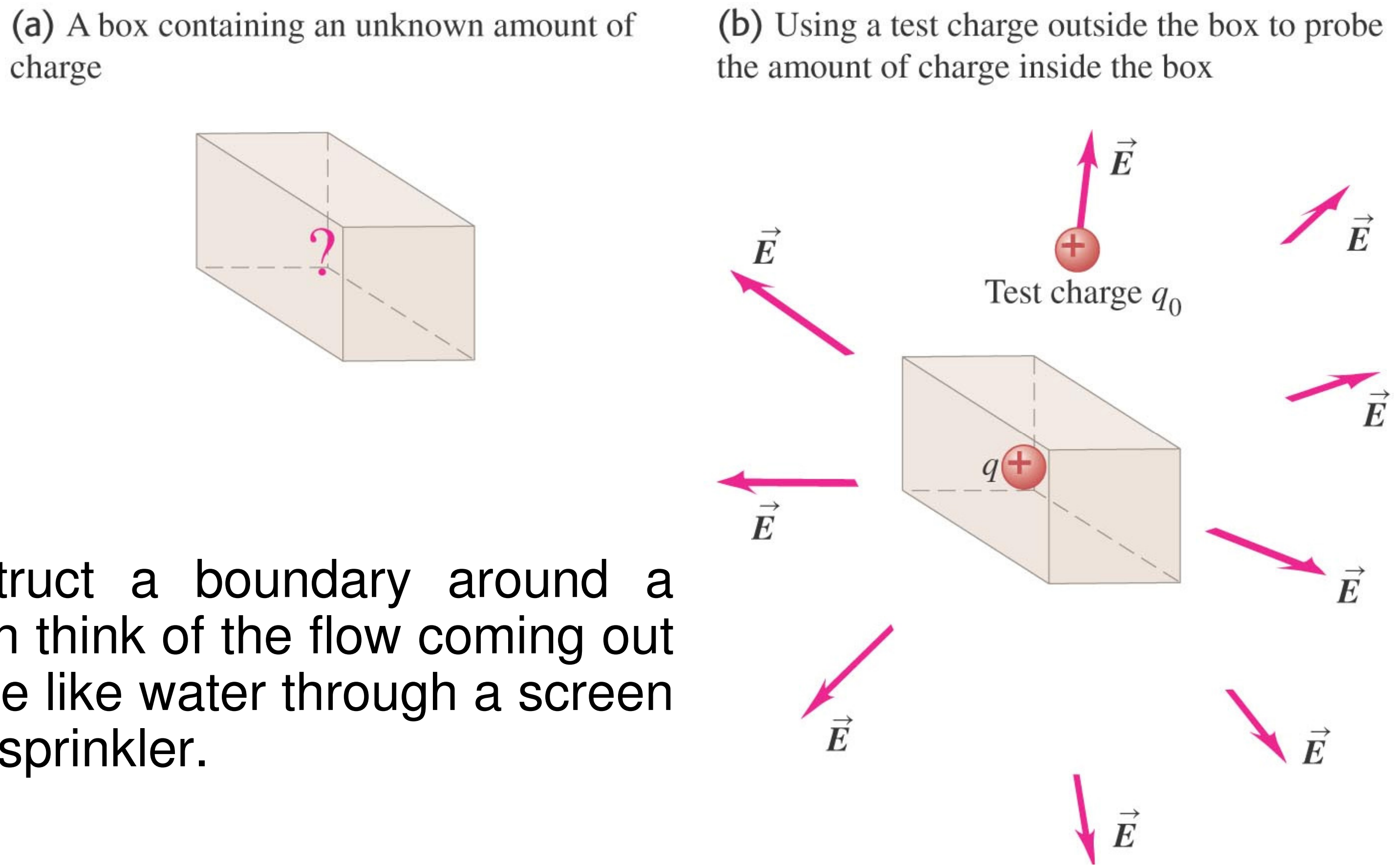
Chapter 22 – Gauss Law

- Charge and Electric flux
- Electric Flux Calculations
- Gauss's Law and applications
- Charges on Conductors

