Chapter 3

Data Interpretation

In this chapter I will discuss the interpretation methods that are common to most of the data used in this thesis. Descriptions of specialized analyses – e.g., of a technique that is applicable to only one of the comets – will be discussed in the relevant comet-specific chapters.

3.1 Philosophy of Thermal Modeling

The energy available to a cometary nucleus comes from the Sun. Internal heating by e.g. radioactive decay is not an important factor owing to the small size of the object. The insolation absorbed by a surface element of the nucleus either is reradiated, is passed along to adjacent elements, or helps to sublimate ice. Currently the numerical value of important factors that heavily influence the nucleus' thermal behavior are unknown, though we hope to achieve some understanding with the cornucopia of spacecraft visits in the coming decade. Detailed models of a cometary nucleus make estimates of such quantities as the thermal conductivity, the porosity, the heat capacity, the surface roughness, the shape, the effective radius, the composition, the structure of the ice/rock matrix, the emissivity, and the rotation state to try to match the observed flux. Only rarely are any of these quantities actually known for a given nucleus *a priori*; the modeler must simplify the situation to make the problem tractable.

The advent of more sensitive IR instrumentation has led to the acquisition of better datasets, and I have attempted to apply some thermal modeling that goes beyond the standard simple methods to some of the datasets in this study. There are models created by others that are more complex, but in my opinion the direct application of a very complicated model to a real nucleus about which we know very little detail may not really help one understand the basic properties of the nucleus any better than a relatively simple model can.

Previous work on understanding the thermal behavior of nuclei has mostly exploited the two popular thermal models for asteroids: the "standard" thermal model (STM), also known as the slow-rotator model (SRM); and the rapid-rotator model (RRM), also known as the isothermal latitude model (ILM) and the fast-rotator model (FRM). As the names imply, the STM assumes the asteroid is rotating slowly compared to the timescale for the thermal wave to penetrate one thermal skin depth into the nucleus, and the RRM assumes it is rotating much faster than that.

For example, for objects 1 AU from the Sun, a "slow" rotator would have a rotation period of roughly 15 hours, whereas a "fast" rotator would spin in roughly 4 hours. Both models assume the object is spherical. The temperature map of a sphere that follows the STM looks like a bull's-eye centered on the subsolar point, the hottest point on the object, with the temperature decreasing as the local solar zenith angle increases. The night side is at absolute zero. For an object following the RRM, the temperature at any point only depends on the distance from the subsolar point's latitude, not the longitude. (This is the origin of the "isothermal latitude" name.) I have displayed in Fig. 3.1 a schematic, based on a similar figure made by Lebofsky and Spencer (1989; their Fig. 4), showing typical temperature maps for the two models.

The STM uses a measured flux and assumes values for the bolometric IR emissivity, the optical geometric albedo, the IR phase function, the optical phase integral, and the roughness of the surface (embodied in a factor diminishing or enhancing the overall observed flux). With these quantities, one finds the effective radius. The RRM uses the measured flux and requires values for the bolometric IR emissivity, the optical geometric albedo, the optical phase integral, and the rotation axis direction to find the effective radius. As an aside, if one assumes a pole orientation pointing toward the Sun, then the RRM and the STM yield the same temperature map.

For this asteroidal model to be applicable to a cometary nucleus, one has to be sure that (a) the nucleus is a slow-rotator; (b) it is not very active, or rather, not much of the solar input energy is going to sublimating gas instead of heating up the rock; and (c) the coma is not providing a secondary source of energy via backwarming, which is only a problem for very active comets like Hale-Bopp. It is not really necessary that the cometary nucleus be spherical, which is advantageous since many are not (Meech 1999), but the output of the STM is then the effective radius, not the radius itself. There is a complication with this, since the radiometric effective radius does not have to be the same as the geometric effective radius: suppose the nucleus were cigar shaped with the long axis pointing toward the Sun. An observer would measure a relatively small thermal flux and derive a small effective radius, since most of the cigar would not be significantly warmed by the Sun. Fortunately, observing the thermal flux over the course of a rotation period, and if possible at several points in the orbit, can assuage most fears about this pathological case skewing the radiometrically-derived size. The uncertainties from other aspects of the model – e.g., the infrared phase effect, and the beaming effect, described below – usually make the uncertainty in the resulting radius estimate large enough so that it engulfs some of this systematic error anyway. Moreover the uncertainty from extracting the nuclear signal from a coma-laden image increases the error estimate.

