Reading from "Chapter 9: Concepts of Energy I: Work and Energy" from R.D. Knight, *Physics: A Contemporary Perspective,* Volume 1, Preliminary Edition, (Addison Wesley, Reading MA, 1997) pp. 290-297.

Concepts of Energy I: Work and Energy

LOOKING BACK

Sections 2.3; 5.2–5.4; 5.6; 8.2

9.1 Introduction

[**Photo suggestion: A photo collage showing the sun, a windmill, a power plant, and an electric transmission line.**]

The concept of *energy* is one of the most important concepts in all of science. Automobiles, rockets, computers, biological organisms, and ecosystems all use and transform energy in a myriad of ways. Energy is also vital to our daily lives. We use chemical energy to heat our homes and bodies, electrical energy to power our lights and computers, and solar energy to grow our crops and forests. Energy—its characteristics, transformations, and conservation—will be a major new theme in this text. Energy will give us a new perspective on motion. In addition, energy will give us a new tool for problem solving

It is difficult, however, to define in a general way just what energy is. The concept of energy has grown and changed with time, and you will come to understand energy better through seeing many examples of how energy is used than by formal definitions. We will start by discussing what seems to be a completely unrelated topic—money. As you will discover, monetary systems have much in common with energy.

The Parable of the Lost Penny

Jose was a hard worker. His only source of income was the paycheck he received each month. Even though most of each paycheck had to be spent on basic necessities, Jose managed to keep a respectable balance in his checking account. He even saved enough to occasionally buy a few stocks and bonds. Jose never cared much for pennies. To him they seemed more trouble than they were worth. So Jose kept a jar by the door and dropped all of his pennies into it at the end of each day. Eventually, he reasoned, his saved pennies would be worth taking to the bank and converting into crisp new dollar bills.

Jose found it fascinating to keep track of these various sources of money. He noticed, somewhat to his dismay, that the amount of money in his checking account did not spontaneously increase overnight. Neither did the number of pennies in his jar. Furthermore, there seemed to be a definite correlation between the size of his paycheck and the amount of money he had in the bank. So Jose decided to embark on a systematic study of money.

He began, as would any good scientist, by using his initial observations to formulate a hypothesis. Because this was a fairly involved hypothesis involving the relationships between various sorts of money, Jose called it a *model* of his monetary system. He found that he could represent the monetary model as a flow chart, as shown in Fig. 9-1.

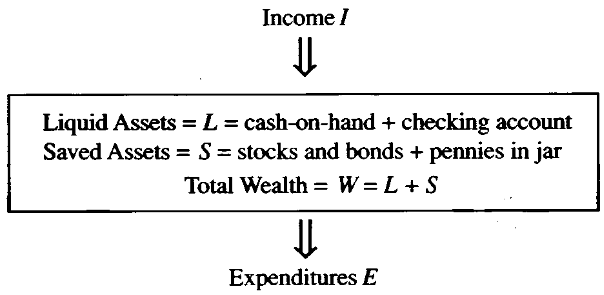

FIGURE 9-1 Jose's model of the monetary system.

As the chart shows, Jose found that it was convenient to divide his money into two basic types—liquid assets and saved assets. The *liquid assets* L, which included his checking account and the cash in his pockets, were moneys available for immediate use. His *saved assets* S, which included his stocks and bonds as well as his jar of pennies, had the *potential* to be converted into liquid assets, but they were not available for immediate use. Jose decided to call the sum total of assets his *wealth*: $W = L + S$.

Jose's assets were, more or less, simply definitions. The more interesting question, he thought, was how his wealth depended on the *monetary transfers* of *income* I and *expenditures* E. These transfers represented both money transferred *to* him by his employer and money transferred *by* him to stores and bill collectors. After painstaking collection and analysis of data, Jose finally deduced the *quantitative* relationship between the monetary transfers and his assets to be

$$I - E = \Delta L + \Delta S = \Delta W.$$

Jose interpreted this equation to mean that the *net* monetary transfer to him, given by $I - E$, was numerically equal to the *change* in his wealth, ΔW. (His data clearly refuted the competing hypothesis of his next door neighbor Bubba, who had asserted that $I - E = W$. That is, that the net monetary transfer *equals* net wealth. After all, Jose noted, his wealth did not drop to zero on days when he had neither income nor expenses.)

