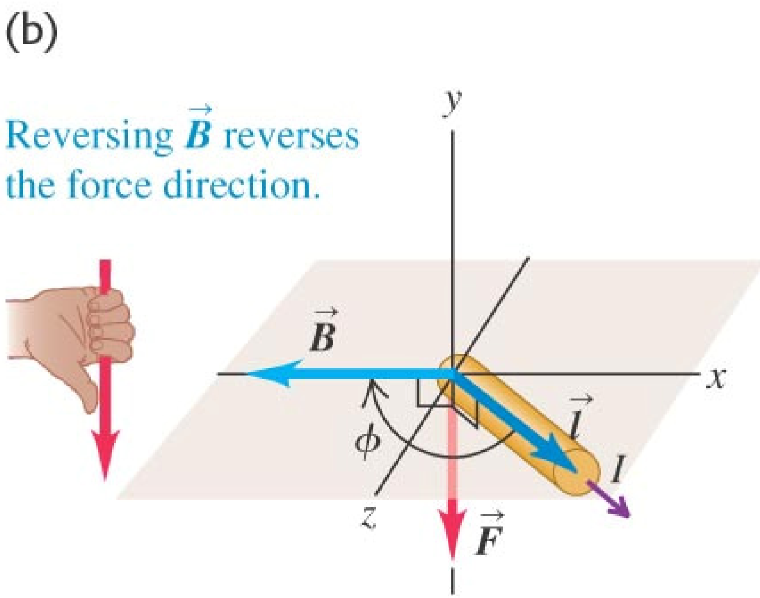
Magnetic force on an infinitesimal wire section:

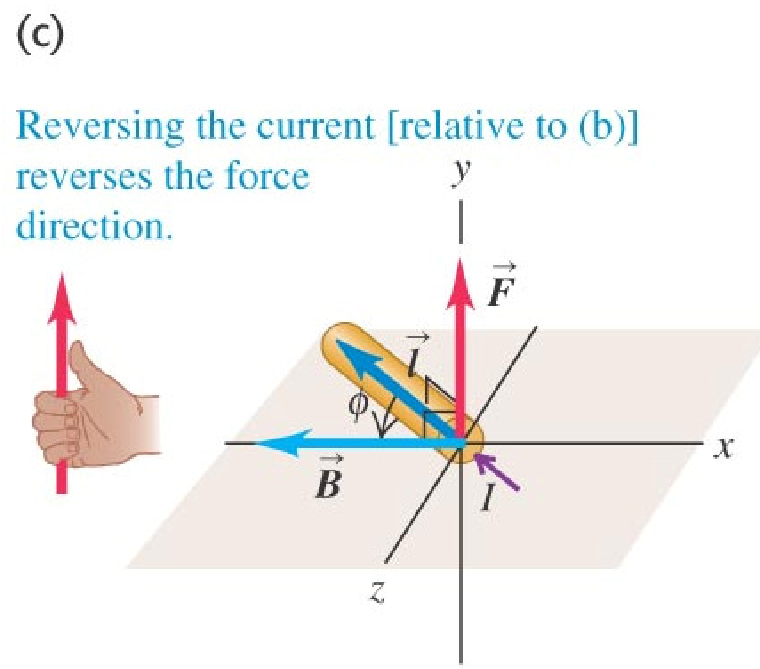
$$d\vec{F} = Id\vec{l} \times \vec{B}$$

- Current is not a vector. The direction of the current flow is given by $\vec{dl}$, not I. $\vec{dl}$ is tangent to the conductor.



$$\vec{F} = I\vec{l} \times \vec{B}$$





7. Force and Torque on a Current Loop

- The net force on a current loop in a uniform magnetic field is zero.

