Chapter 25 – Current, Resistance and Electromotive Force

- Current
- Resistivity
- Resistance
- Electromotive Force and Circuits
- Energy and Power in Electric Circuits
- Theory of Metallic Conduction

1. Current

Electric current: charges in motion from one region to another.

Electric circuit: conducting path that forms a closed loop in which charges move. In these circuits, energy is conveyed from one place to another.

Electrostatics: E = 0 within a conductor → Current (I) = 0, but not all charges are at rest, free electrons can move ($v \sim 10^6$ m/s). Electrons are attracted to + ions in material → do not escape.
Electron motion is random → no net charge flow

Non-electrostatic: E ≠ 0 inside conductor → $\vec{F} = q\,\vec{E}$

Charged particle moving in vacuum → steady acceleration // F

Charged particle moving in a conductor → collisions with "nearly" stationary massive ions in material change random motion of charged particles.
Due to E, superposition of random motion of charge + slow net motion (drift) of charged particles as a group in direction of $\vec{F} = q\,\vec{E}$ → net current in conductor.

Drift velocity (v_d) = 10^{-4} m/s (slow)

Direction of current flow:

- In the absence of an external field, electrons move randomly in a conductor. If a field exists near the conductor, its force on the electron imposes a drift.

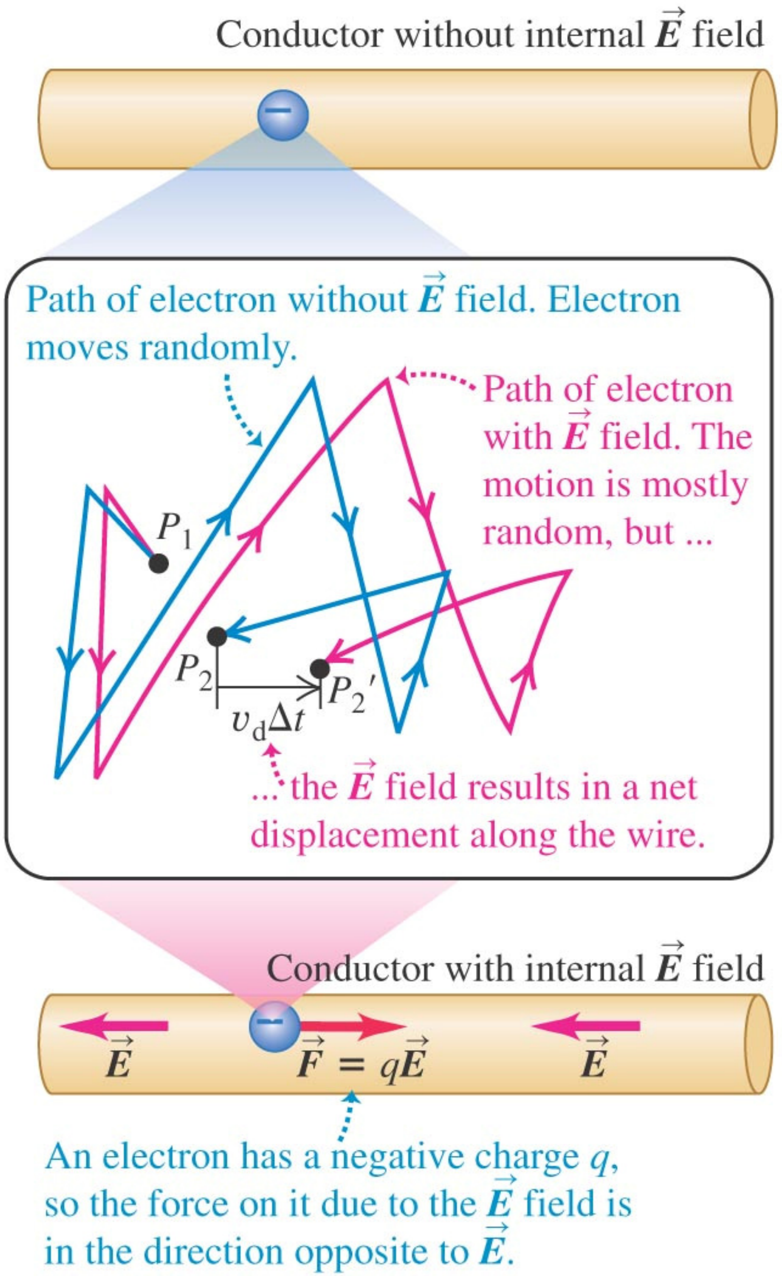
- E does work on moving charges → transfer of KE to the conductor through collisions with ions → increase in vibrational energy of ions → increase T.

- Much of W done by E goes into heating the conductor, not into accelerating charges faster and faster.

Metal: moving charges –

Ionized gas (plasma) or ionic solution: moving charges + or –

Semiconductor: electron + hole (vacancy) conduction

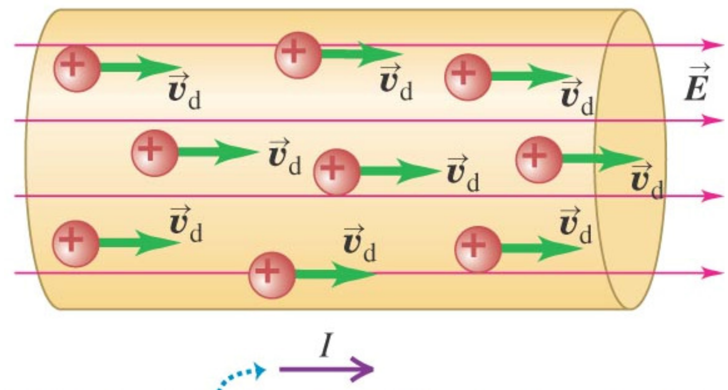


- Positive charges would move with the electric field, electrons move in opposition.

