The purpose of this test is to assess your facility with mathematical concepts needed for PHY2048. While the results of this test will have no bearing on your grade, you will be able to decide for yourself whether you have the math background to successfully pass the course. *Those who fail the test might consider taking remedial courses. No calculator can be used.*

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**Section**

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1. An annulus (in the shape of a washer) has inner radius \( R_1 \) and outer radius \( R_2 \). What is the surface area of the shell?

\[
\pi \left( R_2^2 - R_1^2 \right)
\]

2. A circular disk of radius \( R \) has a mass \( M \). What will be the mass of an annulus (washer) cut out from the disc, of inner radius \( r \) and outer radius \( R \)? The disk has uniformly distributed mass.

\[
\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\pi R^2}
\]

\[\therefore \text{mass} = \pi (R^2 - r^2) * \frac{M}{\pi R^2} = \frac{M}{R^2 h} (R^2 - r^2)\]

3. Solve the system of equations:

\[
\begin{align*}
2x + 3y &= 150 \quad \text{-----------------(1)} \\
3x - 3y + z &= 100 \quad \text{-----------------(2)} \\
x - z &= 50 \quad \text{-----------------(3)}
\end{align*}
\]

Add (1) + (2) to get \( 5x + z = 250 \) ----(4) ;
From (3) \( z = x - 50 \) --------(5)
Substitute 5 in 4 to get: \( 6x = 300 \) or, \( x = 50 \); from (5) \( z = 0 \); from (1) \( y = 50/3 \)

4. The linear mass density \( \lambda \) is defined as mass per unit length.

The linear mass density of a thin rod of length \( l \) is a linear function of \( x \), measured along the length of the rod. If the mass density is \( \lambda_0 \) at \( x = 0 \) and equal to \( 2\lambda_0 \) at \( x = l \), determine the functional form of the mass density \( \lambda(x) \).
5. If the mass of a uniform cylinder of radius $R$ and height $h$ is $M$, what is the mass of a uniform cylindrical shell of the same height, of inner radius $r$ and outer radius $r'$ made of the same material?

Here we first need to find the mass density $\rho$.

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\pi R^2 h}$$

$$\therefore \text{mass} = \pi (r'^2 - r^2) \times \frac{M}{\pi R^2 h} = \frac{M}{R^2 h} (r'^2 - r^2)$$

6. The surface of a sphere is:

a) $4\pi R^2$

b) $4\pi R^3$

c) $(4/3)\pi R^3$

d) $(4/3)\pi R^2$

The correct answer is (a)

7. A party balloon as it is originally inflated has a diameter of 0.5m. It is then further inflated, until its new diameter is 1m. By what factor has its volume changed?

a) Twice the original volume

b) Four times the original volume

c) Six times the original volume

d) Eight times the original volume

e) Twelve times the original volume

The volume of the original balloon is $V = (4/3)\pi R^3$. Therefore if $R$ becomes $2R$, the new volume $V' = (4/3)\pi (2R)^3 = 8V$. The correct answer is (d) Eight times the original volume.
8. For the balloon of the previous problem, how has the surface area changed?

   a) Twice the original area
   b) Four times the original area \[ A = 4\pi R^2. \] When \( R \rightarrow 2R \), \( A \rightarrow 4A \)
   c) Six times the original area
   d) Eight times the original area.
   e) Twelve times the original area

9. Two pizzas are on sale. They have the same thickness and have identical toppings. Which pizza is the best buy?

   (a) A 10 inch pizza for $10
   (b) A 20 inch pizza for $20 \[ \text{This is cheaper in terms of more food!} \]

10. Given \( \cos \theta = a/b \), where \( a \) and \( b \) two given constants, find \( \sin \theta \) in terms of \( a \) and \( b \).

11. Referring to the following figure, which is the correct expression for \( \sin \theta \)?

   \[ \sin \theta = b/c \]

12. Referring to the figure of the previous problem, which is the correct expression for \( a^2 \)?

   a) \( b^2 \sin^2 \theta \)
   b) \( b^2 + c^2 \)
   c) \( c^2 - b^2 \)

13. What is the derivative with respect to \( x \) of \( y = ax^n + b \), where \( a \), \( b \), \( n \) are three constants?

   \[ \frac{dy}{dx} = ax^{n-1} \]
14. In an isosceles triangle ABC, (AB) = (AC), with height h and base b as seen in the following figure, a straight line DEF parallel to the base is drawn. If (AE) = h/3, find the length g of the segment EF in terms of h and b.

\[ \tan \theta = \frac{b}{2h} = \frac{EF}{AE} \]

\[ EF = \frac{b}{2h} \cdot AE = \frac{b}{2h} \cdot \frac{h}{3} = \frac{b}{6} \]

15. Draw a sketch of the following function: \( f(x) = x^2 e^{-x^2} \)

16. Calculate the extrema (maximum/minimum) of the function displayed in question 15.

\[ f(x) = x^2 \exp(-x^2) \]

\[ \frac{df(x)}{dx} = 2x \exp(-x^2) + x^2 (-2x) \exp(-x^2) \]

\[ \frac{df(x)}{dx} = \exp(-x^2) (2x)(1 - x^2) \]

To find the location of maxima or minima, we find the roots of df/dx.
\[ \frac{df(x)}{dx} = 0 \]
\[ 2x(1-x^2) = 0 \]
\[ \therefore x = 0, \pm 1 \]

17. Based on question’s 15 sketch, determine whether the extrema are maxima or minima.

From the graph x=0 correspond to the minimum and x =1 and x=-1 correspond to maxima. You can also verify that by evaluating the 2nd derivative at these points.

18. Solve:

\[ x(x - \frac{5}{4}) + \frac{1}{3}(x + \frac{1}{2}) = \frac{5}{3}(1 + \frac{5}{4}x) - 2x(1 - \frac{1}{4}x) \]
\[ x(x - \frac{5}{4}) + \frac{1}{3}(x + \frac{1}{2}) = \frac{5}{3}(1 + \frac{5}{4}x) - 2x(1 - \frac{1}{4}x) \]
\[ 3x(4x - 5) + 2(2x + 1) = 5(4 + 5x) - 6x(4 - x) \]
\[ 12x^2 - 15x + 4x + 2 = 20 + 25x - 24x + 6x^2 \]
\[ 6x^2 - 12x - 18 = 0 \]
\[ x^2 - 2x - 3 = 0 \]
\[ (x - 3)(x + 1) = 0 \]
\[ \therefore x = 3, or -1 \]

19. Find the x and y components of the position vector \( \vec{r} \), given \( r = 10 \text{ m} \) and \( \Theta = 30^\circ \).

\[ x = r \cos(\Theta) = 10 \cos(30^\circ) = 10 \frac{\sqrt{3}}{2} = 5\sqrt{3} \]
\[ y = r \sin(\Theta) = 10 \sin(60^\circ) = 5 \]