Chapter 23 – Electric Potential

- Electric Potential Energy

- Electric Potential and its Calculation

- Equipotential surfaces

- Potential Gradient

0. Review

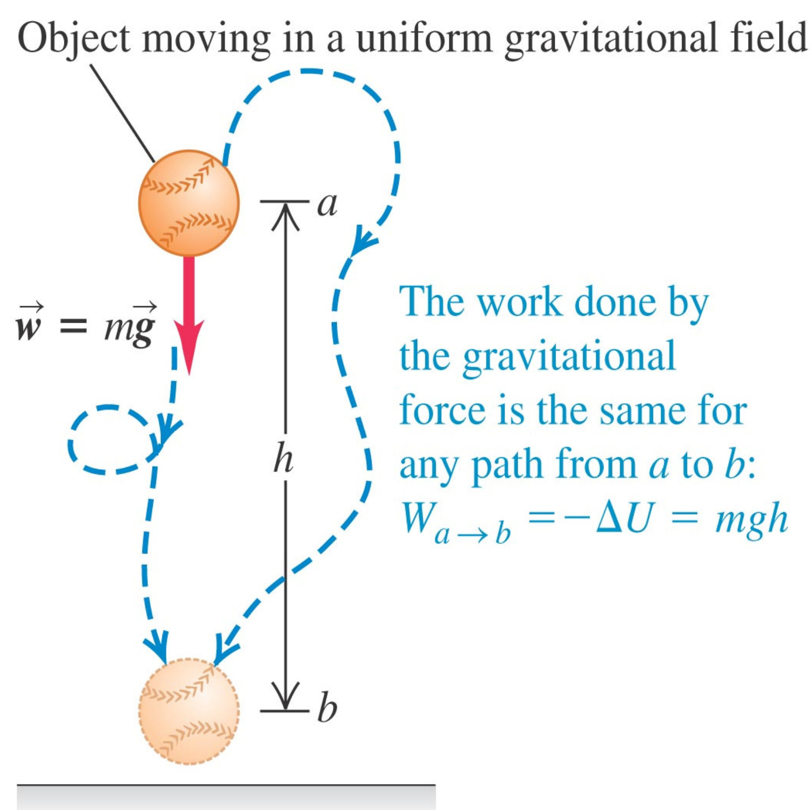
Work: $$W_{a\to b}=\int_a^b \vec{F}\cdot d\vec{l}=\int_a^b F\cdot\cos\varphi\cdot dl$$

Potential energy

- If the force is conservative: $$W_{a\to b}=U_a-U_b=-(U_b-U_a)=-\Delta U$$

Work-Energy: $$K_a+U_a=K_b+U_b$$

The work done raising a basketball against gravity depends only on the potential energy, how high the ball goes. It does not depend on other motions. A point charge moving in a field exhibits similar behavior.



1. Electric Potential Energy

- When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. The work can be expressed in terms of electric potential energy.

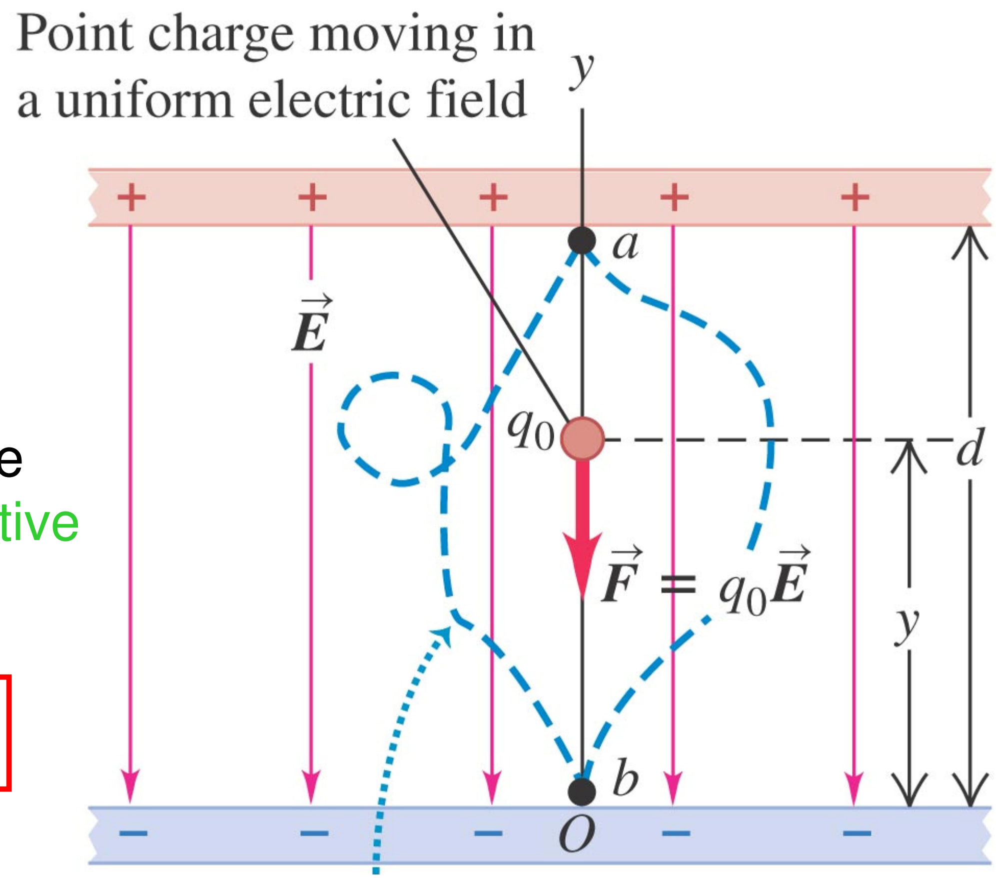
- Electric potential energy depends only on the position of the charged particle in the electric field.