- The motion of electrons in a wire is analogous to water coursing through a river.

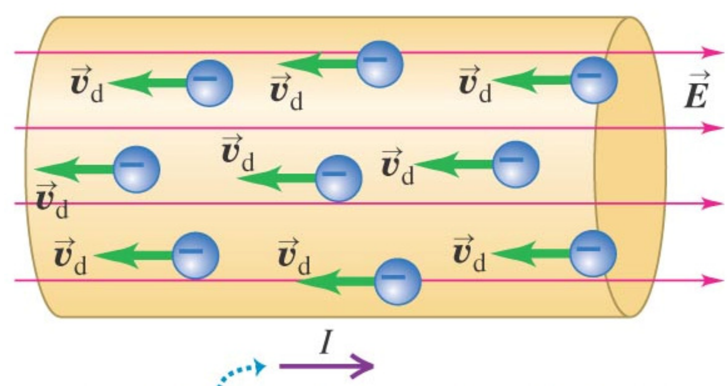
Conventional current (I): direction in which there is a flow of positive charge.
This direction is not necessarily the same as the direction in which charged particles are actually moving.

Current: $$I = \frac{dQ}{dt}$$



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



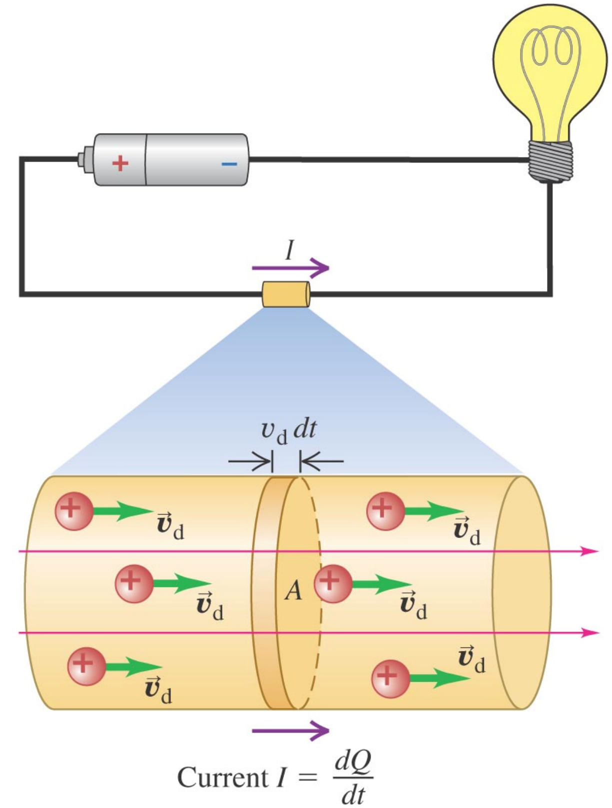
In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

- Current is not a vector! → no single vector can describe motion along curved path.

Current units: 1 A = 1 C/s

Current (I) is the time rate of charge transfer through a cross sectional area.

The random component of each moving charged particle's motion averages to zero → I in same direction as E.



Current, Drift Velocity and Current Density:

$$I = \frac{dQ}{dt} = n|q|v_d A$$

n = concentration of charged particles

v_d = drift velocity

Current Density (J): $$J = \frac{I}{A} = n|q|v_d$$

$$\vec{J} = nq\vec{v}_d$$

J is a vector, describes how charges flow at a certain point.

Steady current (closed circuit): total charge in every segment of conductor is constant → equal rate of flow of charge in and out of segment.

Direct current: direction of current is always the same.

Alternating current: current continuously changes direction.

2. Resistivity

(Intrinsic material property)

Ohm's law → $\vec{J}$ directly proportional to $\vec{E}$.

1 Ohm = 1 Ω = V/A

Resistivity: $$\rho = \frac{E}{J}$$

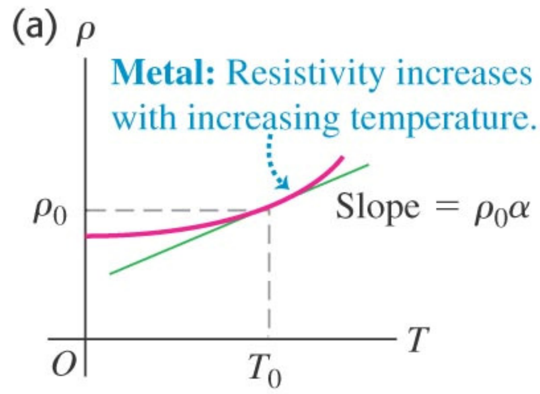
Units: Ω m = (V/m)/(A/m²) = (V/A) m

Substance		$\rho\,(\Omega \cdot m)$	Substance	$\rho\,(\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Conductivity: 1/ρ

Metals: good electrical and thermal conductors. Very large difference in conductivity of metals vs. insulators → possible to confine electric currents.

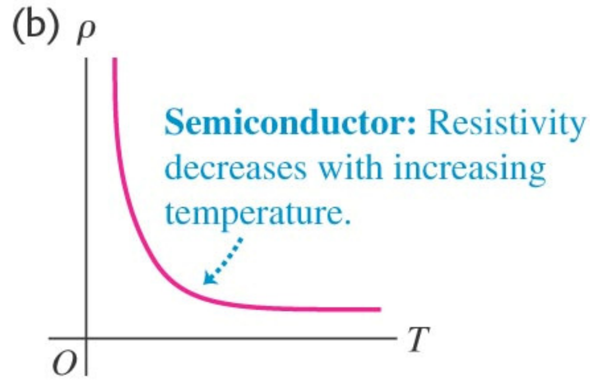
Semiconductors: intermediate resistivity between metal & insulator.