3.2 The Energy of a Nucleus

The STM and RRM model mark the extremes; many objects lie in between. For cometary nuclei, historically the STM has been used because it has been assumed that the thermal inertia, Γ, of nuclei are small; i.e., the nuclei are slow-rotators. The value of Γ is known only for the Moon and a few other satellites, and Spencer *et al.* (1989) point out that the value for an asteroid (or cometary nucleus) could

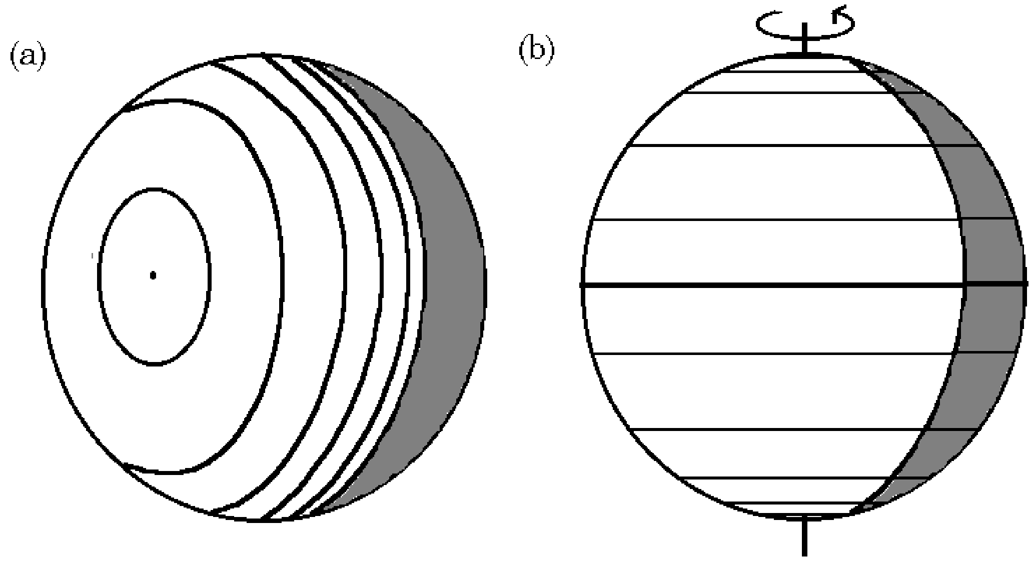

Figure 3.1: Schematic of contour temperature map for (a) slow- and (b) fast-rotators. For each spherical object, the gray-shaded area is unlit by the Sun. In (a), the subsolar point and location of highest temperature is at the dot left-of-center; the temperature decreases toward the terminator in every direction. In (b), I have assumed that the rotation axis is perpendicular to the object's orbit plane, so the subsolar latitutde is at the equator. The temperature is a maximum there and falls off toward the poles. Note that the contours extend beyond the terminator. This figure is based on Figure 4 of Lebofsky and Spencer (1989).

be lower since most of these objects are farther from the Sun so the heat capacity could be lower at the cooler temperatures. Moreover at the lower temperatures the radiative heat transport that is so important in the lunar regolith – and which boosts the effective conductivity – is not necessary. On the other hand the thermal inertia could be higher since the small bodies of the Solar System presumably have less regolith – they simply cannot gravitationally retain it – and the bare rock is a more effective conductor. Harris *et al.* (1998) have claimed that thermal IR data on some NEAs, incorporating some of modeling done by Spencer (1990), seem to indicate a higher thermal inertia than previously supposed.