Electric Potential Energy in a Uniform Field:

$$W_{a\to b} = F \cdot d = q_0 E d$$

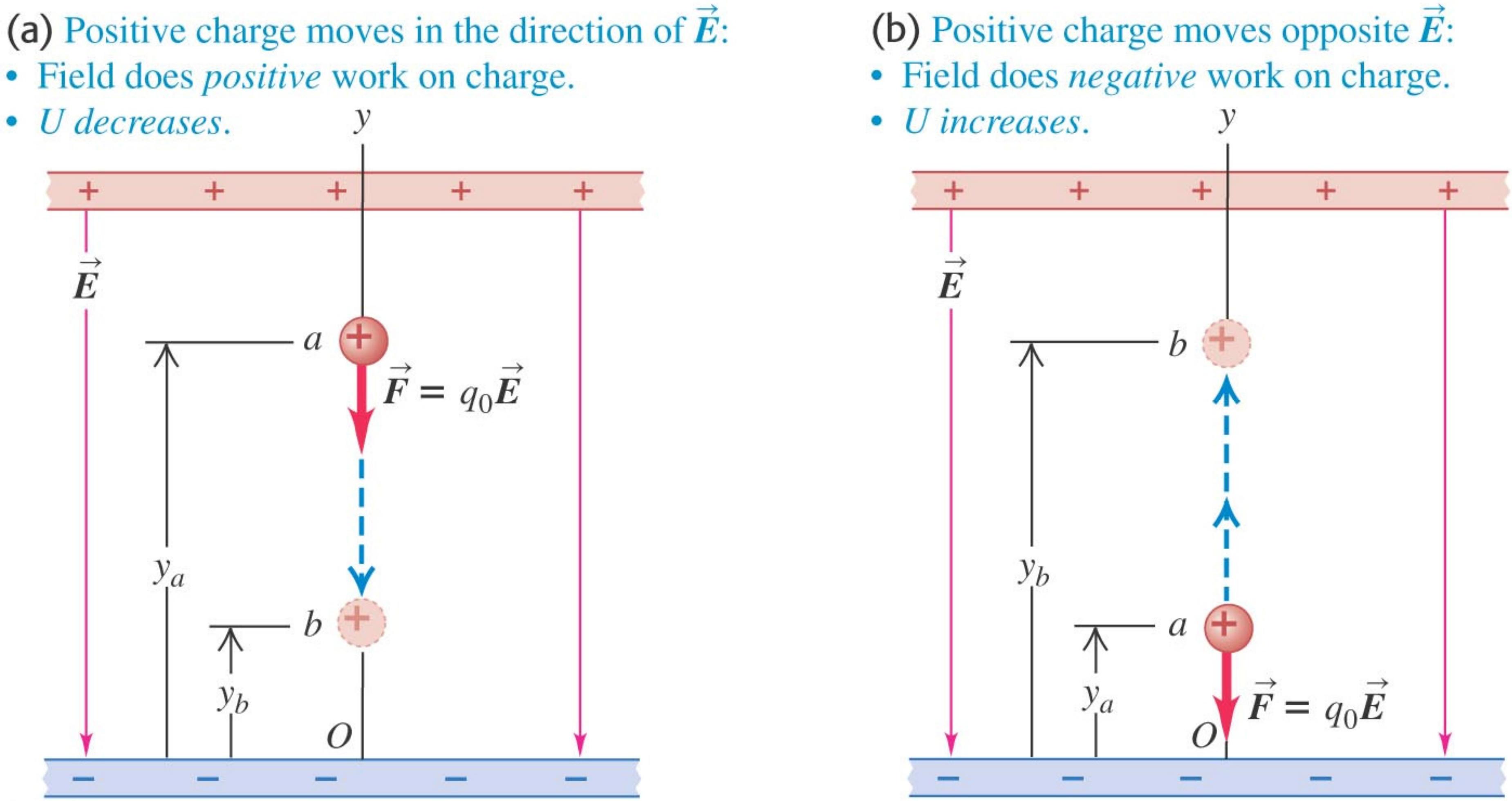
Electric field due to a static charge distribution generates a conservative force:

$$W_{a\to b} = -\Delta U \rightarrow \boxed{U = q_0 E \cdot y}$$



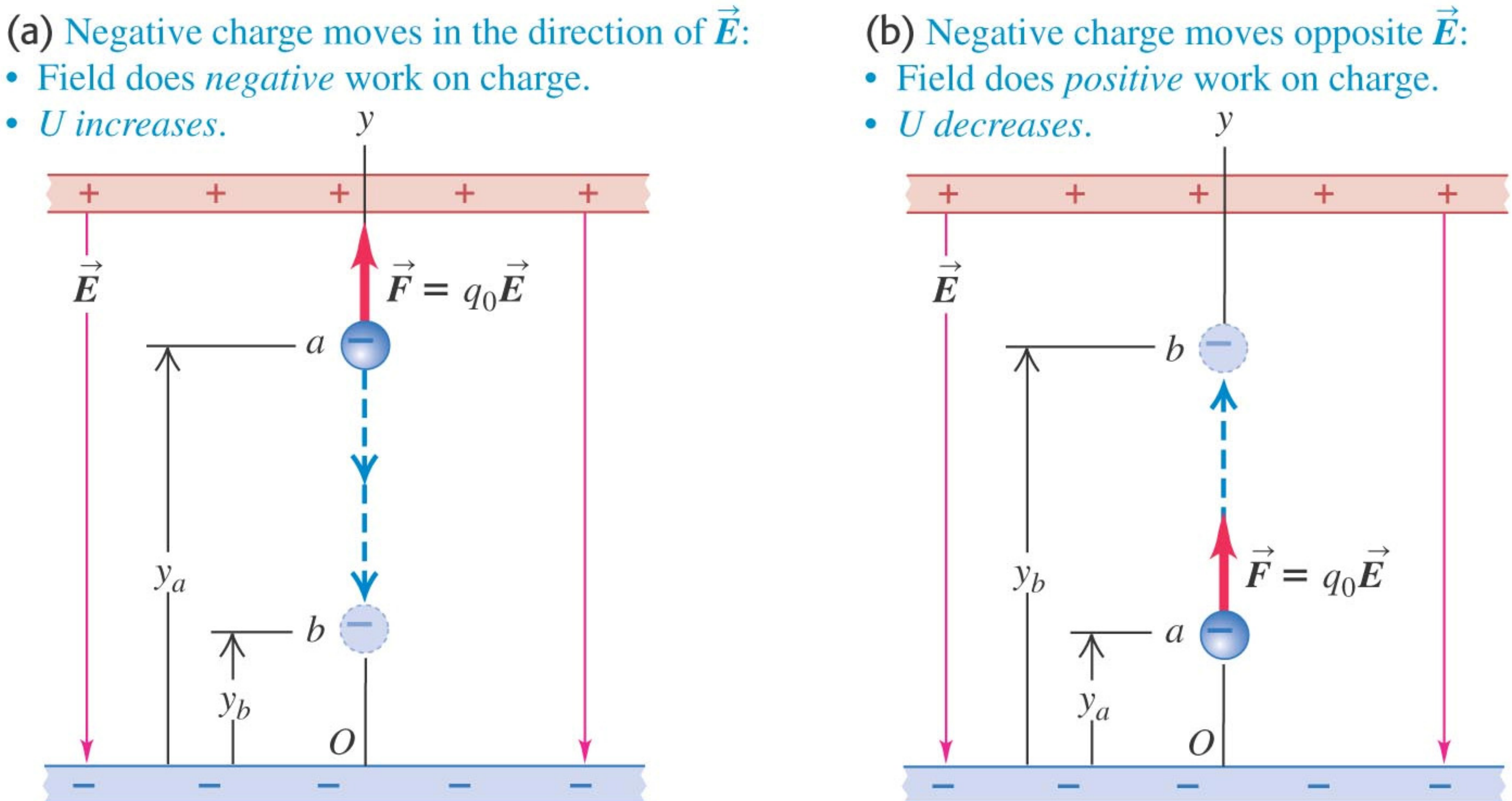
- Test charge moving from height y_a to y_b:

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = q_0 E(y_a - y_b)$$



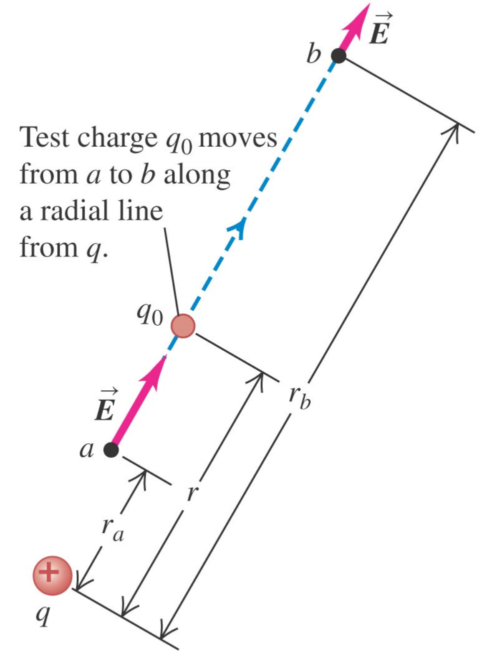
Independently of whether the test charge is (+) or (-):

- U increases if q_0 moves in direction opposite to electric force.
- U decreases if q_0 moves in same direction as $\vec{F} = q_0 \vec{E}$.