Right wire of length “a” → F = I a B (B ⊥ l)

Left wire of length “b” → F’ = I b B sin (90º - φ) (B forms 90º-φ angle with l)

F’ = I b B cos φ

F_{net} = F – F + F’ - F’ = 0

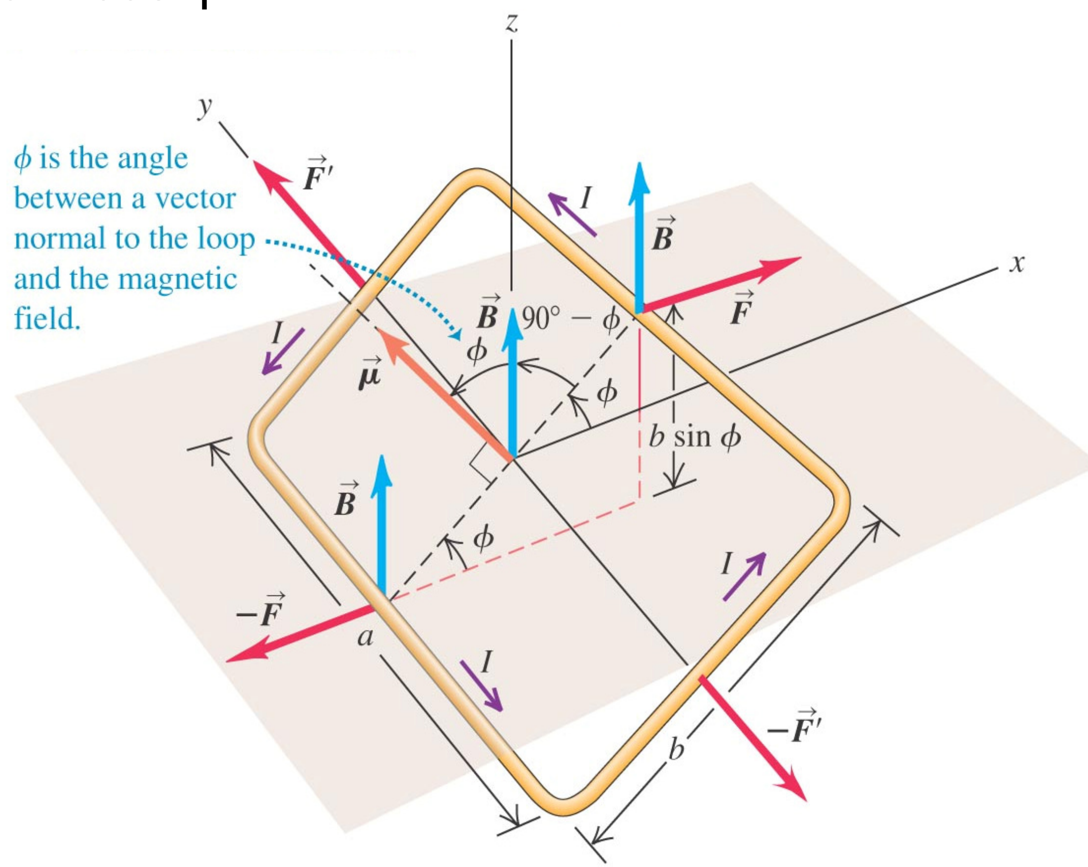
- Net torque ≠ 0 (general).

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r \cdot F \sin\alpha = r_{\perp} F = r F_{\perp}$$