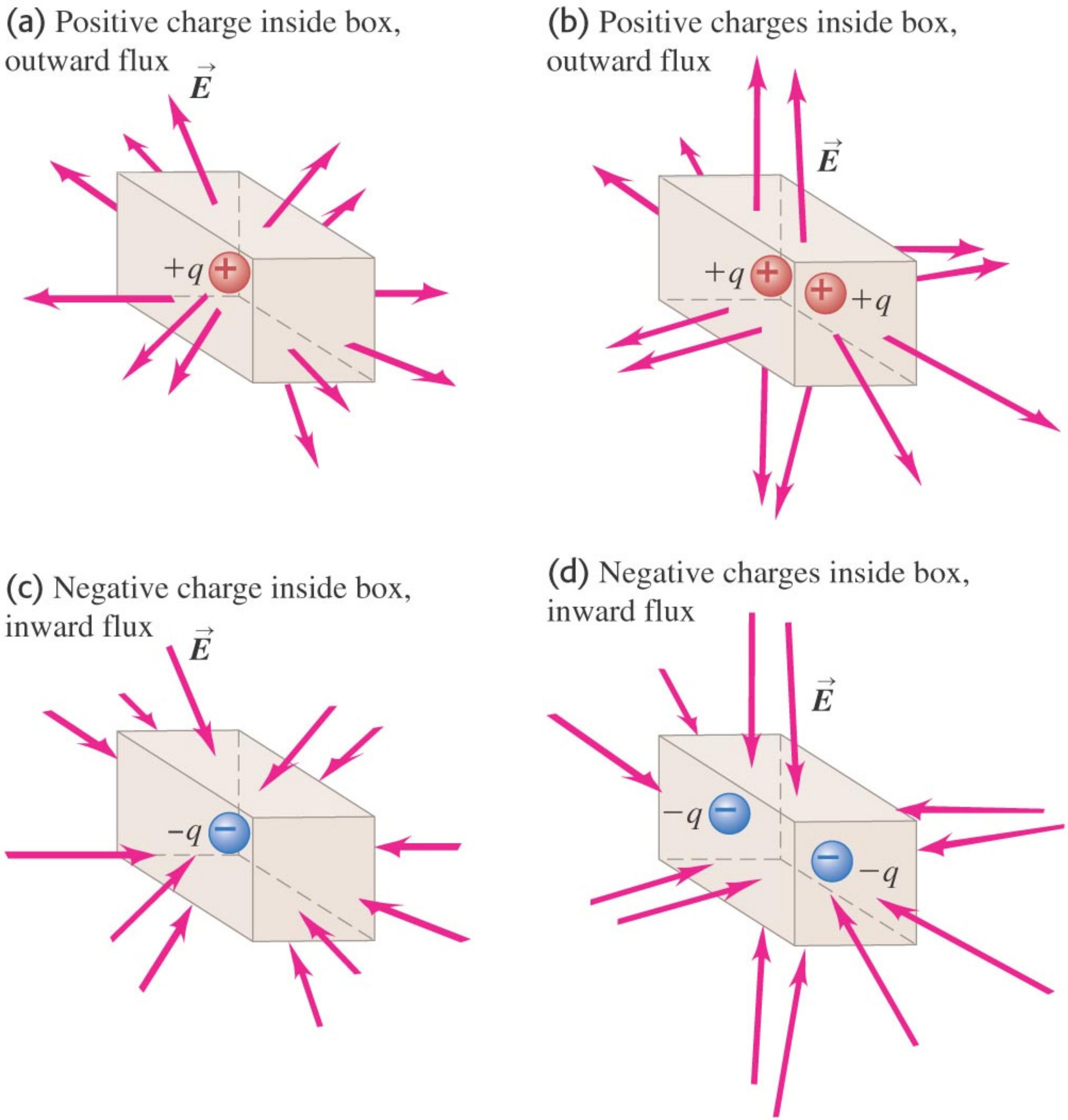
Child acquires electric charge by touching a charged metal sphere. Electrons coat each individual hair fiber and then repel each other.

1. Charge and Electric Flux

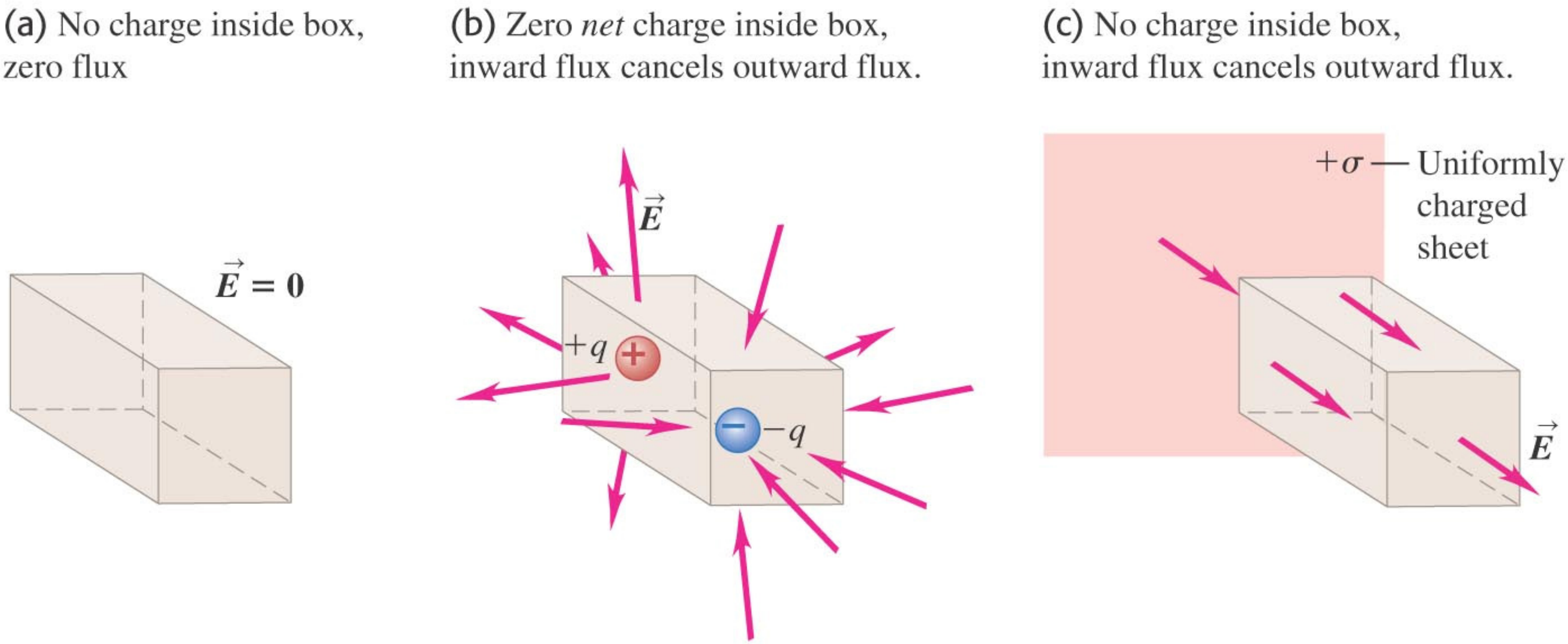
- A charge distribution produces an electric field ($\vec{E}$), and $\vec{E}$ exerts a force on a test charge (q_0). By moving q_0 around a closed box that contains the charge distribution and measuring $\vec{F}$ one can make a 3D map of $\vec{E} = \vec{F}/q_0$ outside the box. From that map, we can obtain the value of q inside box.