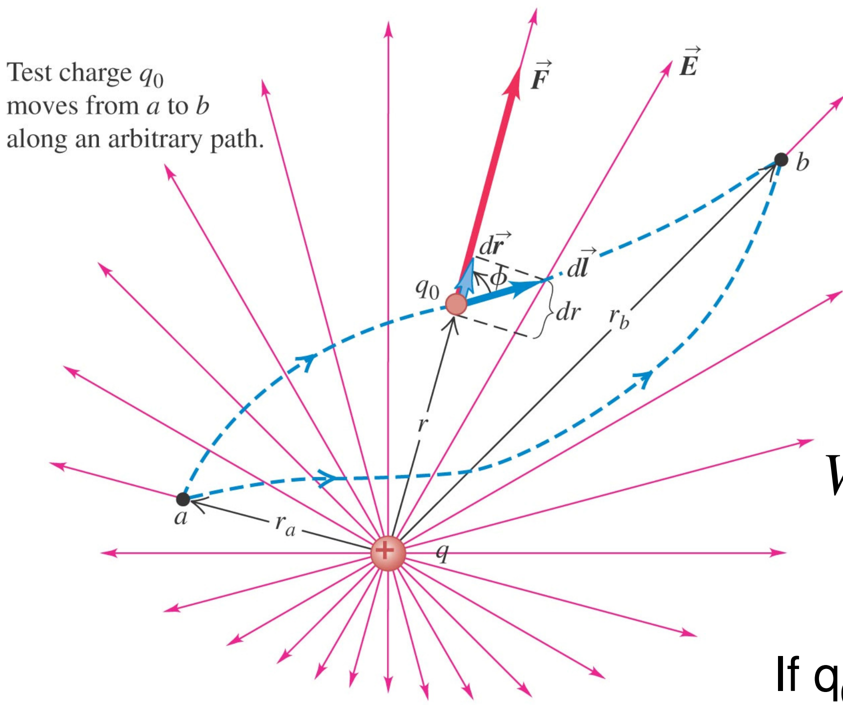


Electric Potential Energy of Two Point Charges:

A test charge (q_0) will move directly away from a like charge q.



$$W_{a\to b} = \int_{r_a}^{r_b} F_r \cdot dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cdot dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



The work done on q_0 by electric field does not depend on path taken, but only on distances r_a and r_b (initial and end points).

$$W_{a\to b} = \int_{r_a}^{r_b} F \cdot \cos\varphi \cdot dl = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cdot \cos\varphi \cdot dl$$

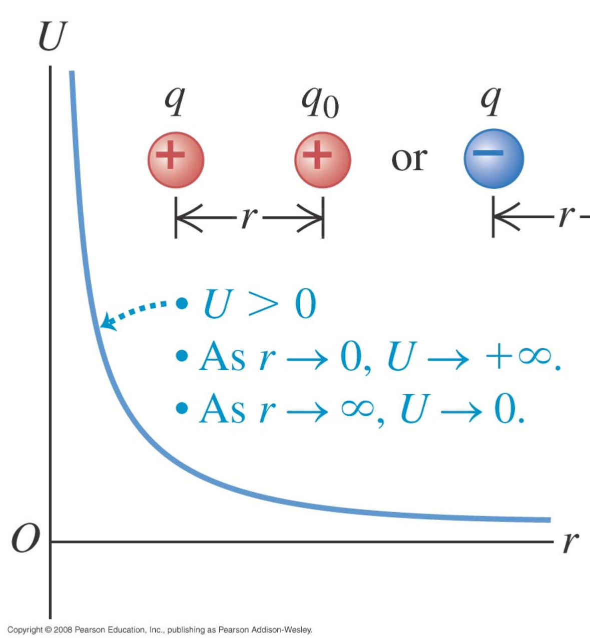
dr = dl cosφ

If q_0 moves from *a* to b, and then returns to *a* by a different path, W (round trip) = 0

- Potential energy when charge q_0 is at distance r from q:

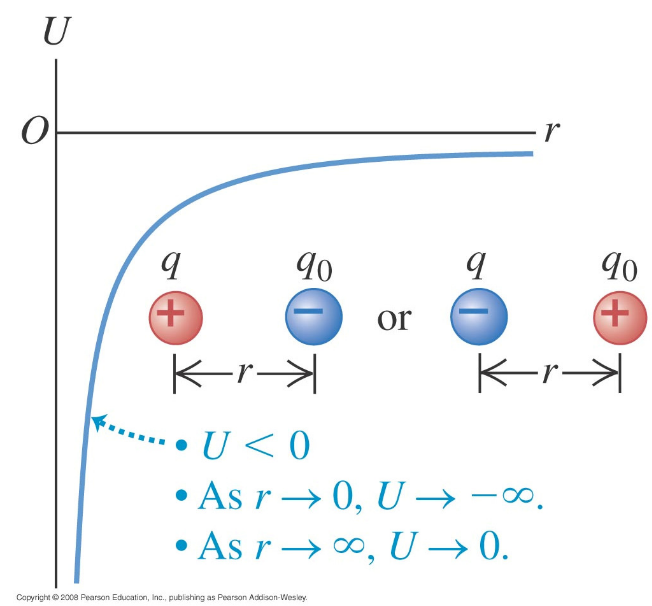
$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = -\Delta U \qquad \rightarrow \qquad \boxed{U = \frac{qq_0}{4\pi\varepsilon_0 r}}$$

(a) q and q_0 have the same sign.



Graphically, U between like charges increases sharply to positive (repulsive) values as the charges become close.

(b) q and q_0 have opposite signs.



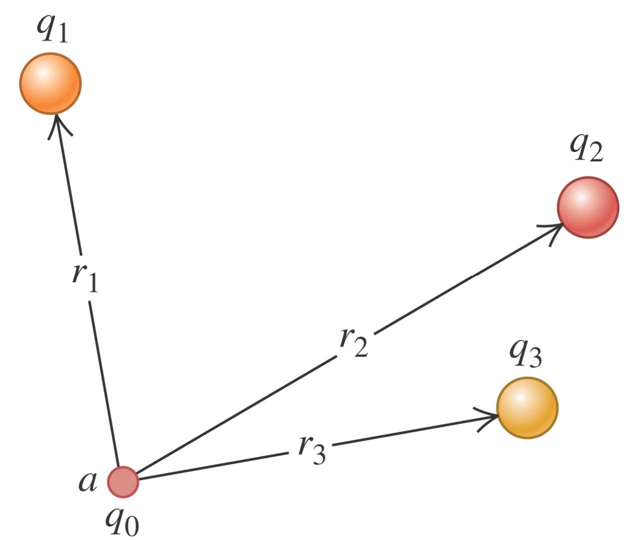
Unlike charges have U becoming sharply negative as they become close (attractive).