Resistivity and Temperature:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

α = temperature coefficient of resistivity



Metal: ρ increases with T

Semiconductor: ρ decreases with T

Superconductor: ρ first decreases smoothly with decreasing T and becomes zero < Tc (critical T)

Highest T_c = 233 K (2009) → $Ta_5Ba_4Ca_2Cu_{10}O_x$

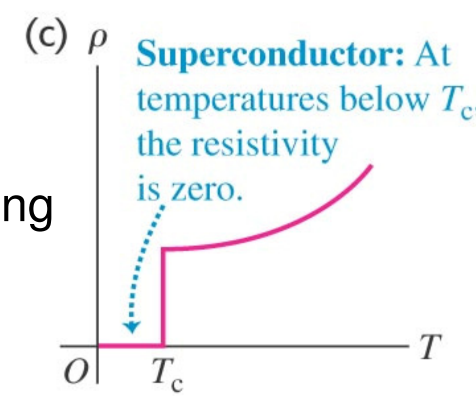


Table 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

Material	$\alpha\,[(°C)^{-1}]$	Material	$\alpha\,[(°C)^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

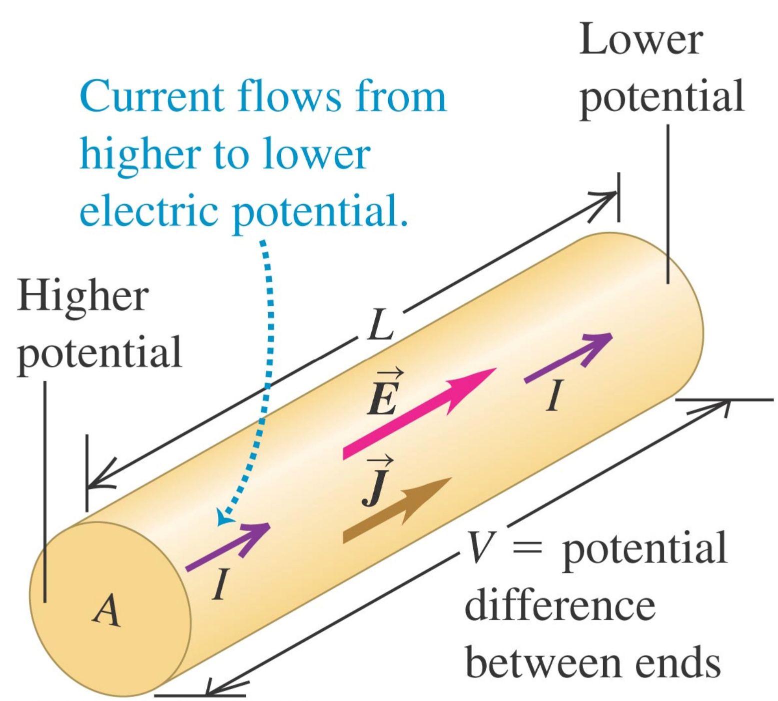
3. Resistance

$$\vec{E} = \rho \cdot \vec{J}$$

Ohm's law → ρ = constant

Current direction: from higher V end to lower V end. Follows E direction, independent of sign of moving charges.

- As the current flows through a potential difference, electric potential energy is lost. This energy is transferred to the ions of conducting material during collisions.



$$I = J \cdot A$$

$$V = E \cdot L$$

R = resistance

$$E = \frac{V}{L} = \rho \cdot J = \rho \frac{I}{A} \quad \rightarrow \quad V = \boxed{\frac{\rho \cdot L}{A}} I$$

Resistance:

$$R = \frac{V}{I} = \frac{\rho \cdot L}{A}$$

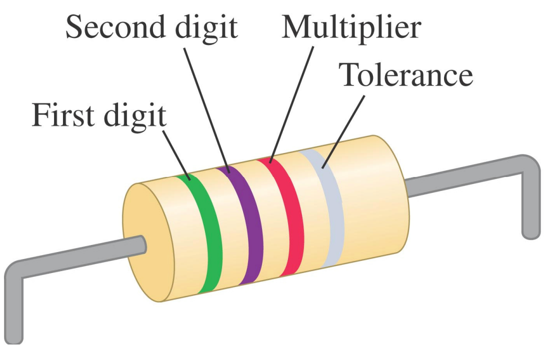
$$\boxed{V = I \cdot R}$$

Ohm's law (conductors)

Units: Ohm = Ω = 1 V/A

$$\boxed{R(T) = R_0[1 + \alpha(T - T_0)]}$$

Resistor: circuit device with a fixed R between its ends.



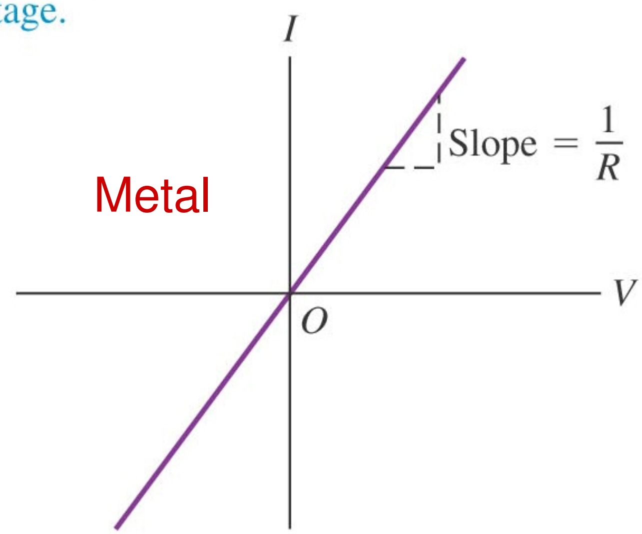
Ex: 5.7 kΩ = green (5) violet (7) red multiplier (100)