I have made an attempt to handle the intermediate case between the STM and RRM with a model that is one or two steps farther in complexity. Further augmentation beyond what I describe here should wait until more elaborate datasets have been collected. As it is I will only apply the model to the Hale-Bopp data, since certain important physical properties of the other comets in this thesis – most notably the spin axis direction – are unknown. First I will describe the basic STM, and then the enhancements that I have supplied. A good discussion of the STM is given by Lebofsky and Spencer (1989).

The energy balance on a facet on the nucleus is:

$$\text{Energy Absorbed} = \text{Energy Emitted}, \tag{3.1}$$

where for a facet at some latitude $\pi/2 - \theta$ and longitude ϕ on a spherical nucleus the l.h.s. is

$$\text{Energy Absorbed} = \int \frac{F_\odot(\lambda)}{4\pi r^2}(1 - A(\theta, \phi, \lambda))R^2 \cos z(\theta, \phi) d\cos\theta d\phi d\lambda, \tag{3.2}$$

and the r.h.s. is

$$\text{Energy Emitted} = \int B(\lambda, T(\theta, \phi))\epsilon(\theta, \phi, \lambda)R^2 d\cos\theta d\phi d\lambda, \tag{3.3}$$

where $F_\odot$ is the solar specific luminosity; r is the comet's heliocentric distance; A is the Bond albedo and is equal to pq, the product of the geometric albedo and the phase integral; R is the nucleus' radius; z is the zenith angle of the Sun as seen from the facet; B is the Planck function; ϵ is the emissivity, which is near unity; and T is the temperature. Since the STM was designed for asteroids, usually A and ϵ are taken to be independent of position, although currently there is no indication of any large albedo spots on cometary nuclei either. In addition, it is assumed that A is independent of wavelength in the optical, where most of the Sun's energy is, and ϵ is independent of wavelength in the mid-IR, where most of its thermal output is. This simplifies the equations to

$$\frac{L_\odot}{4\pi r^2}\pi R^2(1 - A)\cos z(\theta, \phi) = \epsilon R^2 \sigma T^4(\theta, \phi), \tag{3.4}$$

where $L_\odot$ is the solar luminosity and σ is the Stefan-Boltzmann constant. The result is a temperature that depends on the one-fourth power of the local solar zenith angle, with no dependence on R; only T_{SS}, the subsolar point's temperature, is needed to describe the temperature map. By plugging in the temperature map into Eq. 3.3 accounting for the observing geometry, one can find the value of R satisfying what the observer measures with the photometry.

There are two added features to the STM that complicate this picture. First, there is an arbitrary constant multiplied to the r.h.s. (Eq 3.3), η, a beaming factor, to account for the fact that the asteroid is actually not a perfect sphere, but has surface roughness. For example, if at the subsolar point on the asteroid there were a crater, the thermal flux coming out of the asteroid would be higher since the surface of the asteroid in the crater would be hotter (from backwarming by the walls). The value of η seems to be approximately unity, with a known range for a few asteroids and satellites of 0.7 to 1.2 (Spencer *et al.* 1989, Harris 1998). The problem is η is not known *a priori*, so there is some ambiguity akin to the albedo problem with optical data. However it is much less significant since the possible range of η only covers about a factor of 2, and moreover with flux measurements at multiple mid-IR wavelengths it is in principle possible to constrain the value (Harris *et al.* 1998).

The other added feature to the STM is the phase effect. Since we hardly ever observe an object at phase angle α of zero, and often α is $\geq 40°$ when observing nearby comets and NEAs, one needs to know the phase behaviour. One popular model is to have the phase effect in magnitudes proportional to α itself (Matson 1972, Lebofsky *et al.* 1986). The known range for the proportionality constant is 0.005 to 0.017 mag/degree. Another method is to just integrate the amount of light one sees on the Earth-facing hemisphere. This is akin to using a $\frac{1}{2}(1 + \cos\alpha)$ phase law in the optical regime, except that in the mid-IR each differential of area on the surface is weighted by T^4. There is some evidence (Harris 1998) that this latter method describes the phase behavior of asteroids better than the older method, at least for the large asteroids.