- Potential energy is always relative to a certain reference point where U=0. The location of this point is arbitrary. U = 0 when q and q_0 are infinitely apart ($r \rightarrow \infty$).

- U is a shared property of 2 charges, a consequence of the interaction between them. If distance between 2 charges is changed from r_a to r_b, ΔU is same whether q is fixed and q_0 moved, or vice versa.

Electric Potential Energy with Several Point Charges:

The potential energy associated with q_0 at "a" is the algebraic sum of U associated with each pair of charges.



$$U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \quad\right) = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

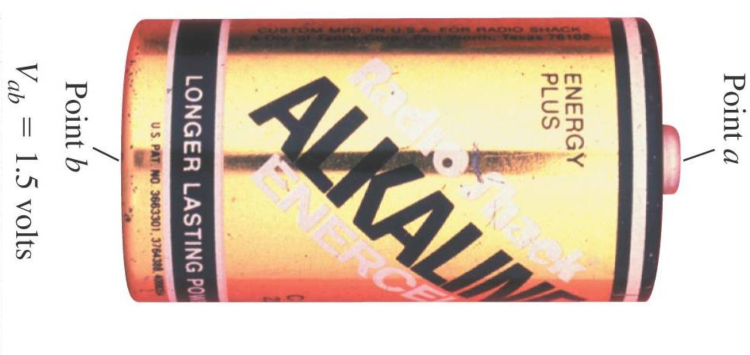
2. Electric Potential

Potential energy per unit charge: $V = \dfrac{U}{q_0}$ V is a scalar quantity

Units: Volt (V) = J/C = Nm/C

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = V_a - V_b = V_{ab}$$ ← Voltage

V_{ab} = work done by the electric force when a unit charge moves from a to b.



The potential of a battery can be measured between point *a* and point *b* (the positive and negative terminals).

Calculating Electric Potential:

Single point charge: $V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Collection of point charges: $V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

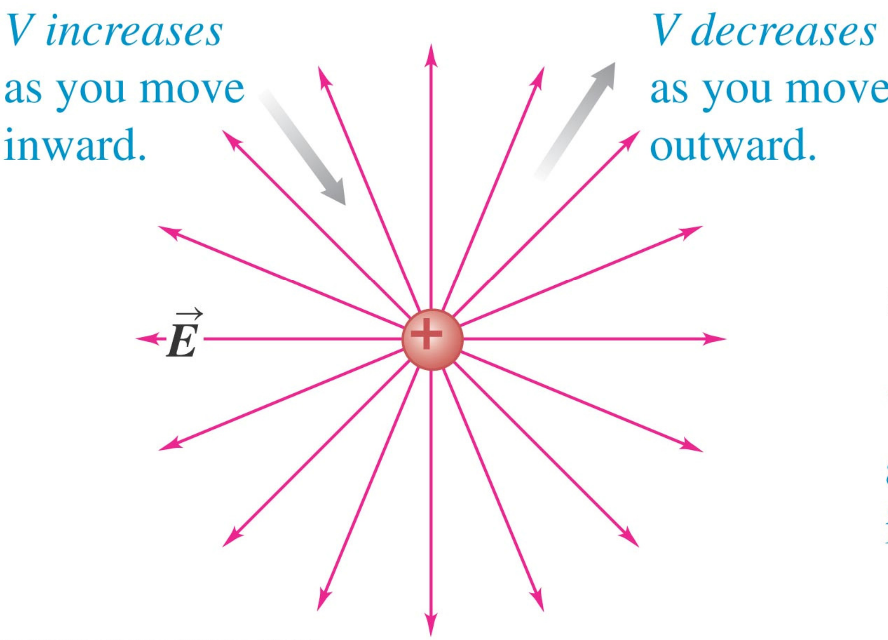
Continuous distribution of charge: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$

Finding Electric Potential from Electric Field:

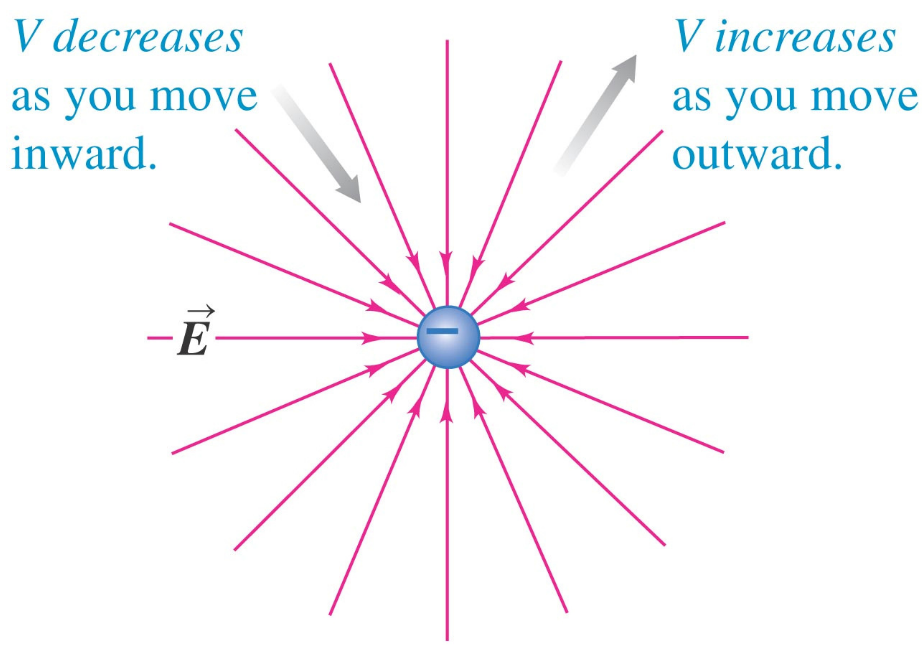
$$\boxed{V_{ab} = V_a - V_b} = \frac{W_{a\to b}}{q_0} = \frac{\int_a^b \vec{F}\cdot d\vec{l}}{q_0} = \frac{\int_a^b q_0\vec{E}\cdot d\vec{l}}{q_0} = \boxed{\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\varphi \cdot dl}$$

