# Chapter 26 – Direct-Current and Circuits

- Resistors in Series and Parallel
- Kirchhoff’s Rules
- Electric Measuring Instruments
- R-C Circuits

# 1. Resistors in Series and Parallel

## Resistors in Series:

$V_{ax} = I R_1$  $V_{xy} = I R_2$  $V_{yb} = I R_3$

$V_{ab} = V_{ax} + V_{xy} + V_{yb} = I (R_1 + R_2 + R_3)$

$V_{ab}/I = R_1 + R_2 + R_3 = R_{eq}$

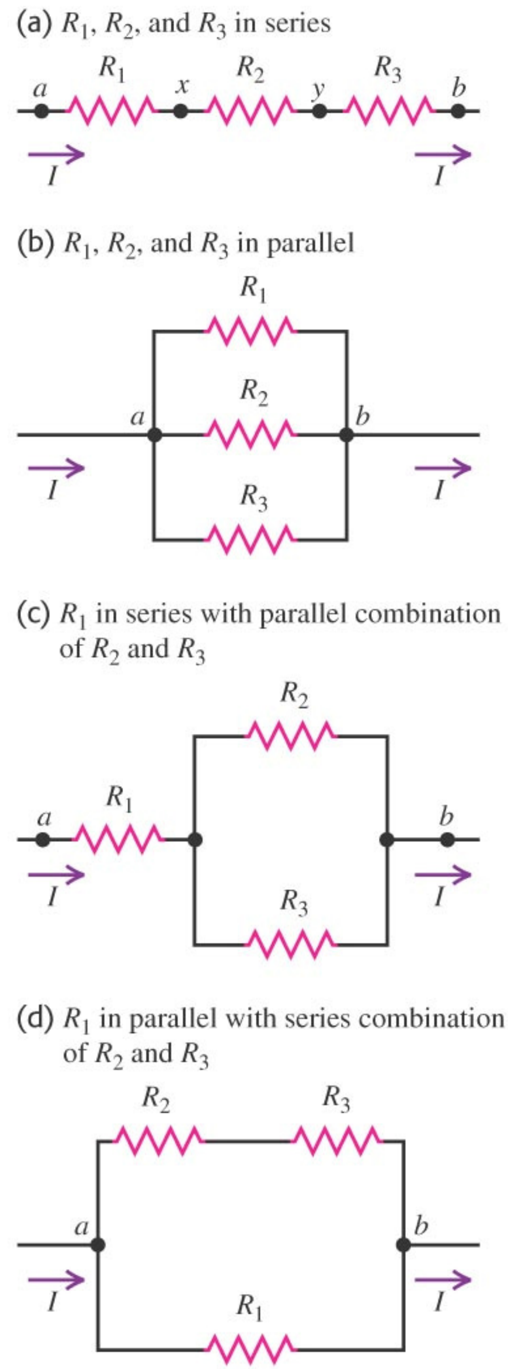
$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

## Resistors in Parallel:

$I_1 = V_{ab}/R_1$  $I_2 = V_{ab}/R_2$  $I_3 = V_{ab}/R_3$

$I = I_1 + I_2 + I_3 = V_{ab} (1/R_1 + 1/R_2 + 1/R_3) \rightarrow I/V_{ab} = 1/R_{eq}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$



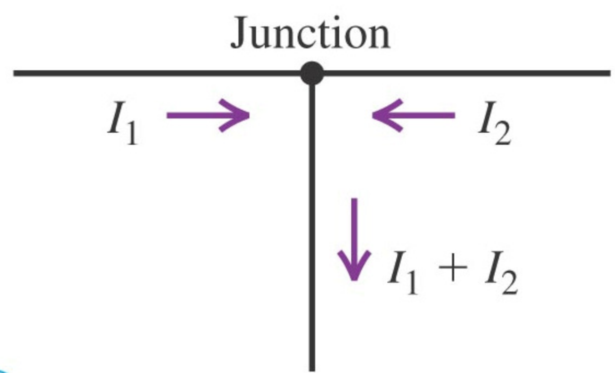
# 2. Kirchhoff's Rules

Junction: point where three or more conductors meet (nodes, branch points).