The optical data enter the analysis for the determination of the albedo A in Eq 3.4, since the optical flux from a spherical object is proportional to pR^2. The phase integral, q, connecting A and p, is roughly known from the optical phase behavior, which has been studied quite a bit more than its IR counterpart. The result is that the problem essentially becomes a system of two equations with two unknowns, R and p. This is the basic method behind the work of Campins *et al.* (1987), Millis *et al.* (1988), and A'Hearn *et al.* (1989) when they made the first ground-based measurements of nuclear albedos in the mid-1980s.

3.3 The Augmented Thermal Model

For the augmentation of the model, I have used two basic equations: the conservation of energy equation, and the one-dimensional heat transport equation, the simple parabolic partial differential equation. Energy conservation is treated with the input being insolation and the outputs being reradiation, volatile vaporization, and conduction into the subsurface layers. I have not attempted to treat lateral heat transport.

Energy conservations dictates

$$\frac{L_{\odot}}{4\pi r^2}(1-A) = \sigma \int\int T^4(\theta,\phi)\epsilon d\cos\theta d\phi + \kappa\frac{dT}{dz} + L(T)\frac{dM}{dt}, \tag{3.6}$$

where κ is the thermal conductivity, $L(T)$ is the latent heat of vaporization, and dM/dt is the gas mass loss rate. Except for comets such as Hyakutake, which had an extremely active nucleus, the contribution of the third term in that equation will usually be only on the few percent level. For this reason, I have simplified the model by having the gas emanate uniformly over the nucleus' surface.

The heat equation is

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho c}\frac{\partial^2 T}{\partial z^2}, \tag{3.7}$$

where ρ is the bulk density and c is the heat capacity. The simultaneous solution of these equations is the basis of my augmented thermal model. The solution is a temperature map from which the expected flux is calculated for a given radius size. The continuum between STM and RRM is sampled simply by altering the thermal inertia $\Gamma = \sqrt{\kappa\rho c}$.

Spencer *et al.* (1989) have done much work on the thermophysical behavior between the STM and the RRM. They formulated the constant Θ, the thermophysical parameter, to indicate when the STM, the RRM, or something in between is applicable, defined as

$$\Theta = \sqrt{\kappa\rho c\omega}/(\epsilon\sigma T_{SS}^3), \tag{3.8}$$

where ω is just 2π divided by the rotation period. It is basically a comparison of the rotation time scale and the timescale for the thermal wave to penetrate one skin depth, where the skin depth l is given by

$$l = \frac{2}{\omega}\frac{\kappa}{\rho c}. \tag{3.9}$$

If $\Theta << 1$, then STM is applicable, whereas if $\Theta >> 1$, the RRM is the one to use. For example, if a cometary nucleus at 1 AU from the Sun has a lunar thermal inertia (50 J K^{-1} m^{-2} s$^{-1/2}$), and spins on its axis in 10 hours, then $\Theta = 0.2$ (since the subsolar point will have $T_{SS} = 390$ K). This places it in the slow-rotator regime, but since 0.2 is of the same order as unity, we would not expect the STM to perfectly describe the object's thermal behavior.

Another aspect of my augmented model is the ability to handle ellipsoidal nuclei. This introduces yet more parameters into the model, since not only are the axial ratios of the nucleus required, but also the rotation state, since the flux observed at Earth will now depend on the sub-earth latitude and longitude. Note that the usual observations of nuclei that measure the varying cross section reveal only the projected axial ratio, not the actual ones, unless the data can be combined with measurements at other points in the comet's orbit. I will show an example of this in Chapter 5. Brown (1985) has studied the effects of ellipticity on the output of the STM, and shown that slight asphericity does not make much difference in the use of the STM, but – as with the cigar-shaped nucleus example that I previously mentioned – serious systematic problems can exist if the objects are significantly elongated.