Table 25.3 Color Codes for Resistors

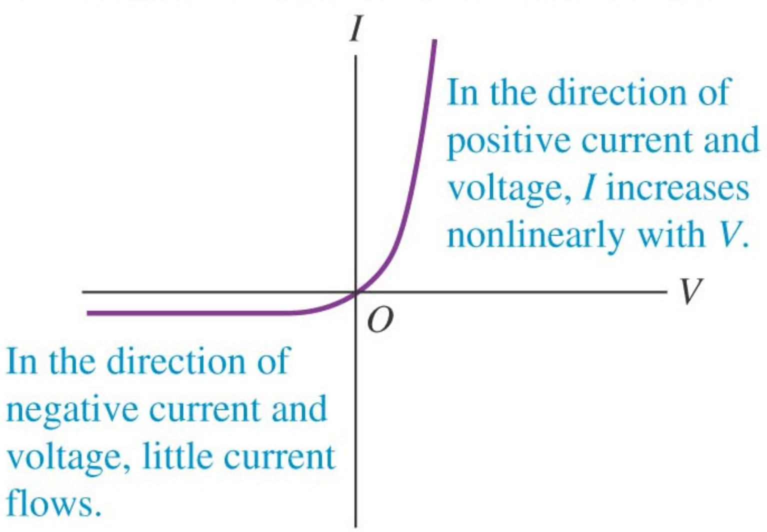
Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Current-voltage curves

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



Semiconductor diode: a nonohmic resistor



4. Electromotive Force and Circuits

- No steady motion of charge in incomplete circuit.

Electromotive Force (emf)

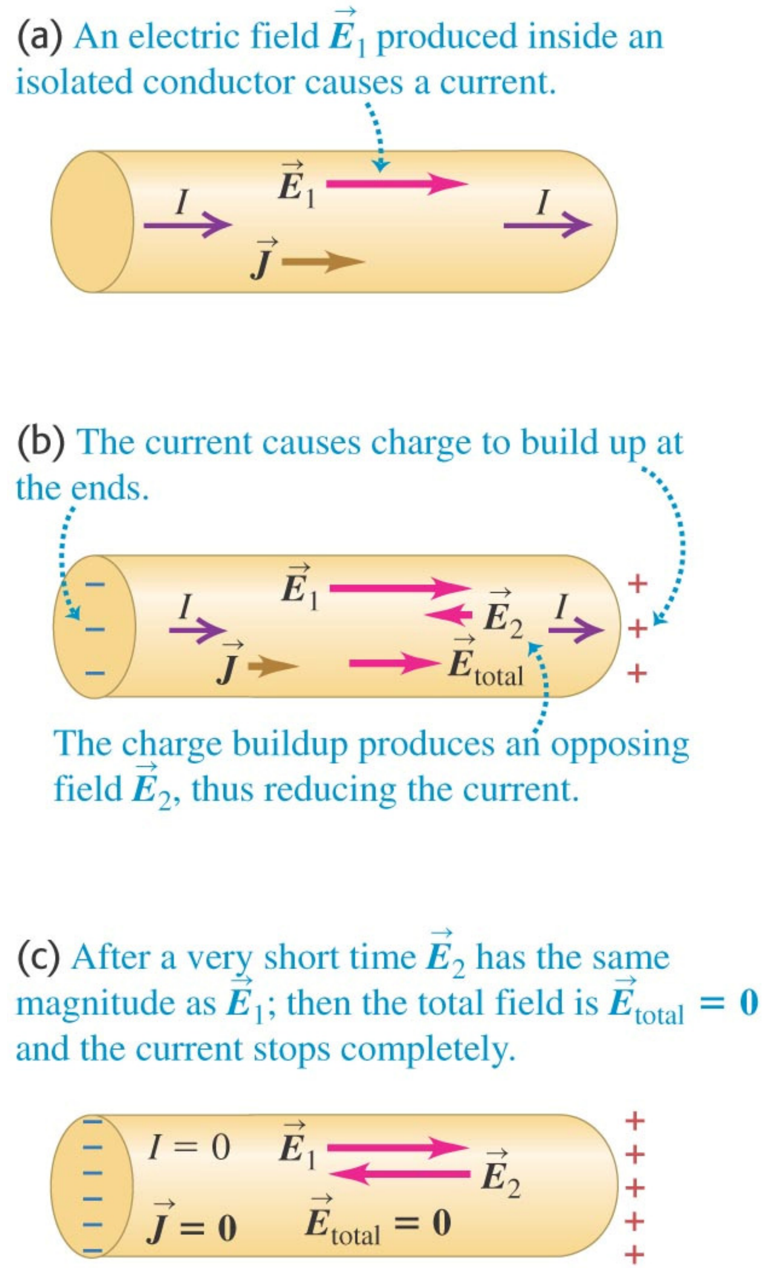
- In an electric circuit there should be a device that acts like the water pump in a fountain = source of emf.

- In this device, the charge travels “uphill” from lower to higher V (opposite to normal conductor) due to the emf force.

- emf is not a force but energy/unit charge

Units: 1 V = 1 J/C

- emf device convert energy (mechanical, chemical, thermal) into electric potential energy and transfer it to circuit.



- Ideal emf device maintains a constant potential difference between its terminals, independent of I.