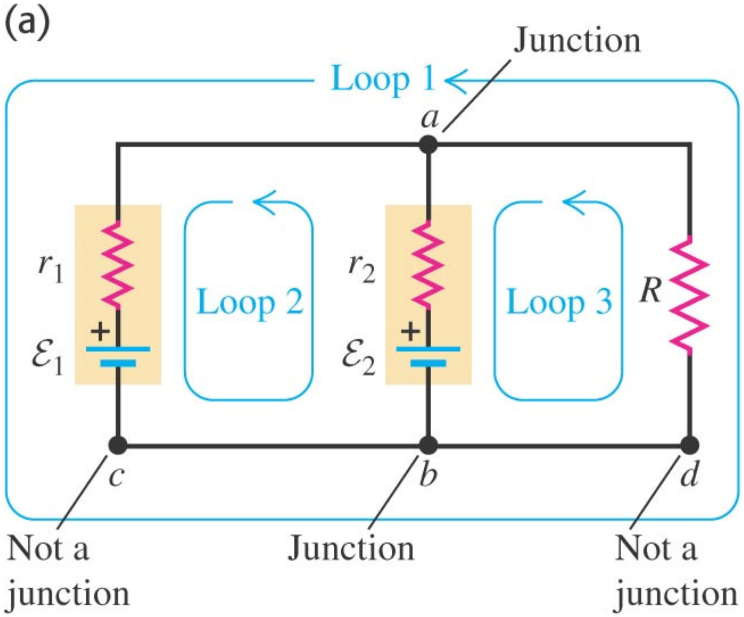
Loop: closed conducting path.

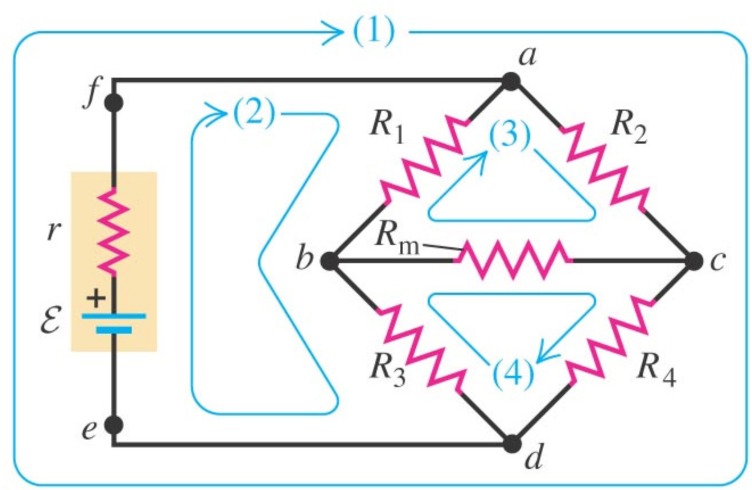
Kirchhoff's junction rule: the algebraic sum of the currents into any junction is zero.

$$\Sigma I = 0$$




(a)



-The junction rule is based on conservation of electric charge. No charge can accumulate at a junction → total charge entering the junction per unit time = total charge leaving.

Kirchhoff's loop rule: the algebraic sum of the potential difference in any loop, Including those associated with emfs and those of resistive elements, must equal zero.

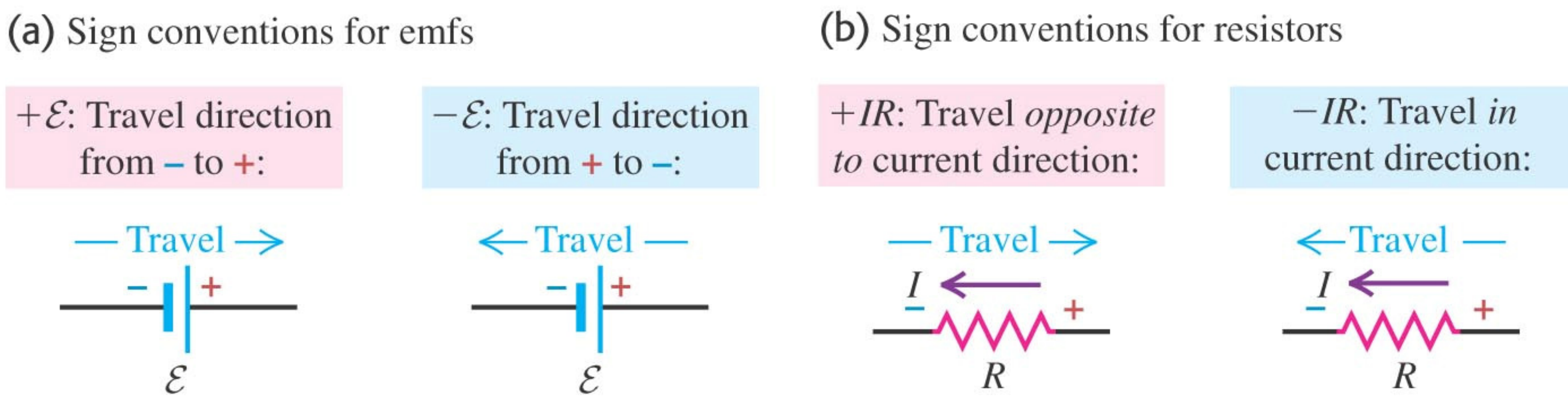
$$\Sigma V = 0$$ (electrostatic force is conservative)