Since the augmented model explicitly calculates the temperature at several layers within the nucleus, the model is able to handle my radio data, which the STM is unable to do. Our microwave observations do not sample the surface temperature, but rather the temperature several "skin depths" below the surface. The subsurface layer that is sampled could be a few wavelengths deep (for relatively rocky material) or a few tens of wavelengths deep (for icier material) (de Pater *et al.* 1985). Regardless of the exact depth, it is clear that the microwave data show a lower temperature than at the surface. Here we see a case where the uncertain nuclear porosity, composition, and conductivity have a very direct effect on the interpretation of data.

3.4 Rotation of the Nucleus

The rotation of cometary nuclei has been studied for the past few decades, as mentioned in Chapter 1. However for only a handful of nuclei are there arguably well-determined rotation periods. A thorough review has been written by Belton (1991), and Meech (1999) has added more information from the 1990s. It is likely that some cometary nuclei are in complex rotation, complicating one's derivation of the rotation state via observations, and it is telling that there is still uncertainty in the rotation state of 1P/Halley's nucleus, one of the most deeply studied comets in all history. In this section I will give a brief description of the easy methods to determine a periodicity in the rotation state of the nucleus, but there is the caveat that it is not the only periodicity.

There are two main methods I employ for determining periodicity, one based on the morphology of the near-nuclear coma, the other based on the photometry of the comet's photocenter. I did not use the zero-date method used by Whipple (1982) because the other two methods are more reliable, as Whipple himself has stated.

The first method, used for Comet Hyakutake and Hale-Bopp, requires taking images over a long enough time baseline to be able to match up when a particular feature in the dust coma – e.g., a jet or an envelope – returns to the same orientation. The time between these witnessed events is an integer multiple of the rotation period. There are pitfalls to this method: there is a basic assumption that an active area on the nucleus that produces the coma feature when it is first seen will stay active long enough for the observer to witness it later. Moreover, even if it does stay active, it may not be easy to tell when that particular feature is back in view: e.g., one could be fooled if there is another similar-looking jet in the coma. Implicit in the use of this method is that the comet itself does not change significantly during the observing interval. For example determining the periodicity could become problematical if, during the observing interval, the comet goes into outburst or splits.

The second method, which was also used on comet Hyakutake and on comet Encke, involves measuring the photometry over a long enough continuous time interval to watch the variation in brightness. In principle one could do this with the post-processed images, where the coma has been removed leaving just the light of the nucleus, but this has not been possible for any of the comets in this study. This method measures the variation due to the changing cross section of the nucleus plus whatever variation is due to the coma. Fortunately for the two comets it was not difficult to tell which component was both dominating the flux value and the variation. The data set for this method is a light curve, a time-series of the flux.

The extraction of a period from these data is a non-trivial problem. For the first method, at the most basic level one matches images by eye, although for good temporal sampling cross-correlation methods may be possible. When employing the second method, common simple algorithms that are used in the cometary science community (as well as other fields of astronomy, e.g., variable star research) are described by Stellingwerf (1978) and Dworetsky (1983); these involve trial-and-error of many potential periods, minimizing the length of a string that connects the time series photometry in a phased light curve plot. An advantage is that the algorithm is perfectly able to deal with data sampled at a non-periodic rate, and also it is not beholden to any assumed shape of the light curve, sinusoidal or otherwise. A related method is to just take the Fourier transform – i.e., get a power spectrum – of the time-series, and find the most important frequency. Since the mathematical process of transforming can introduce extra noise into the data, this method works best when there are many points to the light curve.

One significant problem with the morphological and photometric methods is that an observer usually does not have perfect temporal coverage of the entire rotational phase. An observing night often just does not last long enough to watch a nucleus cycle through one complete rotation. Stringing observations together over several nights helps alleviate this problem, but it is hardly ever completely eradicated: there are usually aliases to the best choice of periodicity P that one finds for the particular observing run, aliases with values like $\frac{3}{2}P$ or $\frac{1}{2}P$, i.e., small whole-number ratios multiplying P. In general the longer the baseline over which one observes, the better one can constrain the period.