- Moving with the electric field → W>0 → $V_a > V_b$ →V decreases.
- Moving against E → W<0 → V increases.

(a) A positive point charge



(b) A negative point charge



Review of units:

Electric charge: C

Electric potential energy: J (1 eV = 1.602 x 10^{-19} J)

Electric potential: V = J/C = Nm/C

Electric field: N/C = V/m

3. Calculating Electric Potential

- Most problems are easier to solve using an energy approach (based on U and V) than a dynamical approach (based on E and F).

Ionization and Corona Discharge:

- There is a maximum potential to which a conductor in air can be raised. The limit is due to the ionization of air molecules that make air conducting. This occurs at $E_m = 3 \times 10^6$ V/m (dielectric strength of air).

Conducting sphere:

$$V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

Max. potential to which a spherical conductor can be raised: $V_m = R\,E_m$

Ex$_1$: if R = 1cm, V_m = 30,000 V → adding extra charge would not raise V, but would cause surrounding air to become ionized and conductive → extra charge leaks into air.

Ex$_2$: if R very small (sharp point, thin wire) → $E = V/R$ will be large, even a small V will give rise to E sufficiently large to ionize air ($E>E_m$). The resulting current and "glow" are called "corona".

Ex$_3$: large R (prevent corona) → metal ball at end of car antenna, blunt end of lightning rod. If there is excess charge in atmosphere (thunderstorm), large charge of opposite sign can buildup on blunt end → atmospheric charge is attracted to lightning rod. A conducting wire connecting the lightning rod and ground allows charge dissipation.

4. Equipotential Surfaces

- 3D surface on which the electric potential (V) is the same at every point.

- If q_0 is moved from point to point on an equip. surface → electric potential energy (q_0V) is constant. U constant → -ΔU = W = 0

$$W_{a\to b} = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cdot\cos\varphi\cdot dl = 0 \;\rightarrow\; \cos\varphi = 0 \;\rightarrow\; \vec{E},\vec{F} \perp d\vec{l}$$

- Field lines (curves) → E tangent

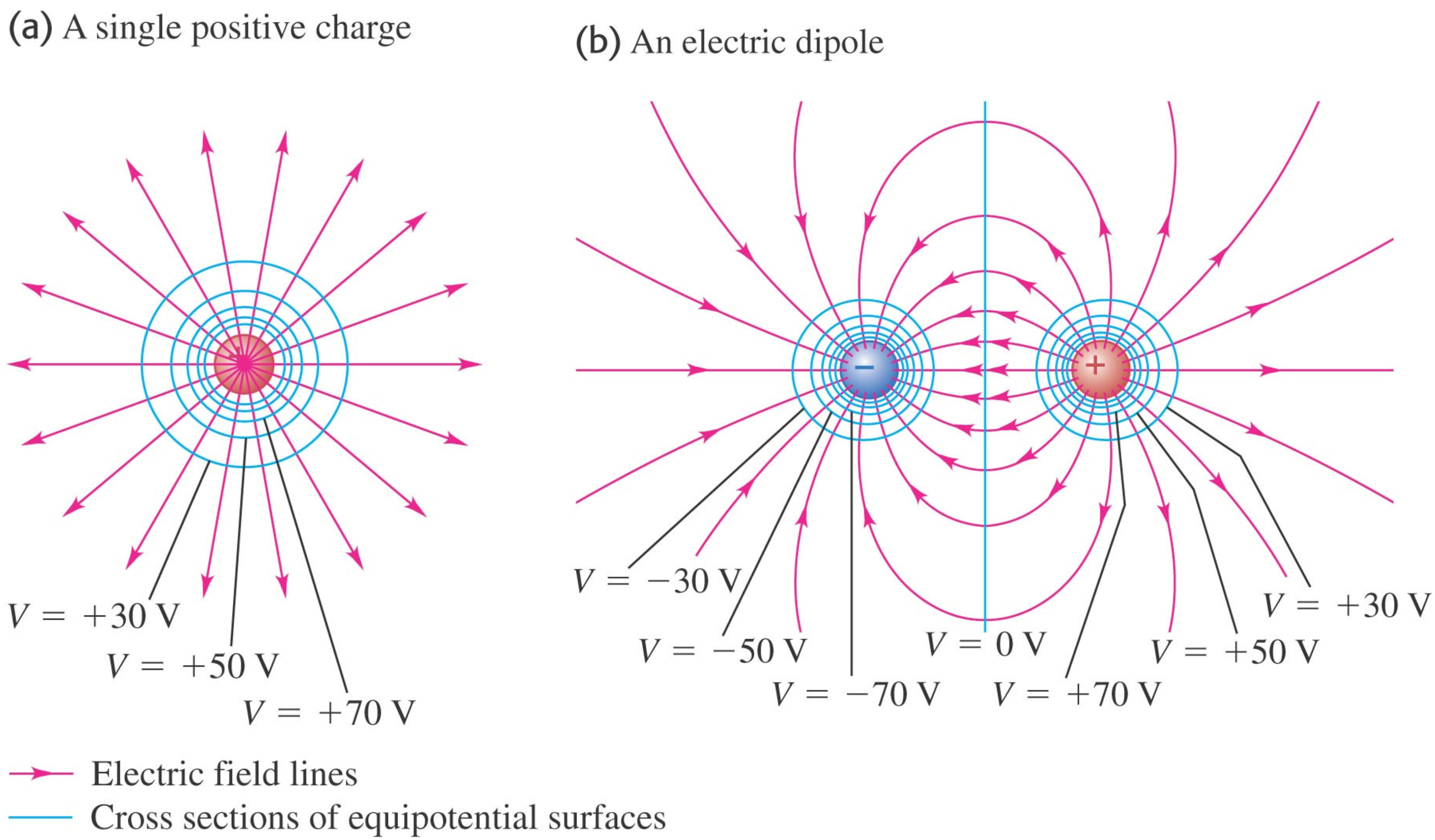
- Equipotential surfaces (curved surfaces) → E perpendicular