- If we construct a boundary around a charge, we can think of the flow coming out from the charge like water through a screen surrounding a sprinkler.


(a) Positive charge inside box, outward flux
$\vec{E}$
$+q$
(b) Positive charges inside box, outward flux
$\vec{E}$
$+q$
$+q$
(c) Negative charge inside box, inward flux
$\vec{E}$
$-q$
(d) Negative charges inside box, inward flux
$\vec{E}$
$-q$
$-q$

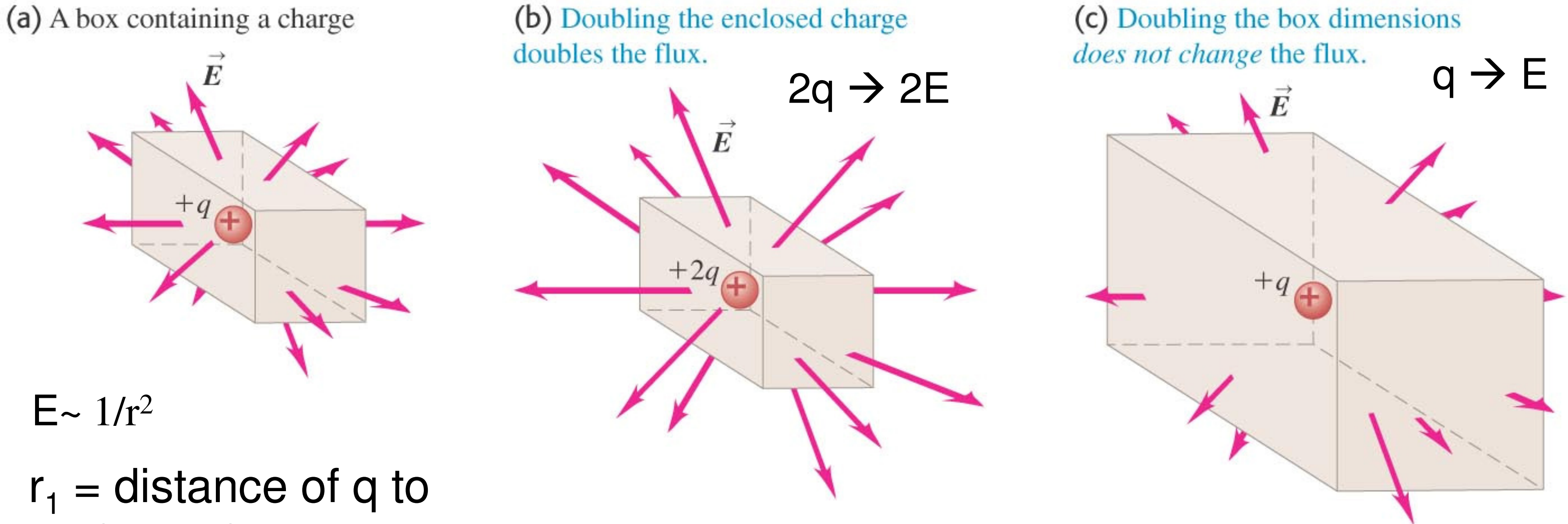
Electric Flux and Enclosed Charge:



-There is a connection between sign of net charge enclosed by a closed surface and the direction of electric flux through surface (inward for -q, outward for +q).

- There is a connection between magnitude of net enclosed charge and strength of net "flow" of $\vec{E}$.

- The net electric flux through the surface of a box is directly proportional to the magnitude of the net charge enclosed by the box.



$E \sim 1/r^2$

r_1 = distance of q to surface of box_1.

$r_2 = 2r_1$ = distance of q to surface of box_2.

In (c), $E_2 = E_1/4$, since $r_2=2r_1$, but $A_2=4A_1$ →net flux constant.

- Electric flux = (perpendicular component of E) · (area of box face)

-The net electric flux due to a point charge inside a box is independent of box's size, only depends on net amount of charge enclosed.

- Charges outside the surface do not give net electric flux through surface.

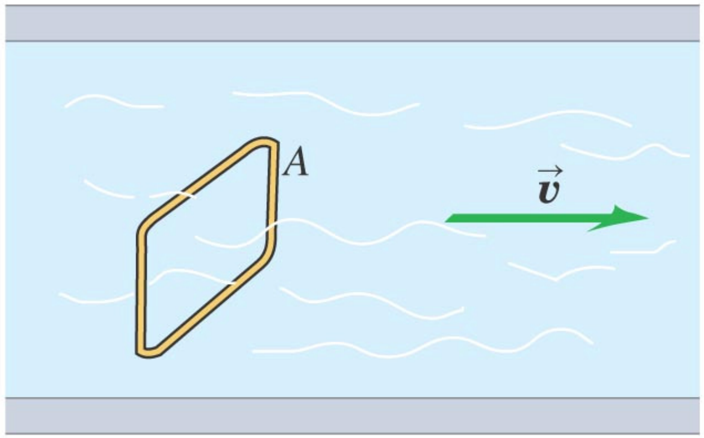
2. Calculating Electric Flux

Flux Fluid Analogy:

- If we considered flux through a rectangle, the flux will change as the rectangle changes orientation to the flow.

$$\frac{dV}{dt} = vA \qquad (v \perp A)$$

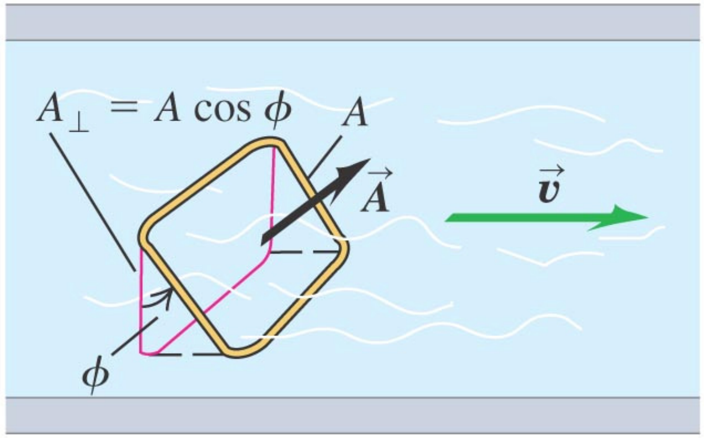
(a) A wire rectangle in a fluid



dV/dt = volume flow rate
v = flow speed

$$\frac{dV}{dt} = vA_{\perp} = vA\cos\varphi = v_{\perp}A = \vec{v}\cdot\vec{A}$$

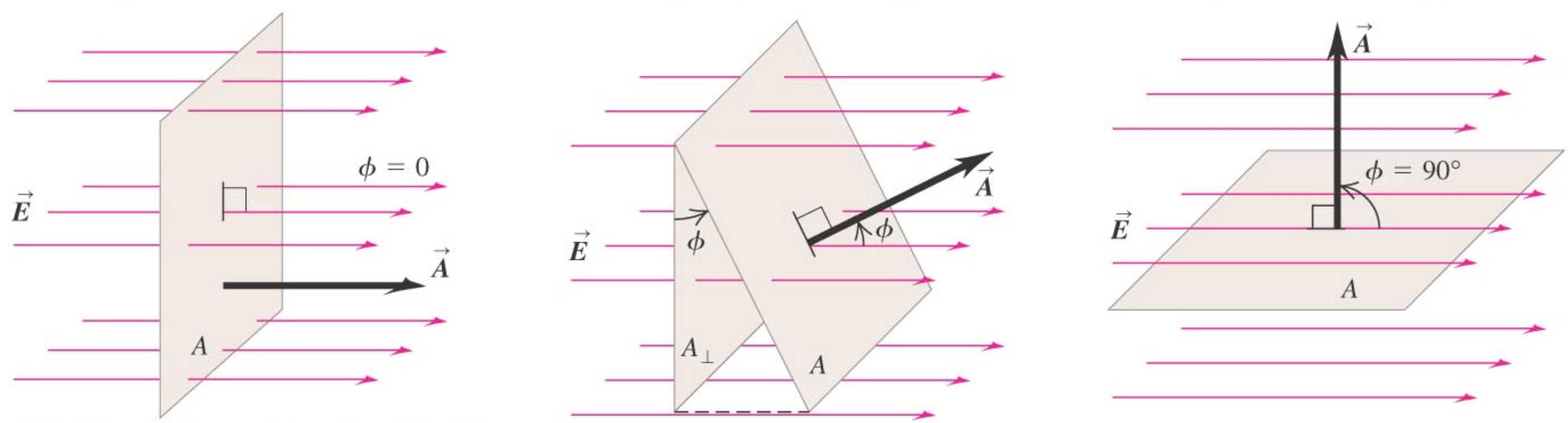
(b) The wire rectangle tilted by an angle ϕ



Flux of a Uniform Electric Field:

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\varphi = E_\perp A$$

Units: N m²/C



$\Phi_E = E \cdot A$ $\Phi_E = E \cdot A \cdot \cos\varphi$ $\Phi_E = 0$

We can define a vector area: $\vec{A} = A \cdot \hat{n}$ with *n* being a unit vector ⊥ A.

Flux of a Non-uniform Electric Field:

$$\Phi_E = \int E\cos\phi \, dA = \int E_\perp \, dA = \int \vec{\boldsymbol{E}} \cdot d\vec{A}$$

3. Gauss's Law

- The total electric flux through any closed surface is proportional to the total electric charge inside the surface.

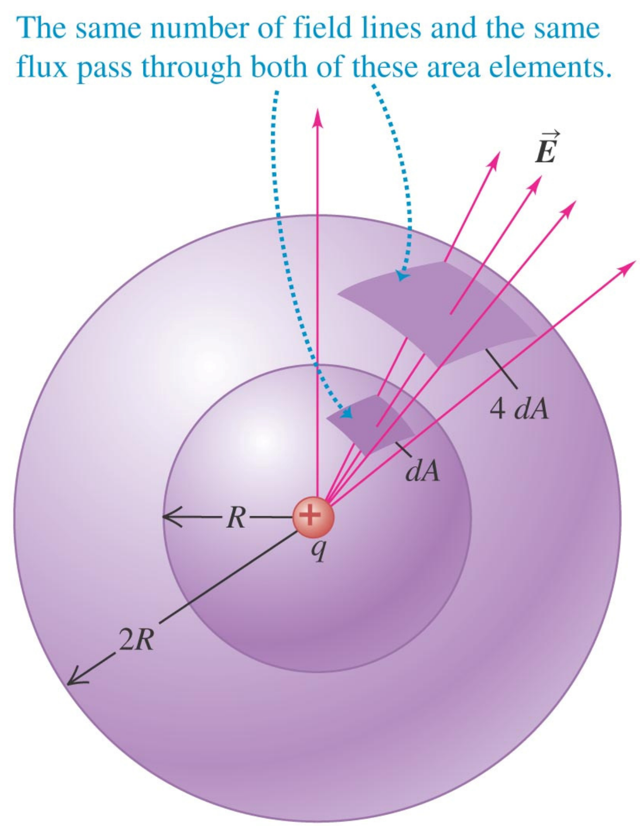
Point Charge Inside a Spherical Surface:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

