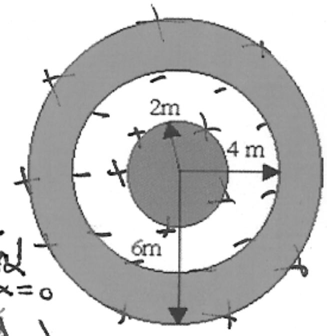
Problem 5 (17 points)

A solid spherical conductor with radius $R_1 = 2$ m is given a net charge of $+Q_0$ and placed inside a spherical conducting shell (with inner radius $R_2 = 4$ m and outer radius $R_3 = 6$ m) with net charge $+q$.



(a) Use Gauss' Law to find a symbolic expression for magnitude of the electric field (if the electric field is zero state that explicitly and show your reasoning) at the following points:

1. $r_1 = 1$ m
2. $r_2 = 3$ m
3. $r_3 = 5$ m
4. $r_4 = 9$ m

+3 $\Phi = \frac{q_{enc}}{\epsilon_0}$ $\Phi = \oint \vec{E}\cdot d\vec{A} = \int |\vec{E}||d\vec{A}| \cdot \cos\alpha$, $\alpha = 0$

$EA = \frac{q_{enc}}{\epsilon_0} \Rightarrow E = \frac{q_{enc}}{\epsilon_0 A} = \frac{q_{enc}}{4\pi r^2 \epsilon_0}$

$\int E \cdot dA = E \int dA$

E is constant on Gaussian surface

+1 1. $q_{enc} = 0 \Rightarrow E = 0$

+2 2. $E = \frac{Q_0}{36\pi\epsilon_0}$ or $\frac{Q_0 k}{9}$

+1 3. $q_{enc} = 0 \Rightarrow E = 0$

+2 4. $E = \frac{Q_0 + q}{4(81)\pi\epsilon_0}$ or $\frac{(Q_0 + q)k}{81}$

(b) Where and how much charge is on each surface of the two conductors?

+1 Inner sphere — $+Q_0$ on ~~outside~~ edge

+1 Shell
Inner edge $-Q_0$

+1 outer edge ~ $+Q_0 + q$

Picture +1 Bonus

(c) Using your expression for the electric field in part a, find the electric potential difference between the two spheres.

$\Delta V = -\int \vec{E}\cdot d\vec{S} = -\int_2^4 \frac{kQ\,dr}{r^2} = \frac{kQ}{r}\Big|_2^4 = \frac{kQ}{4} - \frac{2kQ}{4} = \frac{-kQ}{4}$

+ ↓ +1 +1 +1 +1

$= -\int |\vec{E}||d\vec{s}| \cos\alpha$, $\cos\alpha = 1$

$d\vec{s} = d\vec{r}$

$|\vec{E}| = \frac{kQ_0}{r^2}$ $= -\int_{r=2m}^{r=4m} \frac{kQ}{r^2} dr =$