During a one-week period when Jose stayed home sick, *isolated* from the rest of the world, he had neither income nor expenses: $I - E = 0$. Amazingly, but in grand confirmation of his hypothesis, he found that his wealth W_f at the end of the week was identical to his wealth W_i at the week's beginning: $W_f = W_i$, or $\Delta W = 0$. This occurred despite the fact that he had moved pennies from his pocket to the jar and also, by telephone, had sold some stocks and transferred the money to his checking account. In other words, Jose found that he could make all of the *internal* conversions of assets from one form to another that he wanted, but his total wealth remained constant (W = constant) as long as he was isolated from the world. This seemed such a remarkable rule that Jose named it the law of conservation of wealth.

One day, however, Jose added up both his income and expenditures for the day and the changes in his various assets—and he was 1¢ off! $I - E = \Delta L + \Delta S - 1$ ¢. Jose quickly verified that it wasn't just a math error, but that some money really, and inexplicably, seemed to have vanished. Jose was devastated. All those years of careful research, and now it seemed that his monetary hypothesis was not true, that under some circumstances—yet to be determined—$I - E \neq \Delta W$. Off by a measly penny. A wasted scientific life....

But wait! Jose realized, in a flash of inspiration, that perhaps there were other types of assets that he had not discovered and that, if *all* assets were included, his monetary hypothesis would still be valid. Weeks went by as Jose, in frantic activity, searched fruitlessly for previously *hidden* assets. While his life disintegrated, his girlfriend gave him an ultimatum: "Knock off this manic behavior and clean house, or I'm moving out." As Jose, his spirits low, lifted the cushion off the sofa to vacuum out the potato chip crumbs—lo and behold!—there it was! The missing penny!

Jose raced to complete his theory, now including the sofa (as well as the washing machine) as previously unknown forms of saved assets that needed to be included in S. Other researchers soon discovered other types of assets—particularly the remarkable find of the "cash in the mattress"—that were included in Jose's hypothesis. To this day, when all known assets are included, monetary scientists have never found a violation of Jose's simple hypothesis that $I - E = \Delta W$. Jose was last seen sailing for Stockholm to collect the Nobel Prize for his theory of wealth.

9.2 Energy

Jose, despite his diligent efforts, did not discover a "law of nature." The monetary system is a human construction that, by design, obeys Jose's "laws." Monetary system laws, such as those that say you cannot print money in your basement, are enforced by society, not by nature. Suppose, however, that physical objects possessed a "natural money" that was governed by a theory, or model, similar to Jose's. An object might have several different forms of this "natural money" that could be converted back-and-forth, but the total amount of an object's "natural money" would only *change* as a result of this natural money being *transferred* in or out of the object. Two of the key words here, as in Jose's model, are *transfer* and *change*.

One of the greatest and most significant discoveries of science is that there is such a "natural money," which we call **energy**. A difficulty for many students, however, is that the concept of energy is rather abstract. Force is very tangible, something you can feel, and acceleration is a concept you can visualize. So thinking of motion in terms of forces and acceleration seems, at least after some practice, fairly straightforward. Energy is a more abstract and more subtle concept than is force.

But, if you think about it, money is also a very abstract idea. Money is *not* the pieces of paper or objects of metal you carry in your pocket or purse. The real and tangible "value" of that piece of paper with $100 written on it is essentially zero—the same as any other piece of paper the same size. However, the fact that someone would give you a pair of shoes or large stack of CDs in exchange for that piece of paper implies that the paper has some *intangible* value. This *monetary* value of the paper is an idea that is useful only because we, as a society, have agreed to certain "laws" about how money behaves and about how one form of monetary value (e.g., a piece of paper with numbers on it) is exchanged for another (e.g., real goods). So despite its abstract nature, money is quite "real." Energy is much the same—abstract, intangible, but nonetheless "real." The distinction is that energy obeys natural laws, which we discover, rather than human laws, which we invent.

There is not a single definition of energy. Energy is a concept that has developed over a long span of time, and there are many forms, or types, of energy. You have heard of some of these, such as solar energy or nuclear energy, but others may be new to you. These forms of energy can differ as much as a checking account differs from loose change in the sofa. Much of our study is going to be focused on the *transformation* of energy. A large fraction of modern technology is concerned with transforming energy from one form (e.g., the chemical energy of oil molecules) to another (e.g., electrical energy or the kinetic energy of your car). We will continue to expand the concepts of energy and its transformation properties for the rest of this text.

As we use energy concepts we will be "accounting" for energy that is transferred in or out of a system, or that is converted from one form to another without loss. It is this characteristic of energy that makes the analogy with money so useful. The fact that nature "balances the books" for energy is one of the most profound discoveries of science. The possible behaviors of physical systems are sharply constrained by having to maintain this balance. Behaviors that might otherwise seem plausible are simply found not to occur in nature if the energy cannot be accounted for. The implications of this true "law of nature" extend well beyond physics. Chemistry, biology, engineering, and ecology are all significantly influenced by the laws of energy use. In the long run, the laws of energy are far more wide-ranging and important in other disciplines than are Newton's laws of mechanics.