Sign Conventions for Loop Rule:

emf source (ε) → positive (travel from – to +)
negative (travel from + to -)

resistor (IR) → negative (travel in same direction as I → decreasing V)
positive (travel in contrary direction to I → increasing V)

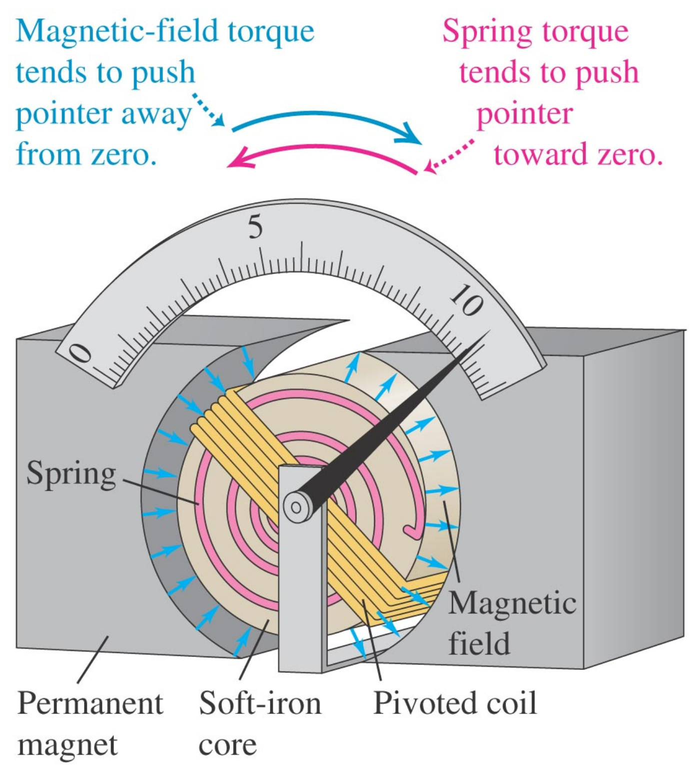
- "Travel" is the direction that we imagine going around the loop, not necessarily the direction of the current.




# 3. Electrical Measuring Instruments

d'Arsonval galvanometer (meter):

Coil of wire mounted next to a permanent magnet. Attached to the coil is a spring. Any current passing through the coil will induce magnetism in the coil (magnetic field exerts a torque on the coil ~ current). When the coil turns, spring makes restoring torque ~ angular displacement ~ current.



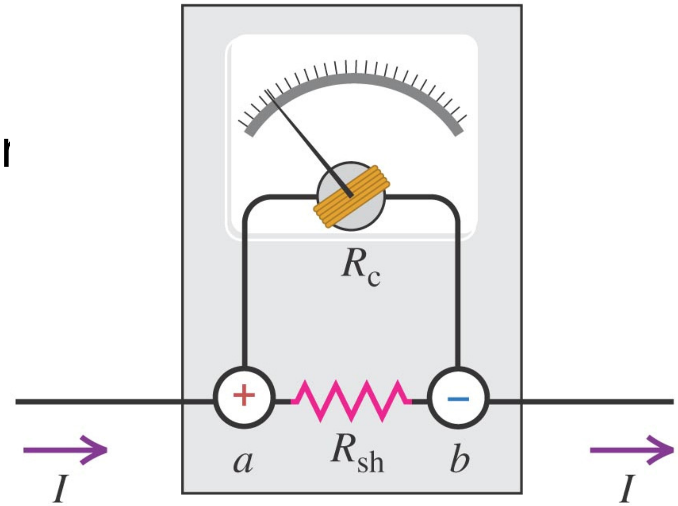

$$V = I_{fs} R_c$$

$I_{fs}$ = current full scale (coil)