$\vec{E} \,//\, d\vec{A}$ at each point

$$\Phi_E = E \cdot A = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\left(4\pi R^2\right) = \frac{q}{\varepsilon_0}$$

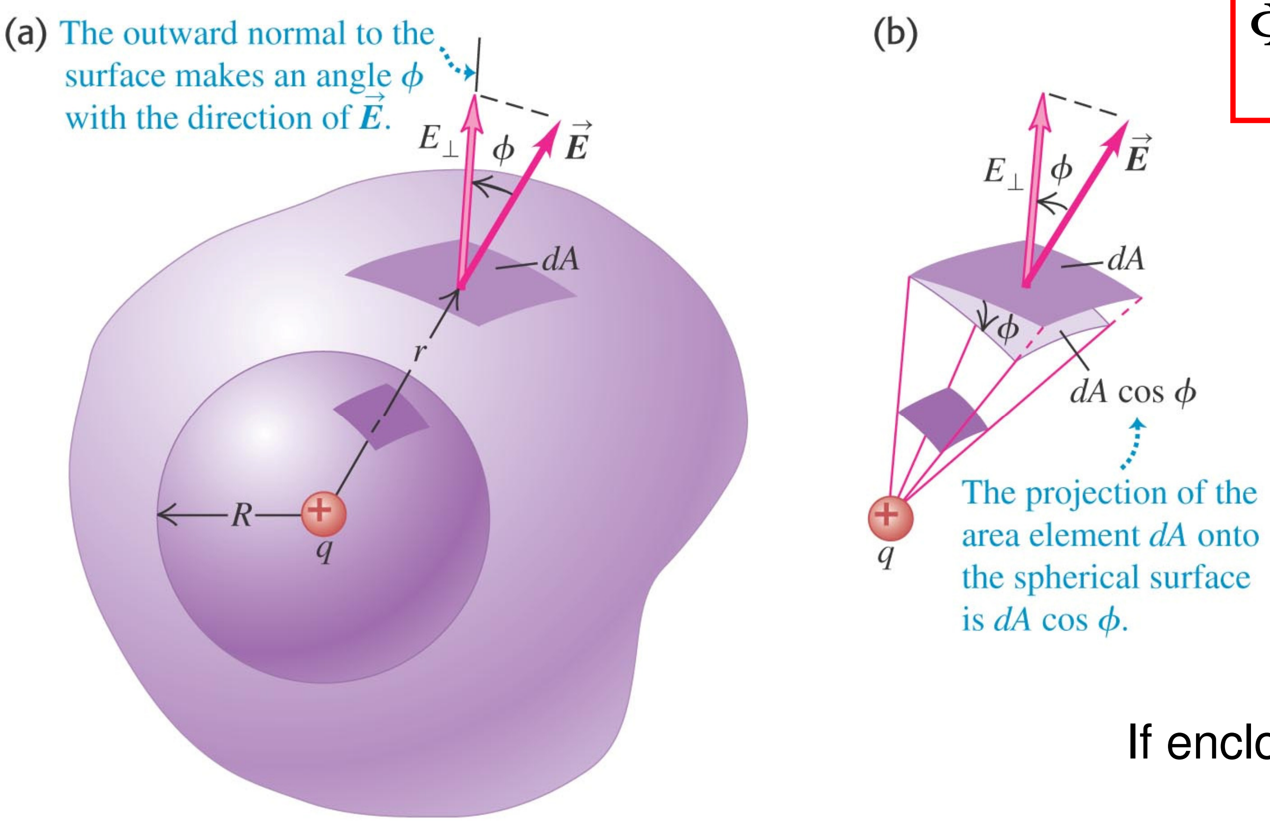
- The flux is independent of the radius R of the sphere.



Point Charge Inside a Nonspherical Surface:

- Divide irregular surface into dA elements, compute electric flux for each (E dA cosφ) and sum results by integrating.

- Each dA projects onto a spherical surface element → total electric flux through irregular surface = flux through sphere.