Even though certain forms of energy were recognized quite early, the *law of conservation of energy* was not recognized until the mid-nineteenth century, long after Newton. The reason, similar to the situation with Jose's "lost penny," was that it took scientists a long time to realize how many types of energy there are and the various ways that energy can be converted from one form to another. The ideas involved go well beyond Newtonian mechanics to include new concepts about heat, about chemical energy, and about the energy of the individual atoms and molecules that comprise a system. The complete statement about energy and its transformation properties is known as the *first law of thermodynamics*. It is a new statement about nature, having more content and meaning than can be deduced from Newton's laws alone. We will defer the full first law until our study of thermodynamics, but these next few chapters will begin the important task of introducing the concepts of energy.

It is worth emphasizing that the law of conservation of energy is a scientific *hypothesis* about nature. Simply writing down a quantitative relationship between various concepts does not make the relationship true. Like Jose, we must first postulate a relationship and *then* seek evidence for its validity. As of today, with 150 years of experimental evidence, we know of no violations of the law of conservation of energy. It has become one of the firmest principles of science. Many scientific discoveries, in fact, have been made as a consequence of experiments where there seemed to be some "missing energy." Rather than believe they had discovered a violation of energy conservation (although that could happen!), the scientists believed so firmly in energy conservation that they searched until they discovered the source of the missing energy.

9.3 The Basic Energy Model

Figure 9-2 provides a basic model of energy that is analogous to Jose's model of money. As a *basic* model it is certainly not complete, and we will add significant new features to our model as we need them. Nonetheless, it is a good starting point.

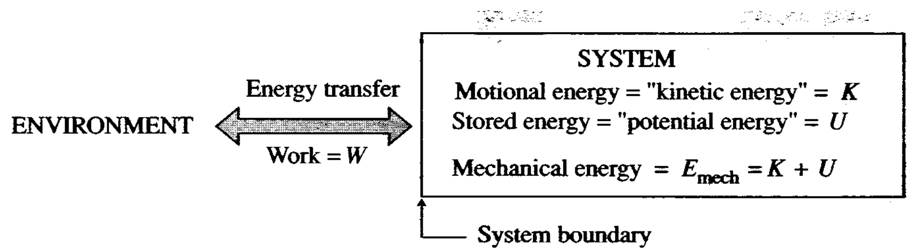

FIGURE 9-2 The basic energy model of a system interacting with its environment.

Once again we are distinguishing between the *system* that we wish to study and its surrounding *environment*. The system can be characterized by two quantities that we call the **kinetic energy** and the **potential energy**. All we need to know about these for now is that kinetic energy (symbol K) is an "energy of motion," while potential energy (symbol U) is a "stored" energy that has the "potential" to be converted to kinetic energy. (Note that such a conversion would be an **energy transformation**.) Kinetic energy is analogous to Jose's liquid assets, while potential energy is analogous to his saved assets. The sum of kinetic and potential energy (analogous to wealth) is called the **mechanical energy** $E_{mech} = K + U$. The term *mechanical energy* designates this form of energy as being due to motion and mechanical effects (like stretching springs) rather than chemical effects or heat effects, which are other forms of energy that we will introduce later. For now, we will omit the subscript and use the symbol E to mean mechanical energy. Later, when it becomes important, we'll use E_{mech} to distinguish mechanical energy from other forms of energy.

Kinetic energy, as you will learn in the next section, depends upon the *speed* $|\vec{v}|$ of the system. Because the speed can be zero but never negative, we require $K \geq 0$. The fact that kinetic energy can never be negative is one of its important characteristics. Potential energy is a bit harder to understand, but a good prototype of potential energy is a stretched rubber band. A stretched rubber band, as you know, is just waiting to be released in order to launch a paper wad. In other words, it has the "potential" for producing kinetic energy. The rubber band's stretch represents "saved" or stored energy, and that is what potential energy is all about. The energy stored in a rubber band depends on how far it is stretched—that is, on the *position* of the ends of the rubber band. So while kinetic energy depends on *speed*, potential energy depends on *position*. As you will see, we can use energy ideas to relate an object's speed to its position.