Electric force: $\vec{F}_e = q\vec{E}$

Non electrostatic force: $\vec{F}_n$

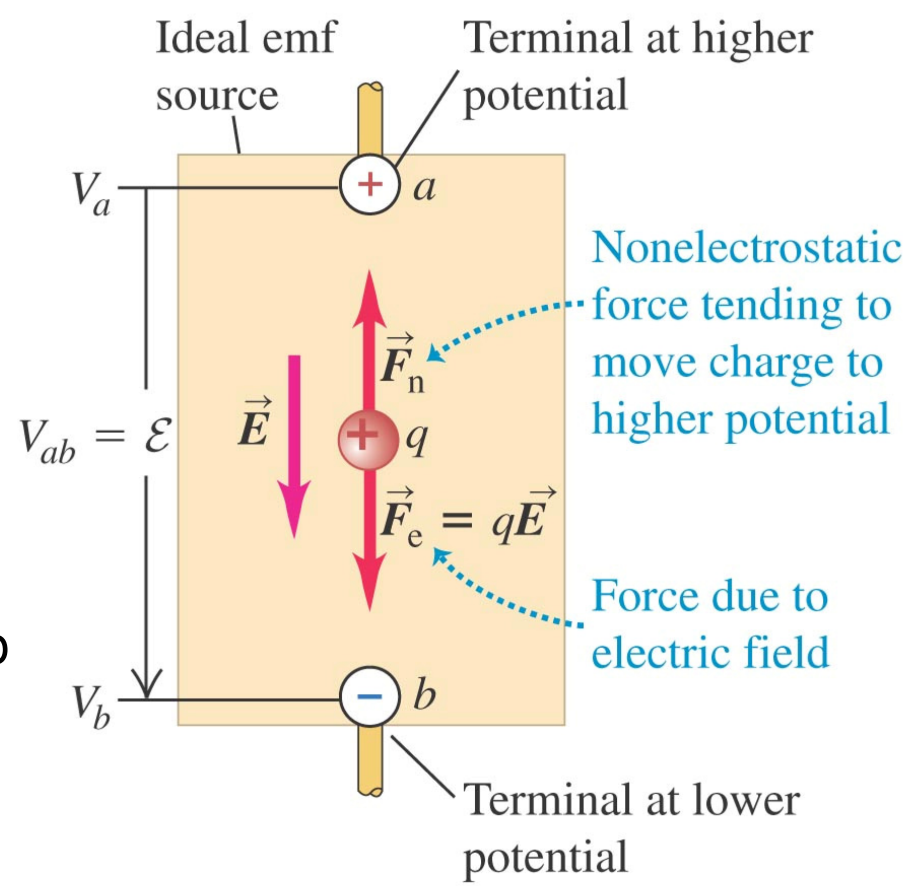
maintains potential difference between terminals. If F_n=0 → charge will flow between terminals until V_{ab}=0

$W_n = q\varepsilon$ displacement opposite to F_e → potential energy Increases by $q \cdot V_{ab}$

$$W_n = \Delta E = q\varepsilon = \Delta K + \Delta U$$
$$= U_a - U_b = q(V_a - V_b)$$

$V_{ab} = \varepsilon$ Ideal source of emf ($F_e = F_n$) Total work on q = 0

Ideal diagram of "open" circuit



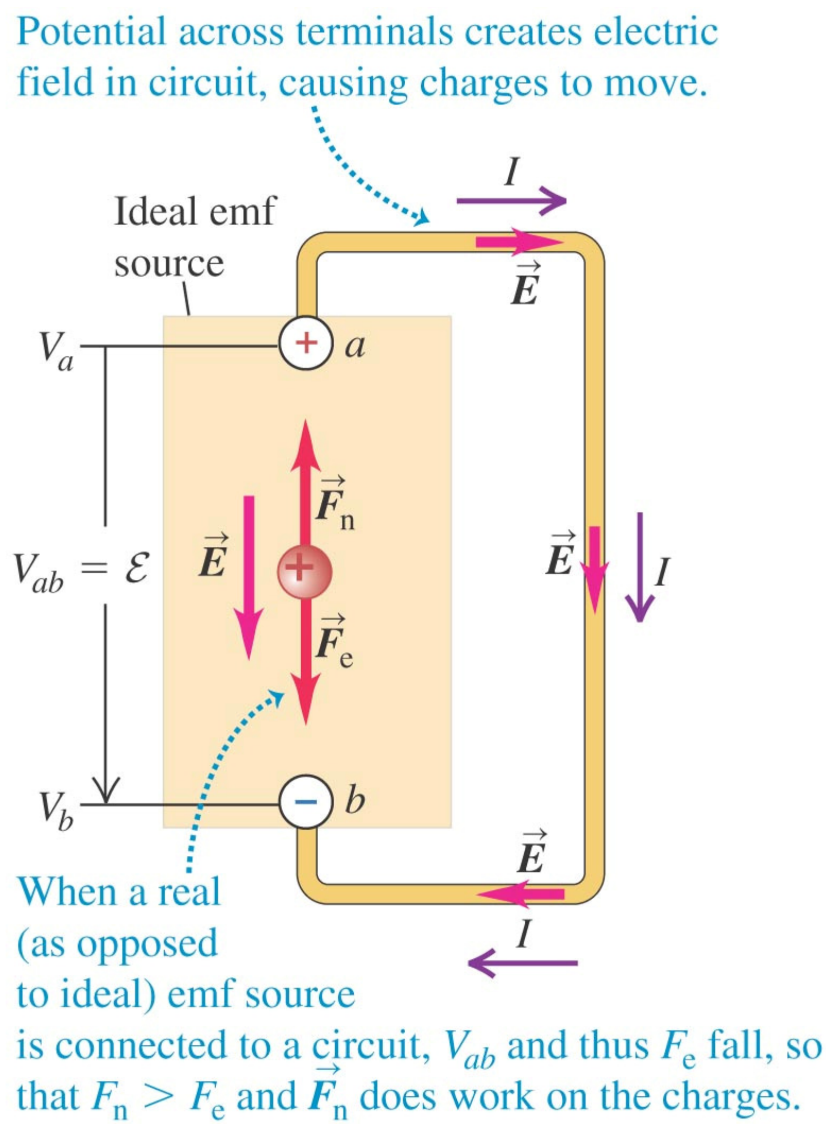
When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