$R_c$ = resistance of coil

Ammeter: device that measures current, R = 0

- It can be adapted to measure currents larger than its full scale range by connecting $R_{sh}$ (shunt resistor) in parallel (some I bypasses meter coil).





$$I_a = I_{sh} + I_{fs}$$

$I_{fs}$ = current through coil

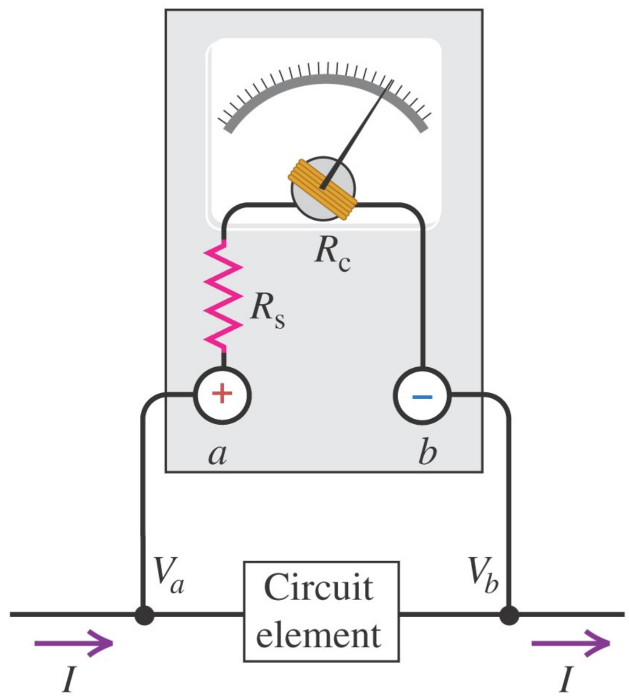
$I_{sh}$ = current through $R_{sh}$

$I_a$ = current measured by ammeter

$$V_{ab} = I_{fs}R_c = I_{sh} R_{sh} = (I_a - I_{fs}) R_{sh}$$

Voltmeter: device that measures voltage, R = ∞

- It can be adapted to measure voltages larger than its full scale range by connecting $R_s$ in series with the coil .

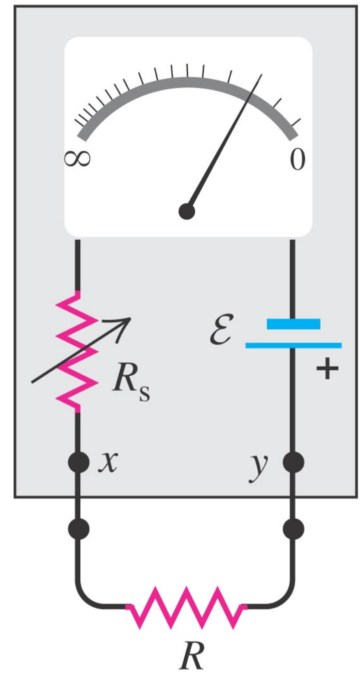




$$V_v = V_{ab} = I_{fs}(R_c + R_s)$$

Ohmeter: device that measures resistance.

- The series resistance $R_s$ is adjusted so that when the terminals x-y are short-circuited (R = 0), the meter deflects full scale (zero). When nothing is connected between x-y (open circuit, $R = \infty$) there is no current (no deflection). For intermediate R values, meter scale is calibrated to read R.

