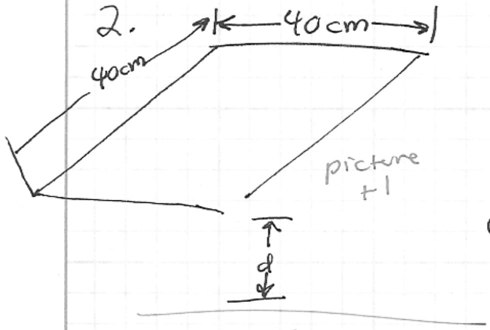
Physics 2049 Test 2 **Spring 2003-Dr. Saul**

Problem 2 (Estimation Problem: 15 points)

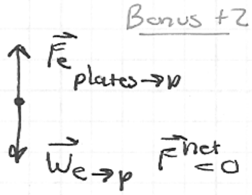
For a science fair, your kid brother wants to levitate a penny between two square metal plates where each side of the square is 40 cm long. If the penny is given an excess charge of 1 nC and the plates are given equal, but opposite charges, what is the magnitude of the charge on each plate. The plates are parallel and placed 3 cm apart.

2.



picture +1

Bonus +2



let $l = 40\text{cm}$ $d = 3\text{ cm}$

Area $= l^2$ +2

$q_{penny} = 1\text{nC}$

find $q_+ = |q_-| = ?$ +1

Use Newton 2nd Law $\vec{\Sigma}\vec{F}'s = 0$ since $a = 0$

$\vec{F} = q\vec{E}$ $+1\frac{1}{2}$

Estimate $m_p \sim 3g$ +1 $1\text{kg} \sim 2.2\text{lb}$

1-10g is reasonable 2 rolls of pennies < 1l

$m_p <$ ~~kg~~ g 0.01lb

Bonus +1

for plates

+1 $E = \frac{\sigma}{\epsilon_0}$

+1 $\vec{F}_e + \vec{W} = 0$

+1 $\vec{F}_e = -\vec{W} = -(-mg\hat{j})$

$F_e = mg\hat{j}$

$q_p E_p = mg$ +1

$q_p \frac{\sigma}{\epsilon_0} = mg \rightarrow \sigma = \frac{mg\epsilon_0}{q_p}$ +1

$8\frac{1}{2}$

~~$q_p \frac{\sigma}{\epsilon_0}$~~ $q_{pl} = \sigma A = \frac{mg\epsilon_0 l^2}{q_p}$ +1 +1

$$q_{pl} = \frac{(3\times10^{-3}\text{kg})(9.8\text{m/s}^2)(8.85\times10^{-12}\frac{C^2}{Nm^2}) \times (40\text{cm})^2 \times \left(\frac{1\text{m}}{100\text{cm}}\right)^2}{10^{-9}\text{ C}}$$

$+1\frac{1}{2}$

Bonus +1

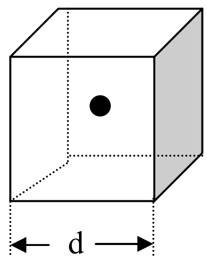
$$[q_{pl}] = \frac{\cancel{kg}\frac{\cancel{m}}{\cancel{s^2}} \frac{C^2}{\frac{\cancel{kg}\cancel{m}}{\cancel{s^2}}\cancel{m^2}} \cdot \cancel{cm^2} \cdot \frac{\cancel{m^2}}{\cancel{cm^2}}}{\cancel{C}} = C$$ ✓

Problem 3 (Essay 10 points) You may use diagrams and equations but no calculations in your response for this problem.

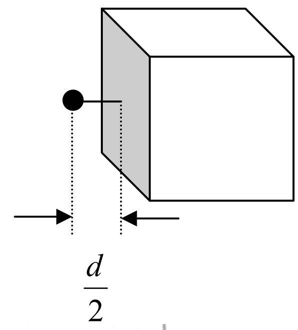
A. Which shaded face has the greater magnitude flux?