$$\tau_{F'} = r \cdot F \sin 0^{\circ} = 0$$

$$\tau_F = F(b/2)\sin\varphi$$

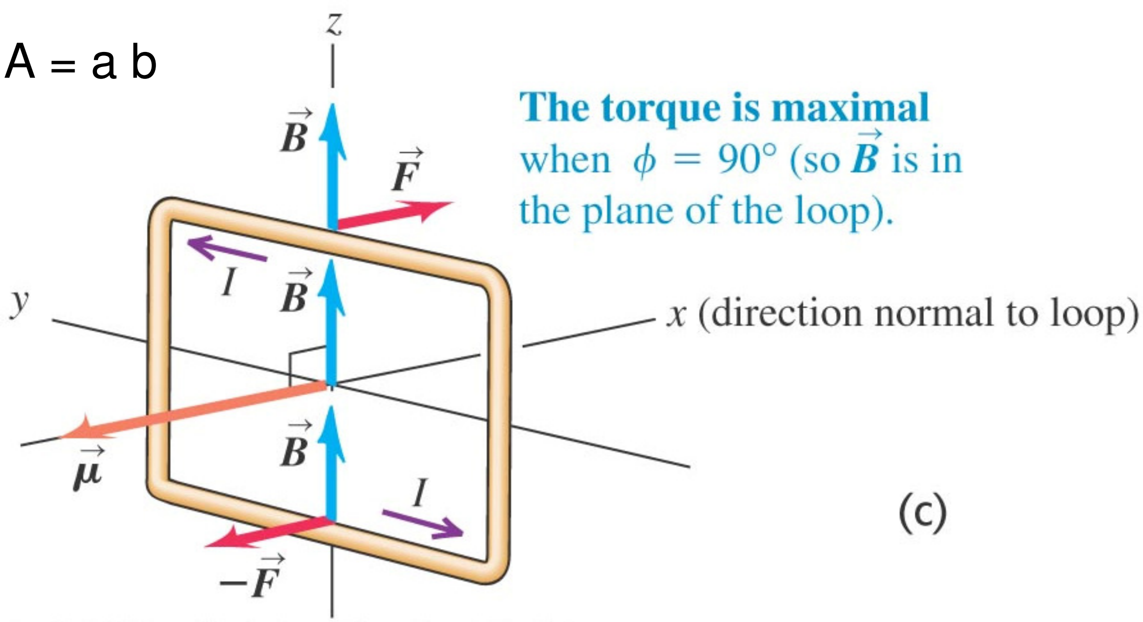


$$\tau_{total} = \tau_{F'} + \tau_{-F'} + \tau_{F} + \tau_{-F} = 0 + 0 + 2(b/2)F\sin\varphi$$

$$\boxed{\tau_{total} = (IBa)(b\sin\varphi) = IBA\sin\varphi}$$

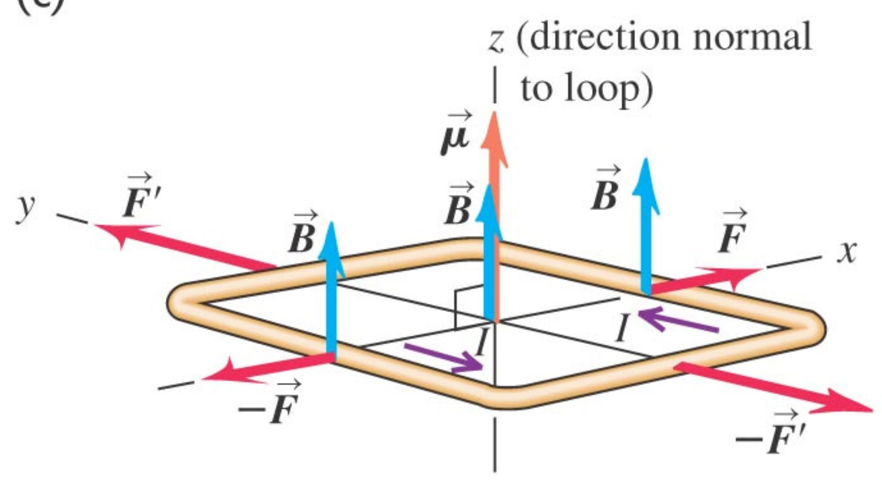
Torque on a current loop

A = a b



Torque is zero, φ = 0º

(c)

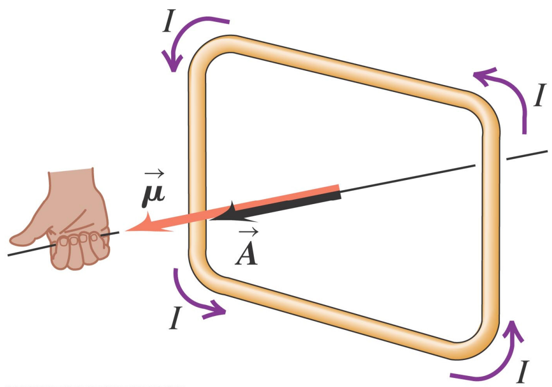


φ is angle between a vector perpendicular to loop and $\vec{B}$

$$\tau_{total} = IBA\sin\varphi$$

Magnetic dipole moment: $\vec{\mu} = I\vec{A}$

Direction: perpendicular to plane of loop (direction of loop's vector area → right hand rule)



$$\tau_{total} = \mu B\sin\varphi$$

Magnetic torque: $\vec{\tau} = \vec{\mu}\times\vec{B}$

Potential Energy for a Magnetic Dipole:

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\varphi$$

Electric dipole moment: $\overline{p} = q\vec{d}$

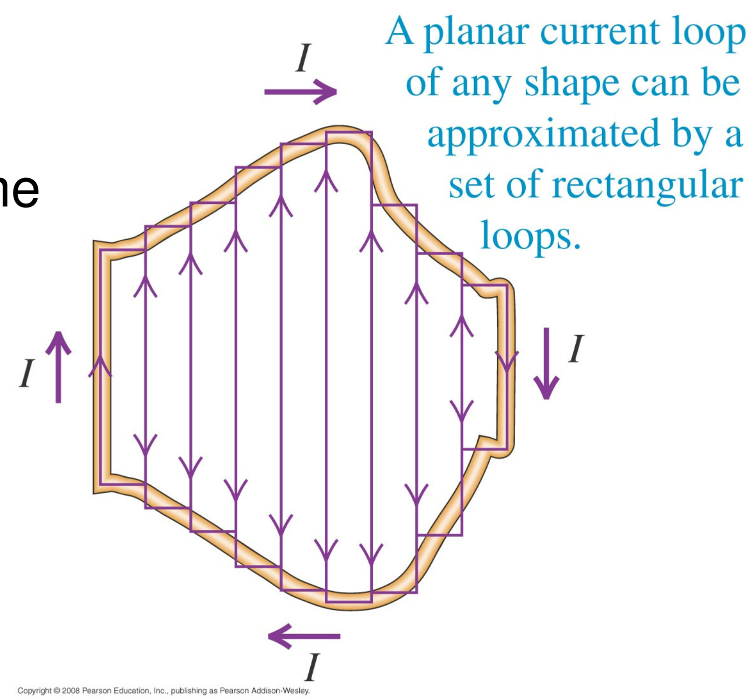
Electric torque: $\vec{\tau} = \vec{p}\times\vec{E}$

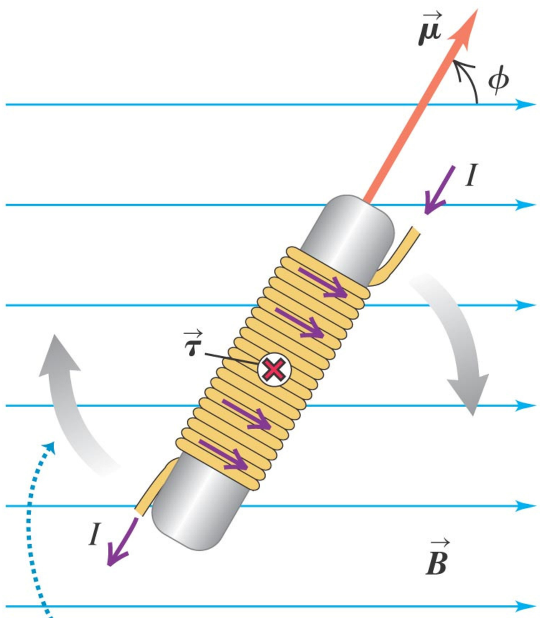
Potential Energy for an Electric Dipole:

$$U = -\vec{p}\cdot\vec{E}$$

Magnetic Torque: Loops and Coils

If these loops all carry equal current “I” in same clockwise sense, F and torque on the sides of two adjacent loops cancel, and only forces and torques around boundary ≠ 0.





The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$.

Solenoid

$$\tau = NIBA\sin\varphi$$

N = number of turns

φ is angle between axis of solenoid and B

Max. torque: solenoid axis ⊥ B.

Torque rotates solenoid to position where its axis is parallel to B.

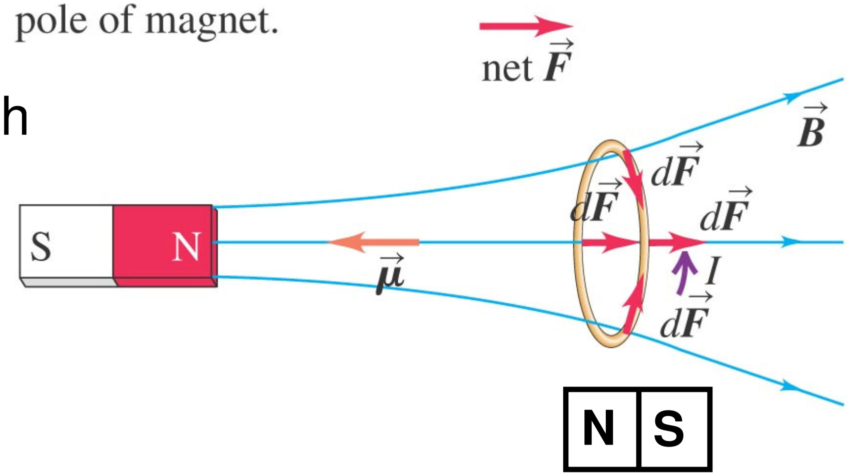
Magnetic Dipole in a Non-Uniform Magnetic Field

- Net force on a current loop in a non-uniform field is not zero.