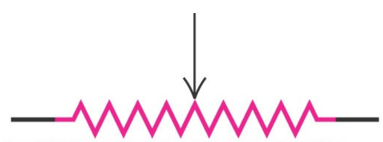


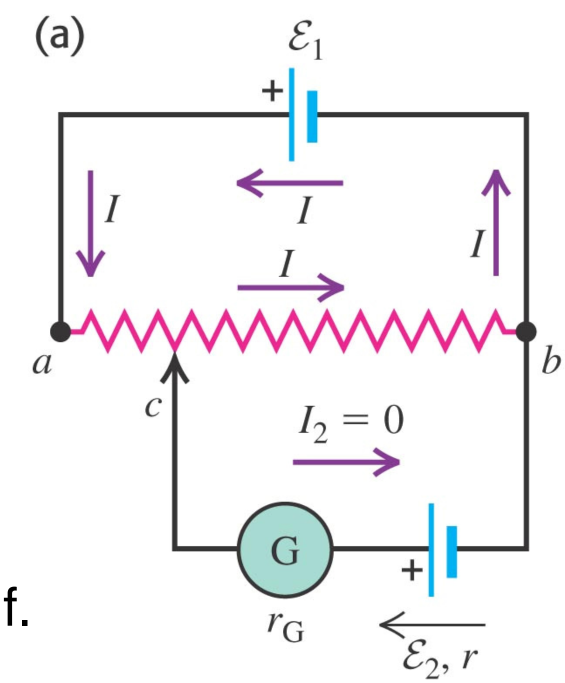


Potenciometer: device that measures emf of a source without drawing any current from it.

- $R_{ab}$ connected to terminals of known emf ($\varepsilon_1$). A sliding contact (c) is connected through galvanometer (G) to unknown source ($\varepsilon_2$). As contact (c) is moved along $R_{ab}$, $R_{cb}$ varies proportional to wire length (c-b). To find $\varepsilon_2$ (c) is moved until G shows no deflection ($I_G = 0$):

$$\varepsilon = I\, R_{cb}$$




- G calibrated by replacing $\varepsilon_2$ by source of known emf.

# 4. R-C Circuits

- Capital letters: V, Q, I (constant)
- Lowercase letters: v, i, q (varying)

Charging a Capacitor:

t = 0 → q = 0 → $v_{bc}$ = 0 → $I_0 = v_{ab} / R = \varepsilon / R$

t = $t_f$ → I = 0 → $v_{ab}$ = 0 → $v_{bc}$ = ε = $Q_f$/C

At an intermediate time, t:

$$\varepsilon = v_{ab} + v_{bc}$$

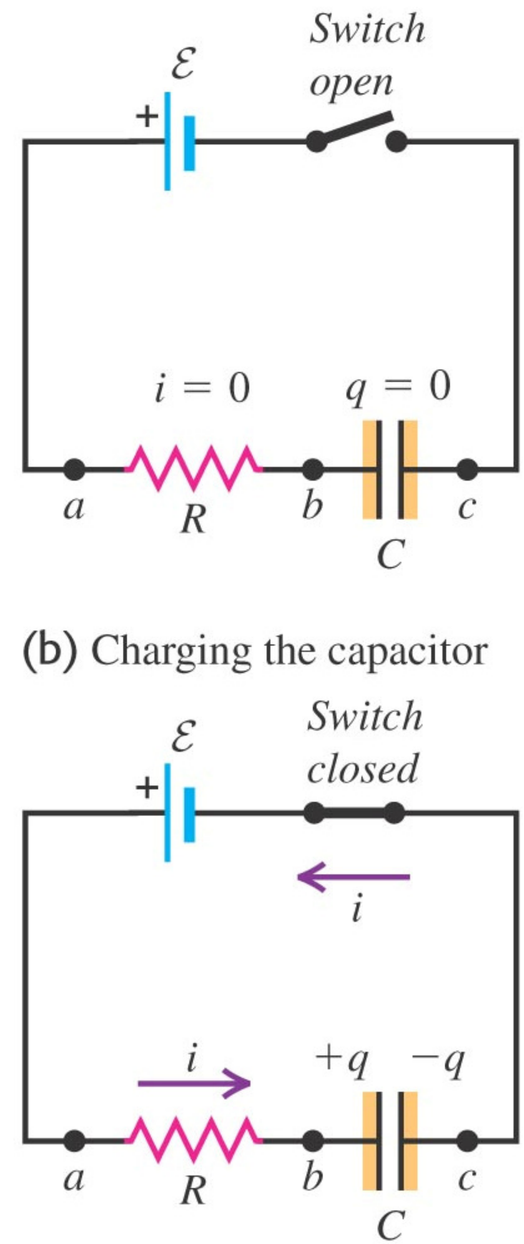
$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