$V_{ab} = \varepsilon = I\,R$

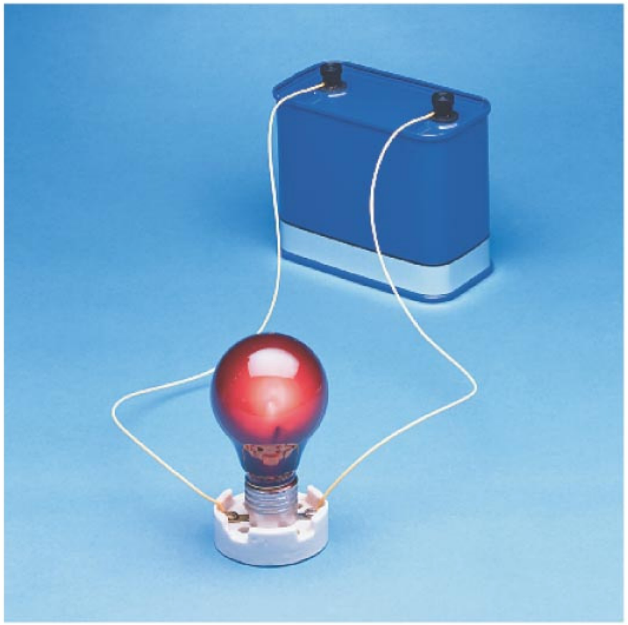
- When a positive charge q flows around a circuit, the potential rise ε as it passes through the ideal source is equal to the potential drop V_{ab} as it passes through reminder of circuit.

-The current is same at every point of a circuit, even if wire thickness different at different points of circuit. Charge is conserved and cannot be accumulated in circuit.

Ideal diagram of "closed" circuit



Internal resistance



- In a battery, you only get 12 V when it isn't connected.

- Making connections allows electrons to flow, but internal resistance within battery delivers incrementally less than 12 V.

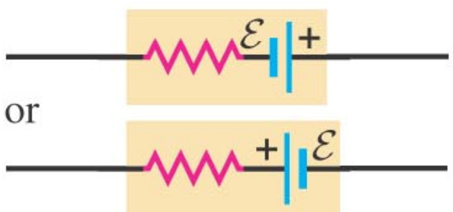
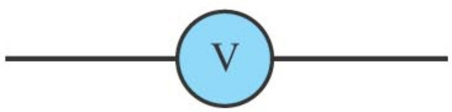
- The potential difference across a real source is not equal to emf. Charge moving through the material of the source encounters internal resistance (r).

Terminal voltage: $$V_{ab} = \varepsilon - Ir$$ Source with internal resistance

- For a real source, $V_{ab} = \varepsilon$ (emf) only if no current flows through source.

$$I = \frac{\varepsilon}{R + r}$$

Table 25.4 Symbols for Circuit Diagrams

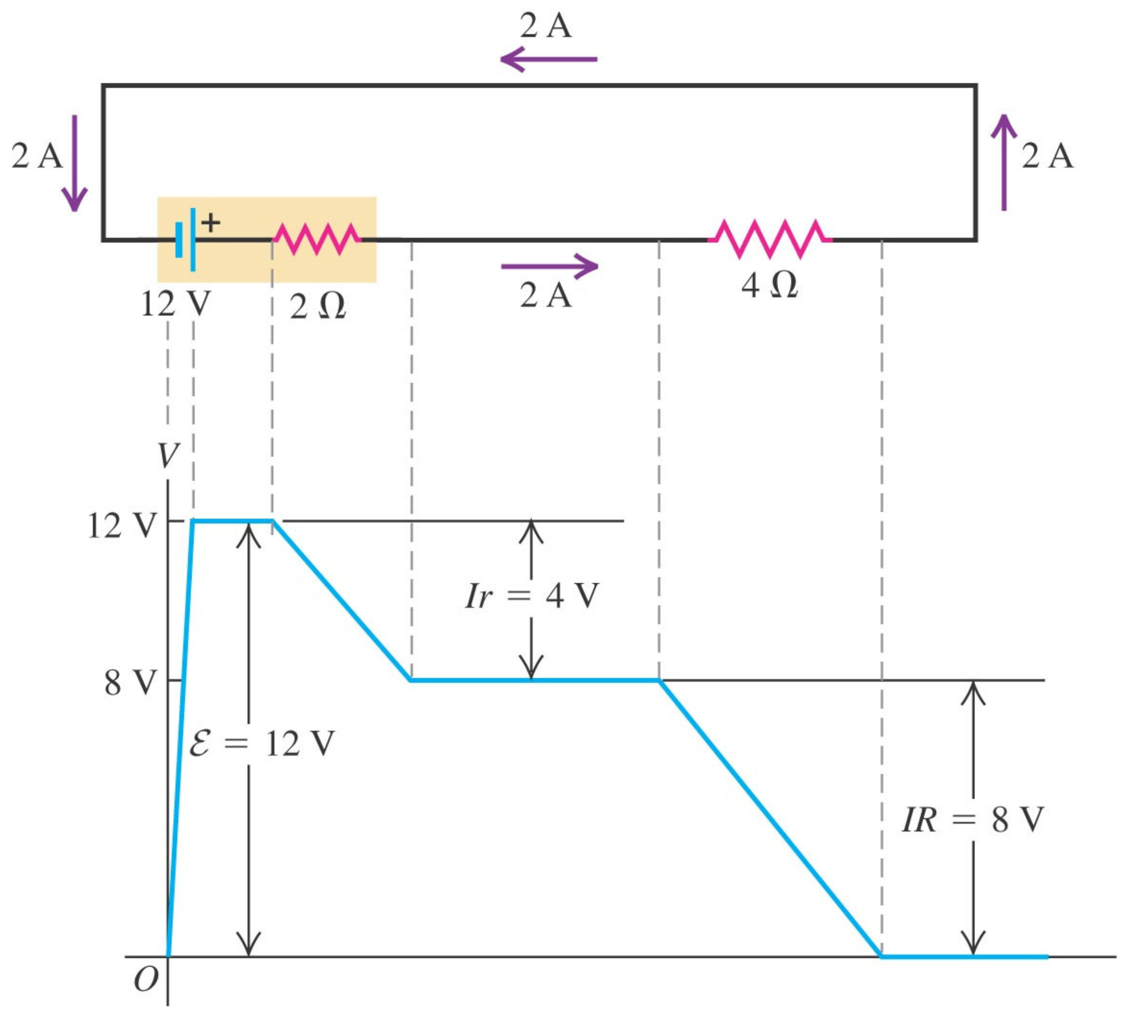
Symbol	Description
	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
	Source of emf with internal resistance r (r can be placed on either side)
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