A system, unless it is completely isolated, has the possibility of exchanging energy with its surrounding environment. There are two primary processes by which this can occur. The first, which is the only one we are going to be concerned with for now, is as a result of forces—pushes and pulls—exerted on the system by the environment. This *mechanical* energy transfer goes by the name **work**. It is also possible for energy to be transferred between the system and its environment, if they are at different temperatures, by a *nonmechanical* energy transfer process called *heat*. Heat is a significant aspect of the energy model that we will add when we get to the study of thermodynamics, but for the time being we will concentrate on the mechanical transfer of energy via work.

The symbol for work is W. (The possibility for confusion once again rears its ugly head, because now you have to make sure that you do not confuse work W with weight $\vec{W}$ or its magnitude $|\vec{W}|$). Notice that the arrow labeled *work* in Fig. 9-2 is bi-directional, rather different than the single-direction arrows of "income" and "expenditures" in Fig. 9-1. This is because work is a quantity that can be either positive or negative, with the interpretation that:

$W > 0 \Rightarrow$ the environment does work on the system and the system's energy increases,

$W < 0 \Rightarrow$ the system does work on the environment and the system's energy decreases.

This is equivalent to considering expenditures—money out—to be a negative income. In fact, that is just how accountants really do handle incomes and expenditures.

Having established our basic quantities, what is the relationship between them? Our hypothesis, which is confirmed by experiment, is:

$$W = \Delta E = \Delta K + \Delta U. \tag{9-1}$$

In words, Eq. 9-1 says that the energy transferred *to* a system via work changes the total mechanical energy *of* the system. Further, the system's change of energy might be a change of kinetic energy, a change of potential energy, or both. Equation 9-1 gives no information about how the total energy change is divided up between kinetic energy and potential energy.

Now consider what happens if you push on an object. As you push, you exert a force on the object and do work on it. That is, you mechanically transfer energy to the object. What happens to the object as a result of this push? One possibility is that the object will accelerate and have a higher speed at the end of the push than it had at the beginning—an increase of kinetic energy. The energy you transferred *to* the system via work ends up, in this case, as an increase *of* the system's kinetic energy. It is also possible that the push causes a rubber band inside the system to be stretched—an increase of potential energy. Here the energy transferred *to* the system via work ends up as an increase *of* the system's potential energy.

The situation could go the other way as well. Suppose a rubber band inside the system is initially stretched and that it is used to pull an object in the environment closer. Now the system is doing work on the environment, by pulling on it. In this case the work is a negative quantity. But the system is also *losing* potential energy as the rubber band retracts, so ΔU is also negative. So in this example the energy *of* the system decreases and that amount of energy is transferred *to* the environment by doing work on it.

It is not just poor typing that has led us to emphasize the terms *of* and *to* in the last three paragraphs. We are making a very significant point about work and energy. Kinetic energy and potential energy are properties that *characterize* the system. Like mass or charge, we could say that they are properties *of* the system. We will often talk about the **state** of the system, by which we mean the specific characteristics of the system at a particular time. In addition to mass, charge, kinetic energy, and potential energy, quantities such as pressure and temperature also characterize the state of the system. They are *of* the system.

Work, on the other hand, is *not* a property of the system. It is a *process*, or an interaction, between the system and the environment. It is something done *to* the system in order to *change the state* of the system. That is why Eq. 9-1 has Δ's on the right side but *not* on the left. The system had a certain amount of kinetic energy K_i and potential energy U_i before work was done on it, and it ends up with some other amount K_f and U_f after the work was done. We can measure the change of state of the system in terms of its change of kinetic energy $\Delta K = K_f - K_i$ and its change of potential energy $\Delta U = U_f - U_i$. But work does not have a before and after—it is simply a measurement of something that was done *to* the system.

So as we interpret Eq. 9-1, we want to say that a *process*, namely doing work on a system, causes a *change* in the state of the system. The work done does not tell us anything about how much total energy E the system has (recall Bubba's mistaken conjecture in the parable), but only by how much the total energy *changes*.

9.4 Kinetic Energy

In the previous section we began to develop the idea that the state of a system can be changed by an external influence or process acting on the system. This is rather vague, so let us see if we can make the idea more precise by associating "state of the system" and "external influence" with specific quantities that we can measure or calculate. We will concentrate, for now, on the motion of a single particle—the simplest possible system. Chapters 10 and 11 will expand these ideas to more complex systems of multiple particles.

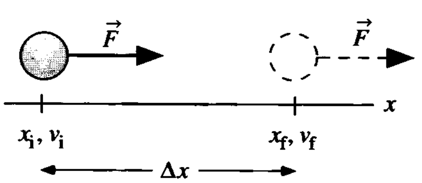

FIGURE 9-3 A particle moving from x_i to x_f under the influence of a *constant* force $\vec{F}$.