$$\varepsilon - iR - \frac{q}{C} = 0$$

$$i = \frac{\varepsilon}{R} - \frac{q}{RC}$$

At t = $t_f$ → i = 0 → $\frac{\varepsilon}{R} = \frac{Q_f}{RC}$

$$Q_f = RC$$



## Charging a Capacitor:

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\varepsilon)$$

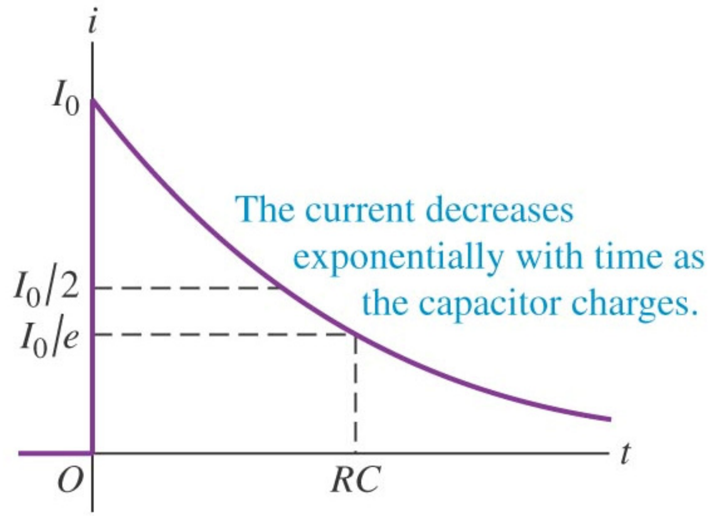
$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

$$\int_0^q \frac{dq'}{q' - C\varepsilon} = -\int_0^t \frac{dt'}{RC}$$ (solve by changing variable x = q' – Cε)

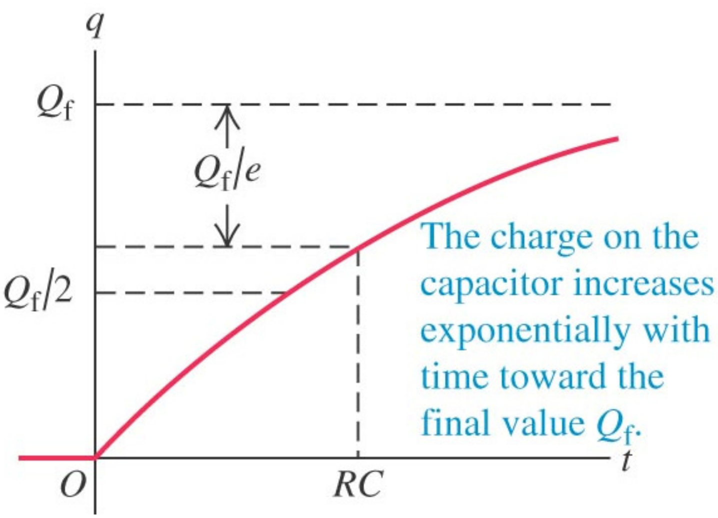
$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC} \qquad \frac{q - C\varepsilon}{-C\varepsilon} = e^{-t/RC}$$

$$q = C\varepsilon\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC} = I_0 e^{-t/RC}$$



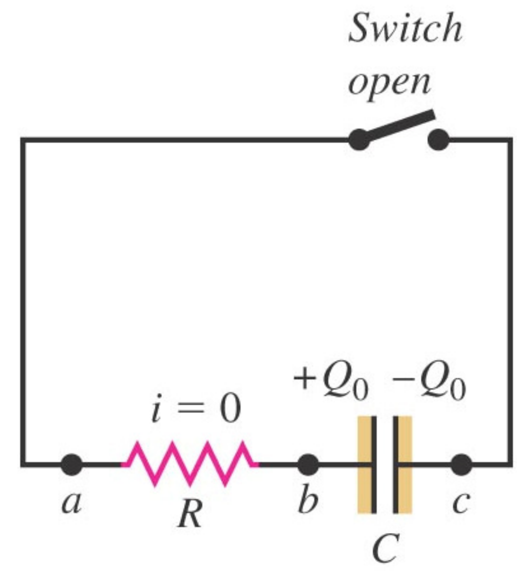

(b) Graph of capacitor charge versus time for a charging capacitor



Time Constant: relaxation time of the circuit → time after which the current in the circuit has decreased to 1/e of $I_0$ and charge has reached (1-1/e) of $Q_f = C\varepsilon$.

$$\tau = RC$$

- If RC small →circuit charges quickly.
- i never becomes exactly 0, and q never becomes exactly $Q_f$ (asymptotic behavior).