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

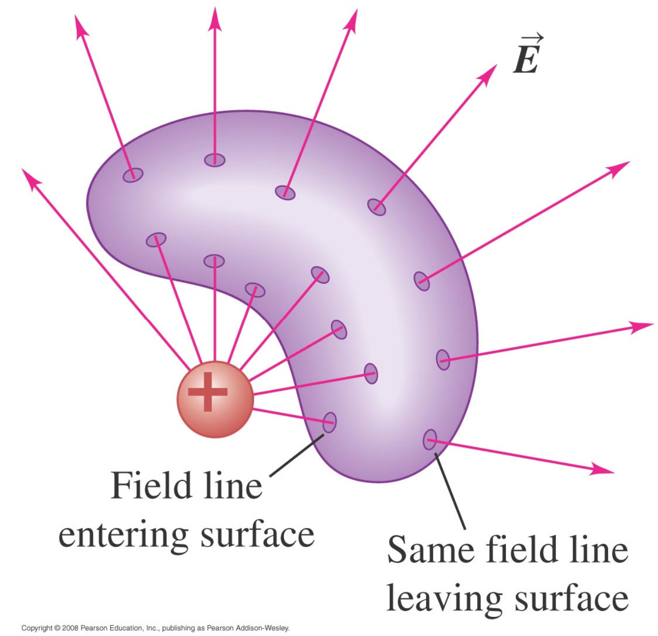
↓ Integral through a closed surface

Valid for + / - q

If enclosed q = 0 → $\Phi_E = 0$

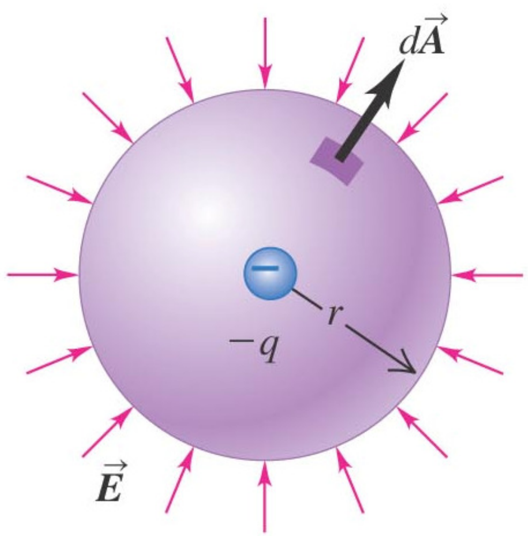
Point charge outside a closed surface that encloses no charge. If an electric field line enters the surface at one point it must leave at another.

- *Electric field lines can begin or end inside a region of space only when there is a charge in that region.*



General form of Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\varphi \, dA = \oint E_\perp dA = \frac{Q_{encl}}{\varepsilon_0}$$



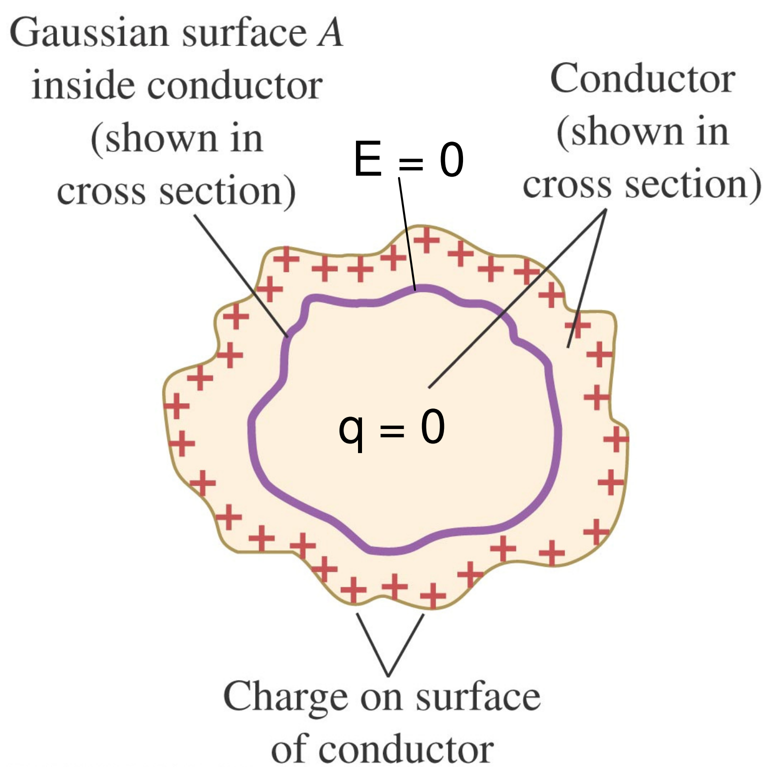
Example: Spherical Gaussian surface around –q (negative inward flux)

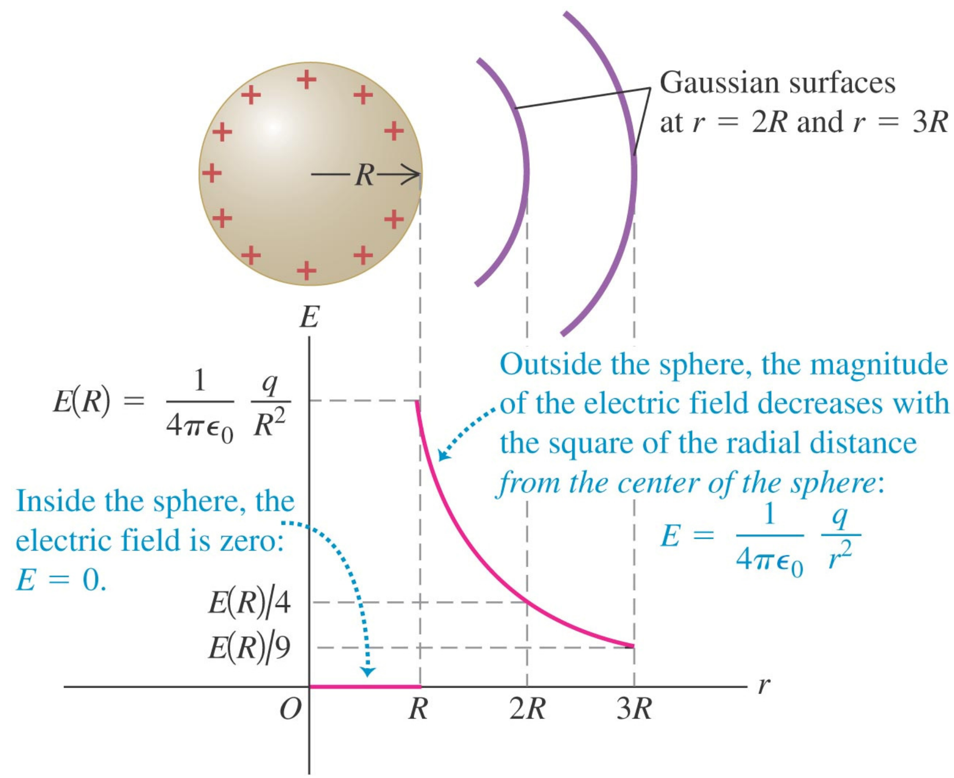
$$\Phi_E = \oint E_\perp dA = \oint \left(\frac{-q}{4\pi\varepsilon_0 r^2} \right) dA = \left(\frac{-q}{4\pi\varepsilon_0 r^2} \right) \oint dA = \left(\frac{-q}{4\pi\varepsilon_0 r^2} \right) \left(4\pi r^2\right) = \frac{-q}{\varepsilon_0}$$