Figure 9-3 shows a particle of mass m that moves along the x-axis from an initial position x_i to a final position x_f under the influence of a *constant* force F. The force acts steadily on the particle as it moves—that is, the force is not an impulse force that acts briefly on the particle at x_i, but is a force that is applied throughout the particle's motion. Such a force will cause a constant acceleration $a = F/m$. Recall, from one-dimensional kinematics, that a particle accelerating from initial velocity v_i to final velocity v_f with constant acceleration a obeys

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(x_f - x_i) \\ \Rightarrow v_f^2 - v_i^2 &= 2a\Delta x. \end{aligned} \tag{9-2}$$

Substituting $a = F/m$ and doing a bit of rearranging gives

$$\begin{aligned} v_f^2 - v_i^2 &= \frac{2F\Delta x}{m} \\ \Rightarrow \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 &= F\Delta x. \end{aligned} \tag{9-3}$$

We can rewrite Eq. 9-3 in the form

$$\Delta(\tfrac{1}{2}mv^2) = F\Delta x. \tag{9-4}$$

What does Eq. 9-4 tell us? The basic quantity on the left, $\frac{1}{2}mv^2$, is a characteristic *of* the particle. It depends, at any particular point of the motion, only on the particle's mass and velocity. Thus the quantity $\frac{1}{2}mv^2$ measures the *state* of the system. The right-hand side, however, measures something being done *to* the system. A force, from an agent somewhere in the environment, reaches in and pushes the particle through a displacement Δx. The quantity $F\Delta x$ thus represents a *process* that happens. With these ideas, we can interpret Eq. 9-4 as saying that the *process* of a force F pushing the particle through a displacement Δx causes a *change* in the *state* of the system, as measured by the quantity $\frac{1}{2}mv^2$. This is exactly the idea behind the basic energy model.

It is worth noting that the quantity $F\Delta x$ tells us nothing at all about the *value* of $\frac{1}{2}mv^2$, but only about how $\frac{1}{2}mv^2$ *changes*. That change could be either positive—if the particle speeds up—or negative—if it slows down. How could it be negative? The quantities F and Δx, as in our earlier analyses of one-dimensional motion, are vector components and, accordingly, have signs. They are the "force component along the axis of motion" and the "displacement," both of which are signed quantities. They are *not* the magnitude of the force $|\vec{F}|$ or the distance $|\Delta x|$, which are always positive. If, for example, the force in Fig. 9-3 points to the *left* as the particle moves toward the right, it would act as a braking force that slows the particle. The component F would be negative, because the vector $\vec{F}$ points in the $-x$-direction, while Δx would continue to be positive. This would make the product $F\Delta x$ negative, exactly what is needed to match the negative sign of $\Delta(\frac{1}{2}mv^2)$ for a particle slowing down. (The *change* in a quantity, recall, is *always* the final value minus the initial value.)

The quantity $\frac{1}{2}mv^2$ is the kinetic energy K of the particle. It is defined as

$$K = \tfrac{1}{2}m|\vec{v}|^2, \tag{9-5}$$

where m is the mass of the particle. In terms of vector components,

$$K = \begin{cases} \tfrac{1}{2}m({v_x}^2 + {v_y}^2 + {v_z}^2) & \text{general formula} \\ \tfrac{1}{2}mv^2 & \text{one-dimensional motion.} \end{cases} \tag{9-6}$$

By its definition, kinetic energy can never be a negative number: $K \geq 0$. The *change* in kinetic energy can, of course, be negative if K decreases in value. If you find, in the course of solving a problem, that K is negative—stop! You have made an error somewhere. Don't just "lose" the minus sign and hope that everything turns out OK.

One of the most important characteristics of kinetic energy is that it is a scalar, rather than a vector. It depends on the speed $|\vec{v}|$, but not on the velocity's direction. The kinetic energy of a particle will be the same regardless of whether it is moving up or down, or left or right. Consequently, the mathematics of an energy solution to a problem is often much easier than the vector mathematics required by a force and acceleration solution.

The unit of energy is that of mass times velocity squared. In the SI system of units, this is kg m^2/s^2. The unit of energy is so important that is has been given its own name: the **joule**. We define:

$$1 \text{ joule} = 1 \text{ J} \equiv 1 \text{ kg m}^2/\text{s}^2.$$

To give you an idea about the size of a Joule, consider a 0.5 kg mass (weight on earth of ≈1 pound) moving with a speed of 2 m/s (≈4 mph). The mass's kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.5 \text{ kg})(2 \text{ m/s})^2 = 1 \text{ J}.$$