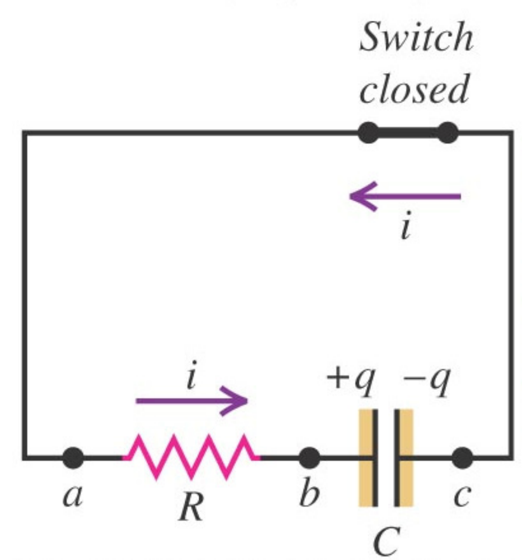

## Discharging a Capacitor:

$t = 0 \rightarrow q = Q_0$ , $\varepsilon = 0$ (capacitor discharges through R)

$$-iR - \frac{q}{C} = 0 \qquad i = \frac{dq}{dt} = -\frac{q}{RC}$$

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt' \qquad \ln\frac{q}{Q_0} = -\frac{t}{RC}$$

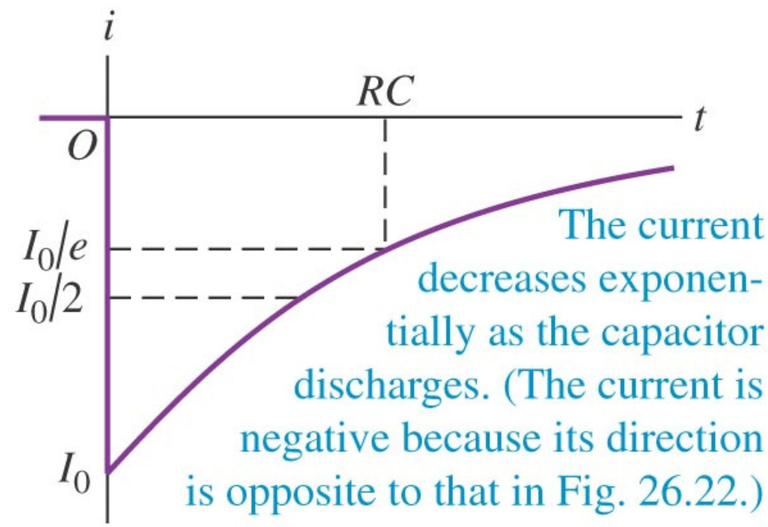
(b) Discharging the capacitor



## Discharging a Capacitor:

$$q = Q_0 e^{-t/RC}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

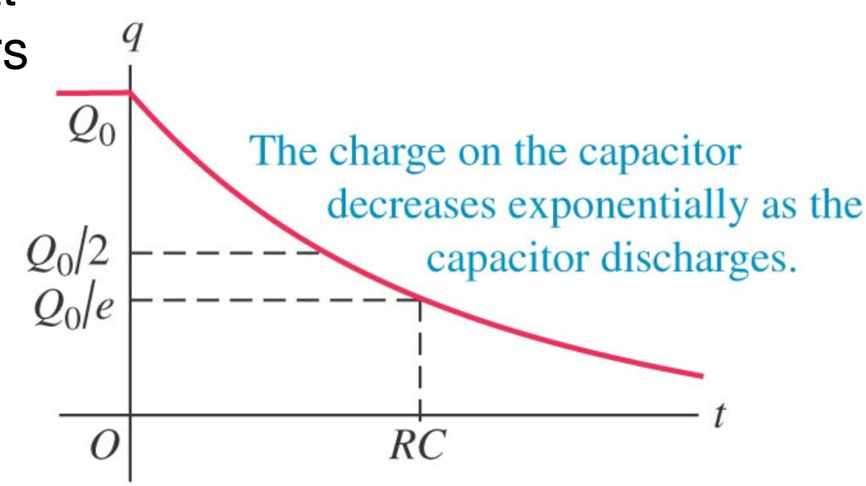



- During charging:

$$\varepsilon \cdot i = i^2 R + \frac{iq}{C}$$

Instantaneous rate at which battery delivers energy to circuit

(b) Graph of capacitor charge versus time for a discharging capacitor




$i^2R$ = power dissipated in R

i q/C = power stored in C

Total energy supplied by battery: $\varepsilon Q_f$

Total energy stored in capacitor: $Q_f \varepsilon/2$