4. Applications of Gauss's Law

- *When excess charge* (charges other than ions/e^- making up a neutral conductor) *is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior of the material.*

- Electrostatic condition (charges at rest) → E = 0 inside material of conductor, otherwise excess charges will move.





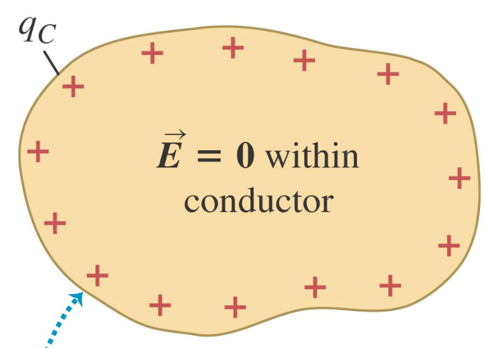
5. Charges on Conductors

- Excess charge only on surface.

- Cavity inside conductor with q = 0 → E = 0 inside conductor, net charge on surface of cavity = 0.

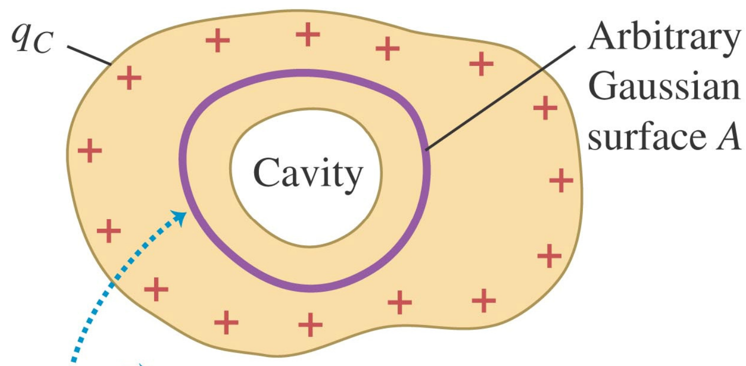
- Cavity inside conductor with +q →E = 0 inside conductor, -q charge on surface of cavity (drawn there by +q). Total charge inside conductor = 0 → +q on outer surface (in addition to original q_c).

(a) Solid conductor with charge q_C



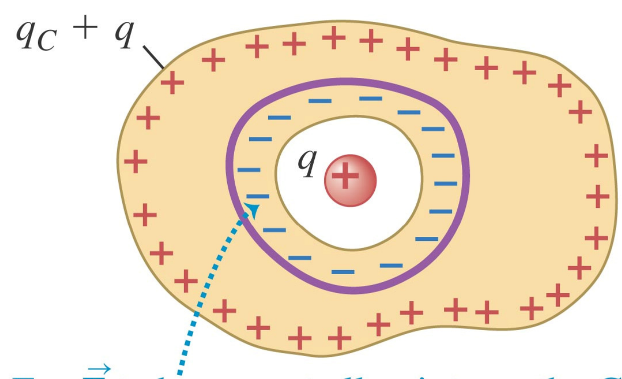
The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E} = \mathbf{0}$ within the conductor.

(b) The same conductor with an internal cavity



Because $\vec{E} = \mathbf{0}$ at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.

(c) An isolated charge q placed in the cavity



For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

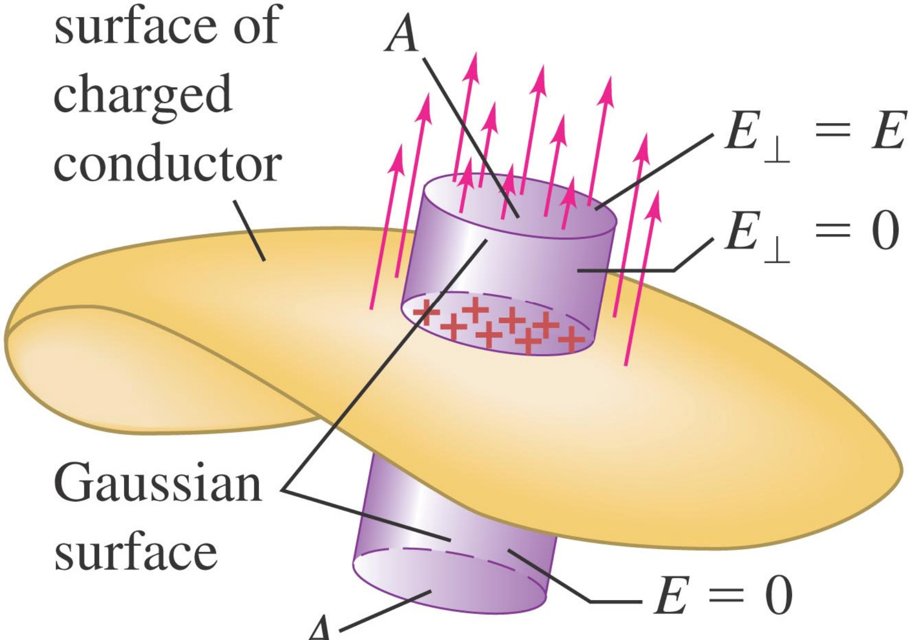
Field at the surface of a conductor:

$$E_{\perp} A = \frac{\sigma A}{\varepsilon_0} \quad \text{and} \quad \boxed{E_{\perp} = \frac{\sigma}{\varepsilon_0}}$$

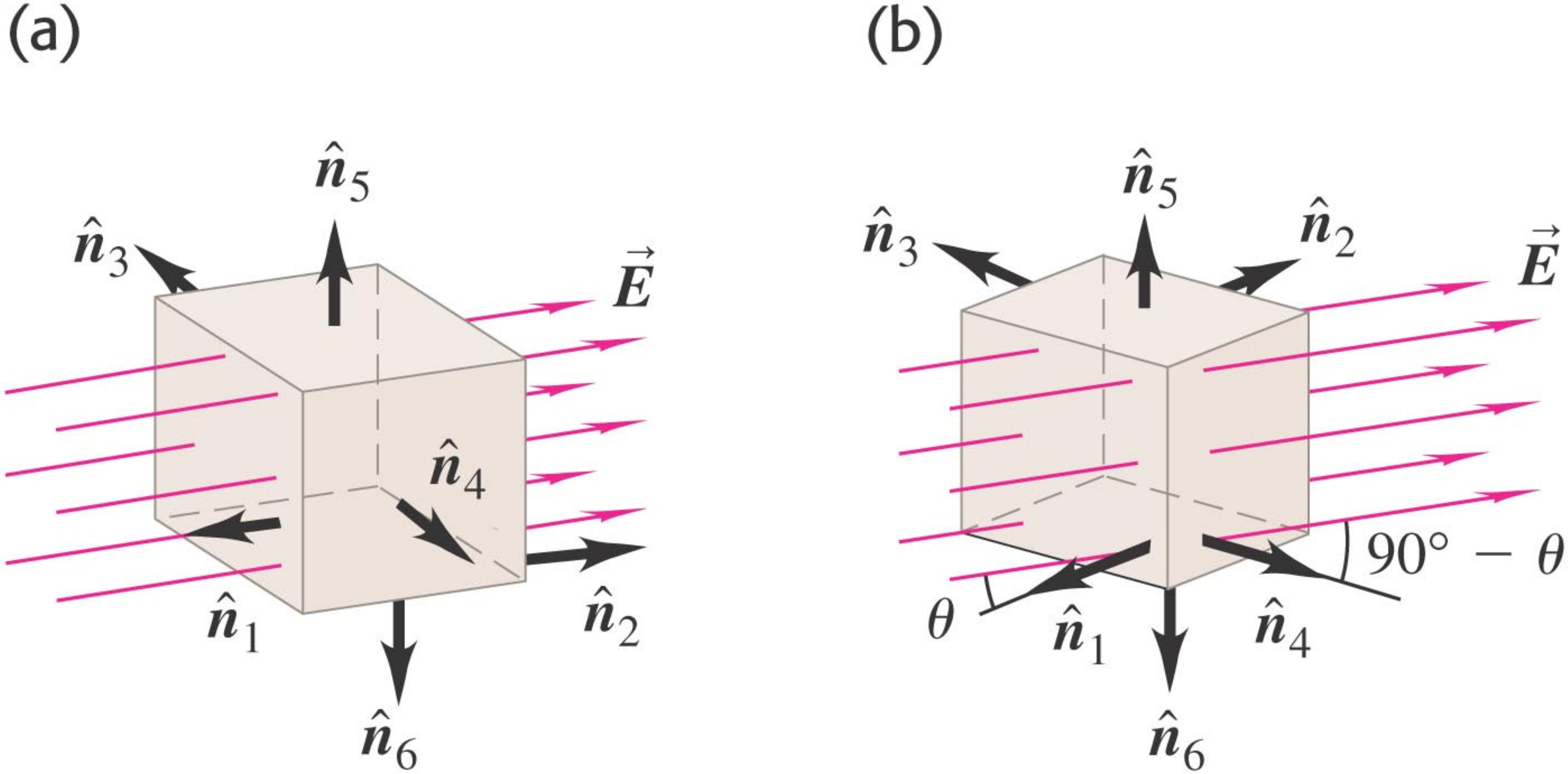
Field outside a charged conductor is perpendicular to surface.

↑

$\sigma = q/A \rightarrow q = \sigma A$



Ex. 22.2



(a)
$\hat{n}_5$
$\hat{n}_3$
$\vec{E}$
$\hat{n}_4$
$\hat{n}_1$
$\hat{n}_2$
$\hat{n}_6$
(b)
$\hat{n}_5$
$\hat{n}_3$
$\hat{n}_2$
$\vec{E}$
$90° - \theta$
θ
$\hat{n}_1$
$\hat{n}_4$
$\hat{n}_6$