- The meters do not disturb the circuit in which they are connected.
- Voltmeter → infinite resistance → I = V/ R → I =0 (measures V)
- Ammeter →zero resistance → V = I R = 0 (measures I)

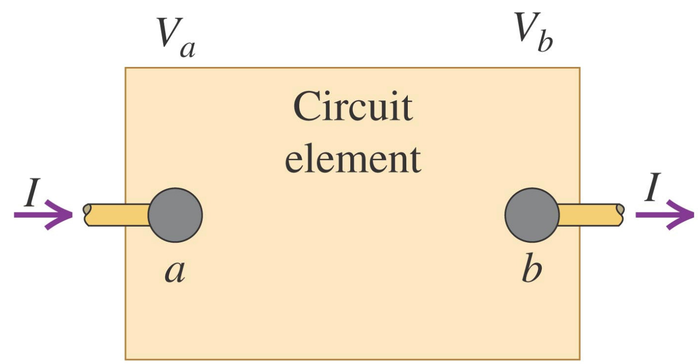
Potential changes around a circuit

- The net change in potential energy for a charge *q* making a round trip around a complete circuit must be zero.

- Local differences in potential occur.



5. Energy and Power in Circuits



Power: rate at which energy is delivered to or extracted from a circuit element.

$$P = V_{ab}\, I = (V_a - V_b)\, I$$

Units: 1 Watt = W = V A = (J/C) (C/s) = J/s

Potential Input to a Pure Resistance

$$P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}$$

Rate of transfer of electric potential energy into the circuit ($V_a > V_b$) → energy dissipated (heat) in resistor at a rate I^2 R.

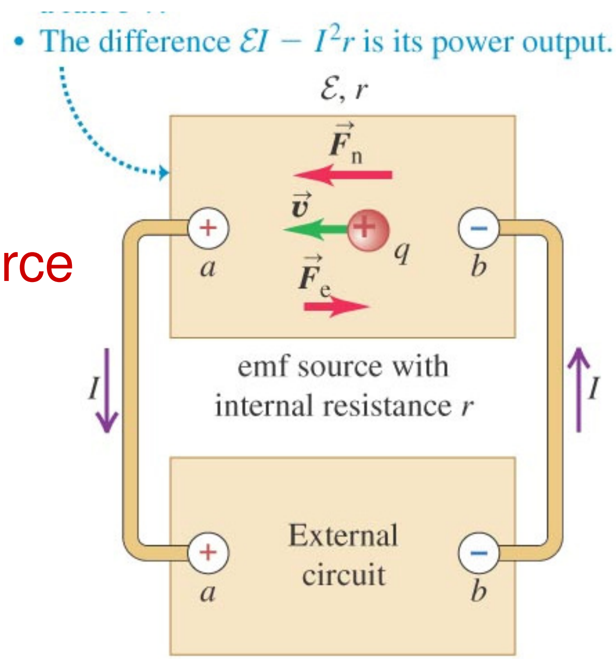
Potential Output of a source

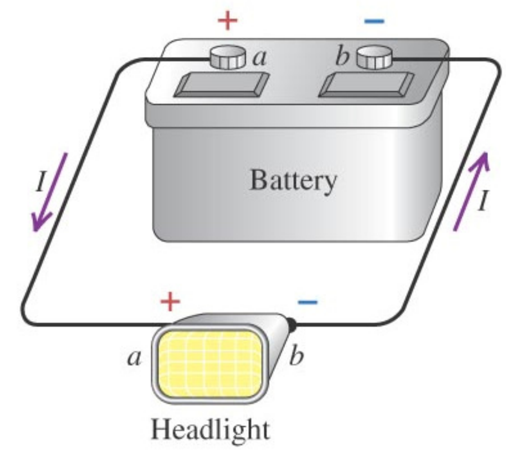
$$P = V_{ab} I = (\varepsilon - Ir)I = \varepsilon \cdot I - I^2 r$$

ε I = rate at which the emf source converts nonelectrical to electrical energy.

I^2 r = rate at which electric energy is dissipated at the internal resistance of source.

