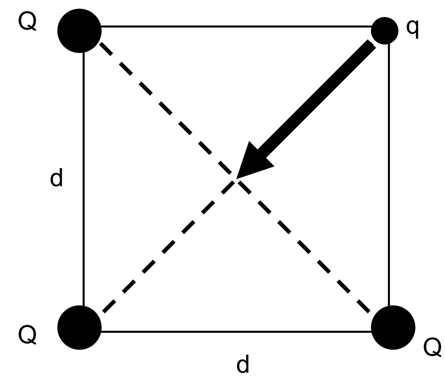
Problem 4 (18 points)



Three point charges, each with charge Q, are located at the three corners of a square as shown in the diagram on the right. Each side of the square is of length d. A forth charge q is then located at the fourth corner. You may use $k=1/4\pi\varepsilon_0$. Write carefully so your work can be followed

(a) Derive the potential for the point charge in the upper left corner.

Derive the potential of a point charge

$|\vec{E}| = k\frac{q}{r^2} = \frac{kQ}{r^2}$ $\qquad d\vec{s} = d\vec{r}$

$\Delta V_{r\to\infty} = -\int_r^\infty \vec{E}\cdot d\vec{s} = -\int_r^\infty |\vec{E}||d\vec{s}|\cos\alpha$ $\quad \alpha = 0$

$\Delta V_{r\to\infty} = -\int_r^\infty \left(\frac{kQ}{r^2}\right)(dr)(1)$

$= -kQ\int_r^\infty \frac{dr}{r^2} = kQ\left[\frac{1}{r}\right]_r^\infty$

$= kQ\left[\cancel{\frac{1}{\infty}}^{0} - \frac{1}{r}\right]$

$= -\frac{kQ}{r}$ $\qquad \Delta V_{\infty\to r} = -\Delta V_{r\to\infty} = -\left(-\frac{kQ}{r}\right)$

$\Delta V_{\infty\to r} = V_r - \cancel{V_\infty}^{0} = \frac{kQ}{r}$ $\qquad \Delta V_r = \frac{kQ}{r}$

(b) How much energy was required to assemble the initial 3 Q charges?

$PE_{total} = PE_1 + PE_2 + PE_3$

1st Q $\quad \Delta V = 0$ so $PE_1 = 0$

2nd Q $\quad PE_2 = \Delta V_1 Q_2 = \left(\frac{kQ}{r}\right)(Q) = \frac{kQ^2}{d}$ $\qquad r = d$

3rd Q $\quad PE_3 = Q_3(\Delta V_1 + \Delta V_2)$

$= Q\left[\left(\frac{kQ}{\sqrt{2}d}\right) + \left(\frac{kQ}{d}\right)\right] = \frac{kQ^2}{d}\left(\frac{1}{\sqrt{2}} + 1\right)$

$PE_{total} = 0 + \frac{kQ^2}{d} + \frac{kQ^2}{d}\left(\frac{1}{\sqrt{2}} + 1\right) = \frac{kQ^2}{d}\left(2 + \frac{1}{\sqrt{2}}\right)$

(c) How much work is required to move a charge q from the fourth corner to a point where the diagonals of the square intersect?

To move the charge q, you would have to do work

$W_{you} = \Delta PE = q\,\Delta V = q(V_f - V_i)$

$V_i = V_{1i} + V_{2i} + V_{3i} = \frac{kQ}{d} + \frac{kQ}{\sqrt{2}d} + \frac{kQ}{d} = \frac{kQ(4+\sqrt{2})}{2d}$

$V_f = V_{1f} + V_{2f} + V_{3f} = \frac{kQ}{\frac{\sqrt{2}}{2}d} + \frac{kQ}{\frac{\sqrt{2}}{2}d} + \frac{kQ}{\frac{\sqrt{2}}{2}d} = \frac{3\sqrt{2}kQ}{d}$

$\frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$\Delta PE = q\left(\frac{3\sqrt{2}\,kQ}{d} - \frac{(4+\sqrt{2})\,kQ}{2d}\right)$

$\Delta PE = \frac{5\sqrt{2}-4}{2}\frac{kqQ}{d}$