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

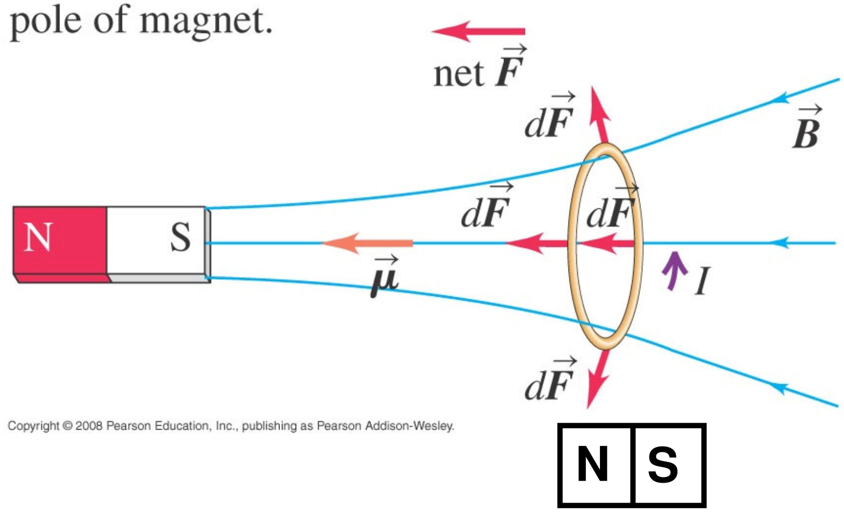
Radial force components cancel each Other → F_{net} to right.

(a) Net force on this coil is away from north pole of magnet.



If polarity of magnet changes → F_{net} to left.

(b) Net force on same coil is toward south pole of magnet.



Magnetic Dipole and How Magnets Work

A solenoid and a magnet orients themselves with axis parallel to field.

Electron analogy: "spinning ball of charge" → circulation of charge around spin axis similar to current loop → electron has net magnetic moment.

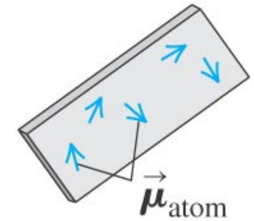
- In Fe atom, large number of electron magnetic moments align to each other → non-zero atomic magnetic moment.

- In unmagnetized Fe piece → no overall alignment of μ of atoms → total μ = 0.

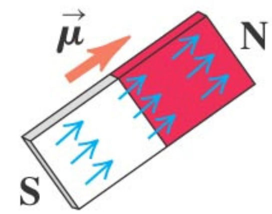
- Iron bar magnet → magnetic moments of many atoms are parallel → total μ ≠ 0.

- A bar magnet tends to align to B, so that line from S to N is in direction of B.

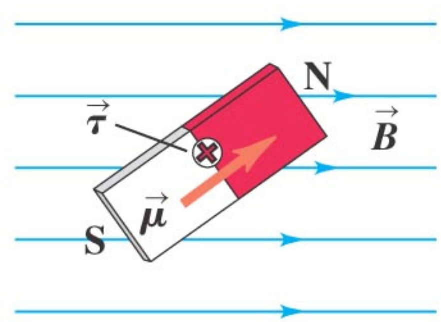
(a) Unmagnetized iron: magnetic moments are oriented randomly.



(b) In a bar magnet, the magnetic moments are aligned.



(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the $\vec{B}$ field.



- South and North poles represent tail and head of magnet's dipole moment, μ.

How can a magnet attract an unmagnetized Fe object?

1) Atomic magnetic moments of Fe try to align to B of bar magnet → Fe acquires net magnetic dipole moment // B.

2) Non-Uniform B attracts magnetic dipole.

The magnetic dipole produced on nail is equivalent to current loop (I direction right hand rule) → net magnetic force on nail is attractive (a) or (b) → unmagnetized Fe object is attracted to either pole of magnet.