- Field lines and equipotential surfaces are mutually perpendicular.

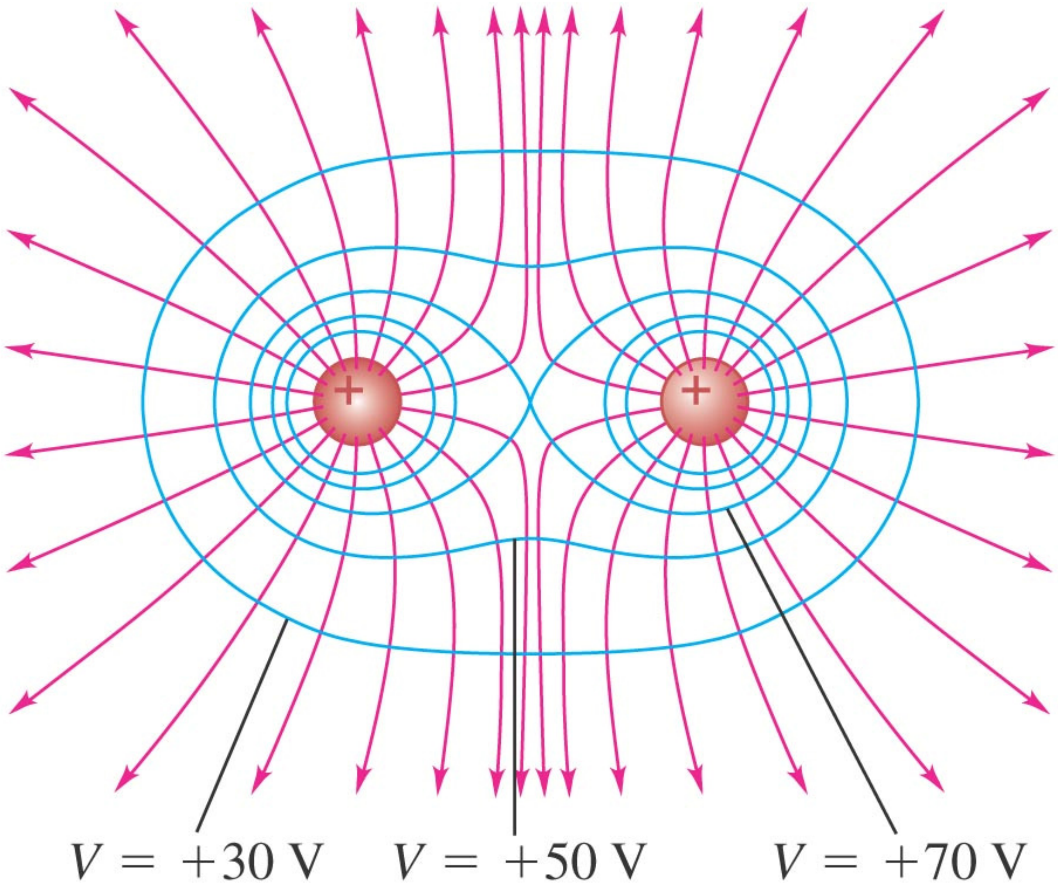
- If electric field uniform → field lines straight, parallel and equally spaced.
  equipotentials → parallel planes perp. field lines.

- At each crossing of an equipotential and field line, the two are perpendicular.

- Important: E does not need to be constant over an equipotential surface. Only V is constant.



(c) Two equal positive charges

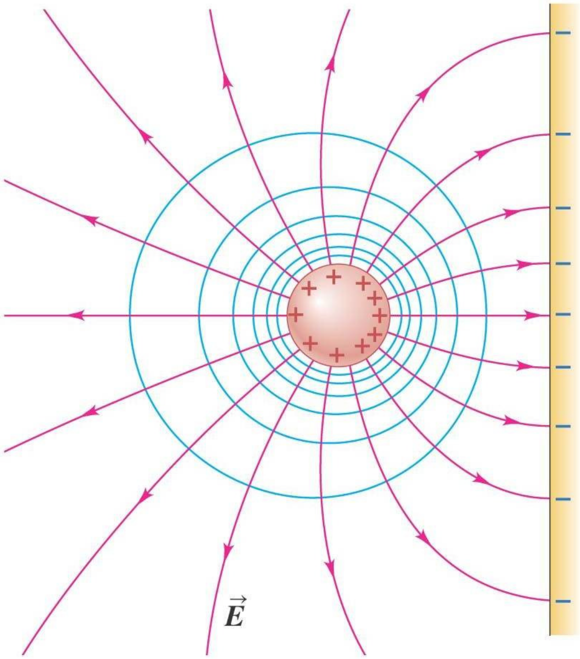


Electric field lines
Cross sections of equipotential surfaces

- E is not constant → E=0 in between the two charges (at equal distance from each one), but not elsewhere within the same equipotential surface.