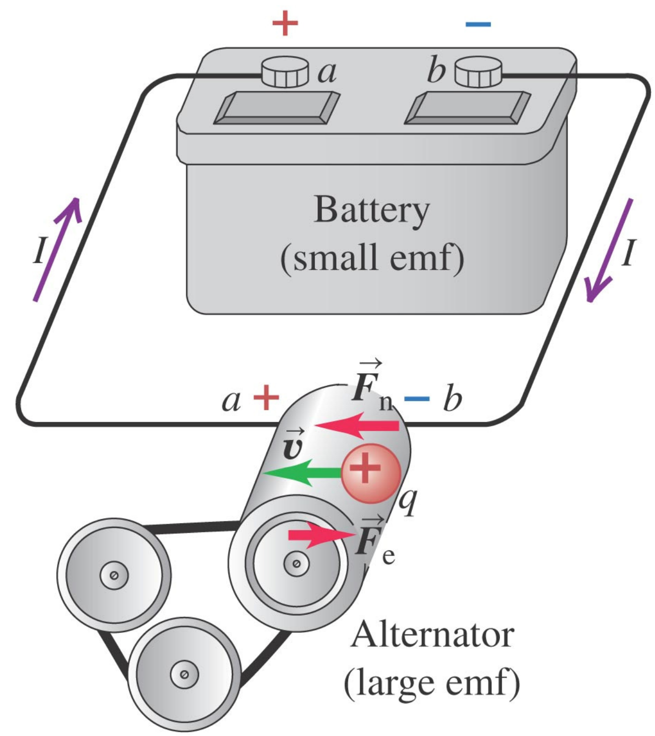
Potential Output of a source





Potential Input to a source

$$P = V_{ab} I = (\varepsilon + Ir)I = \varepsilon \cdot I + I^2 r$$



Conversion of electrical energy into non-electrical energy in the upper source at a rate ε I.

$I^2 r$ = rate of dissipation of energy.

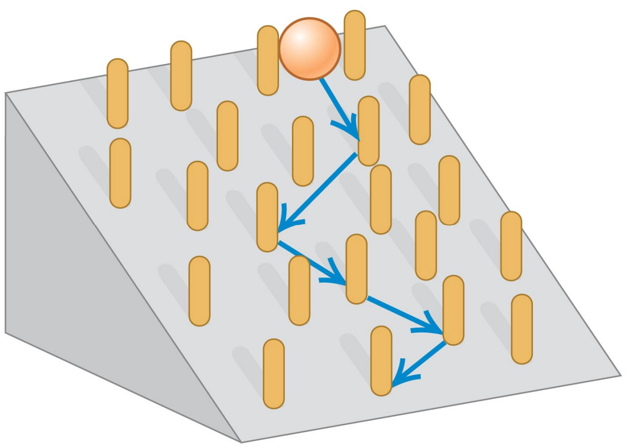
Lower source pushing current upward through upper source.

6. Theory of Metallic Conduction

- If no E → free e- move in straight lines between collisions with + ions
  → random velocities, in average, no net displacement.

- If E → e- path curves due to acceleration caused by F_e → drift speed.

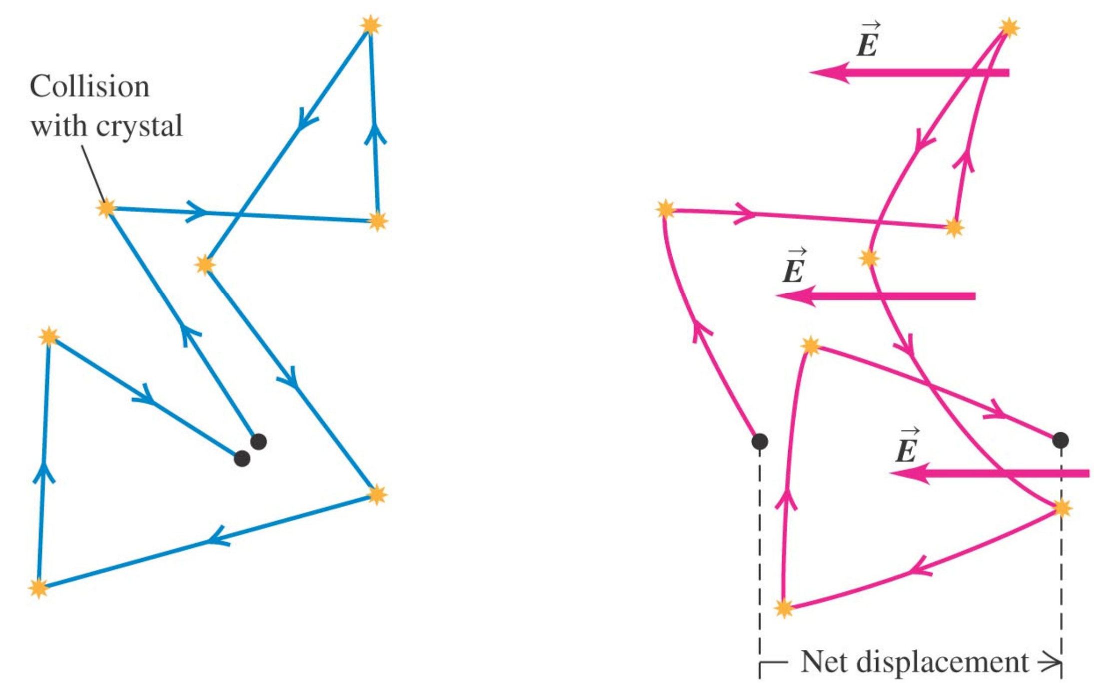
Mean free time (τ): average time between collisions.



Analogy to motion of e- with E.

(a) Typical trajectory for an electron in a metallic crystal *without* an internal $\vec{E}$ field

(b) Typical trajectory for an electron in a metallic crystal *with* an internal $\vec{E}$ field



$$\rho = \frac{E}{J} \qquad \vec{J} = n \cdot q \cdot \vec{v}_d \qquad \vec{a} = \frac{\overline{F}}{m} = \frac{q\vec{E}}{m}$$

$$\vec{v} = \vec{v}_0 + \vec{a}\tau \qquad \vec{v}_{avg} = \vec{a}\tau = \frac{q\tau}{m}\vec{E} = \vec{v}_d$$

$$\vec{J} = n \cdot q \cdot \vec{v}_d = \frac{nq^2\tau}{m}\vec{E}$$

$$\boxed{\rho = \frac{E}{J} = \frac{m}{q^2 n\tau} = \frac{m}{e^2 n\tau}}$$