Equipotentials and Conductors:

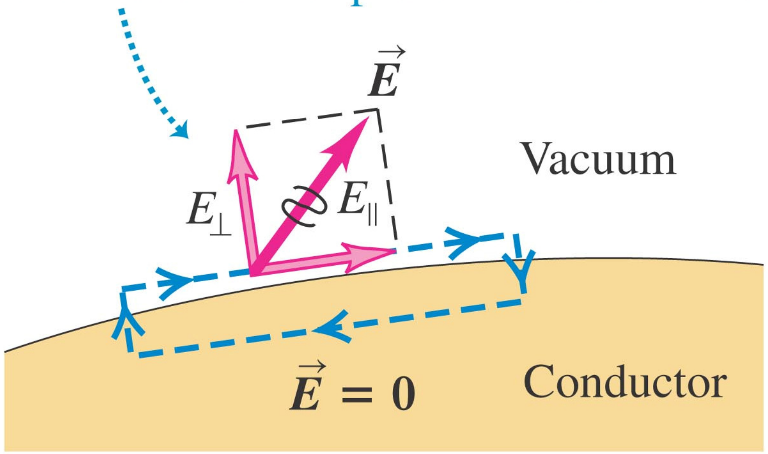
-When all charges are at rest, the surface of a conductor is always an equipotential surface → E outside a conductor ⊥ to surface at each point



Demonstration:

E= 0 (inside conductor) → E tangent to surface inside and out of conductor = 0 → otherwise charges would move following rectangular path.

An impossible electric field
If the electric field just outside a conductor had a tangential component $E_{\parallel}$, a charge could move in a loop with net work done.



$\vec{E}$ ⊥ to conductor surface

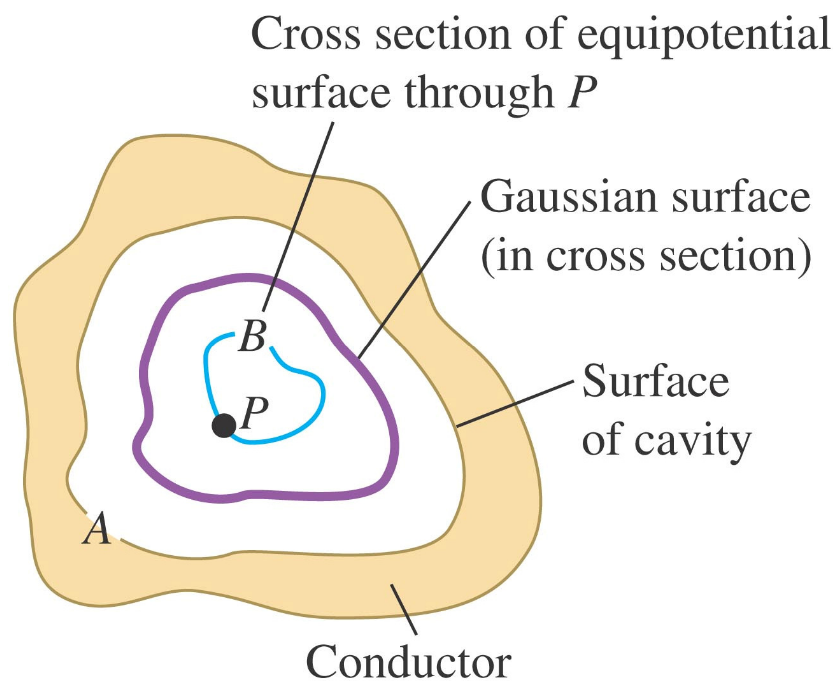
Equipotentials and Conductors:

- In electrostatics, if a conductor has a cavity and if no charge is present inside the cavity → there cannot be any charge on surface of cavity.

Demonstration: (1) prove that each point in cavity must have same V → If P was at different V, one can build a equip. surface B.

(2) Choose Gaussian surface between 2 equip. surfaces (A, B) → E between those two surfaces must be from A to B (or vice versa), but flux through S_{Gauss} won't be zero.

(3) Gauss: charge enclosed by S_{Gauss} cannot be zero → contradicts hypothesis of Q=0 →V at P cannot be different from that on cavity wall (A)→ all cavity same V →E inside cavity = 0



5. Potential Gradient

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b dV \rightarrow \; -dV = \vec{E} \cdot d\vec{l}$$

$$-dV = E_x dx + E_y dy + E_z dz$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$\boxed{\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial x}\hat{j} + \frac{\partial V}{\partial x}\hat{k} \right) = -\vec{\nabla} V}$$

- The potential gradient points in the direction in which V increases most rapidly with a change in position.

- At each point, the direction of $\vec{E}$ is the direction in which V decreases most rapidly and is always perpendicular to the equipotential surface through point.

- Moving in direction of $\vec{E}$ means moving in direction of decreasing potential.