B. Which Gaussian cube has the larger Φ_{Total}?

Cube A



Cube B



Case A Definition of flux $\Phi \equiv \int \vec{E} \cdot d\vec{A}$

Since both charges have charge q_o, both the shaded areas have the magnitude area, and in both cases the relative positions of the charge and shaded area's are the same (including the distance between the charge and the area), the E-field and the flux ~~are~~ on the shaded Areas is the same magnitude.

Another way to look at flux through the shaded areas is to draw a 3rd cube just to the left of cube B (so the shaded area forms the right-hand side wall of the new cube. Then you can see that you have identical point charges at the center of identical cubes, so the flux through one side of each cube has to be the same magnitude

Case B Gauss Law say $\oint \vec{E} \cdot dA = \frac{q_{enclosed}}{\epsilon_o}$

That is the net flux through a surface that completely encloses a volume = the charge enclosed by that surface divided by ϵ_o.

Cube A encloses a charge $q_1 = q_o$

for cube A $\Phi_A = \oint_A \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_o} = \frac{q_o}{\epsilon_o}$

$\Phi_A = \frac{q_o}{\epsilon_o}$

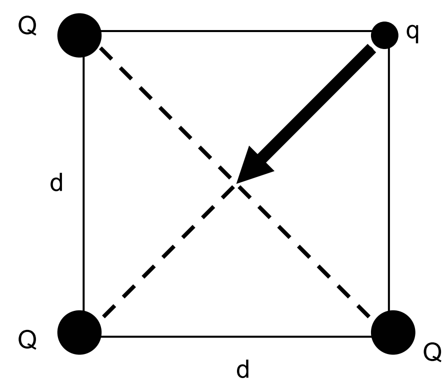
or the flux through cube A $= \frac{q_o}{\epsilon_o}$

Cube B does not enclose a charge so $\Phi_B = 0$

So if $q_o > 0$, then $\Phi_A > \Phi_B$

if $q_o < 0$, then $\Phi_A < \Phi_B$

Problem 4 (18 points)



Three point charges, each with charge Q, are located at the three corners of a square as shown in the diagram on the right. Each side of the square is of length d. A forth charge q is then located at the fourth corner. You may use $k=1/4\pi\varepsilon_0$. Write carefully so your work can be followed

(a) Derive the potential for the point charge in the upper left corner.

Derive the potential of a point charge

$|\vec{E}| = k\frac{q}{r^2} = k\frac{Q}{r^2}$ $\quad d\vec{s} = d\vec{r}$

$\Delta V_{r\to\infty} = -\int_r^\infty \vec{E}\cdot d\vec{s} = -\int_r^\infty |\vec{E}||d\vec{s}|\cos\alpha$ $\quad \cos\alpha = 1,\ \alpha = 0$

$\Delta V_{r\to\infty} = -\int_r^\infty \left(\frac{kQ}{r^2}\right)(dr)(1)$

$= -kQ\int_r^\infty \frac{dr}{r^2} = kQ\left[\frac{1}{r}\right]_r^\infty$

$= kQ\left[\cancel{\frac{1}{\infty}}^{0} - \frac{1}{r}\right]$

$= -\frac{kQ}{r}$ $\quad \Delta V_{\infty\to r} = -\Delta V_{r\to\infty} = -\left(-\frac{kQ}{r}\right)$

$\Delta V_{\infty\to r} = V_r - \cancel{V_\infty}^{0} = \frac{kQ}{r}$ $\quad \Delta V_r = \frac{kQ}{r}$

(b) How much energy was required to assemble the initial 3 Q charges?

$PE_{total} = PE_1 + PE_2 + PE_3$

1st Q $\quad \Delta V = 0$ so $PE_1 = 0$

2nd Q $\quad PE_2 = \Delta V_1 Q_2 = \left(\frac{kQ}{r}\right)(Q) = \frac{kQ^2}{d}$ $\quad r = d$

3rd Q $\quad PE_3 = Q_3(\Delta V_1 + \Delta V_2)$

$= Q\left[\left(\frac{kQ}{\sqrt{2}d}\right) + \left(\frac{kQ}{d}\right)\right] = \frac{kQ^2}{d}\left(\frac{1}{\sqrt{2}} + 1\right)$

$PE_{total} = 0 + \frac{kQ^2}{d} + \frac{kQ^2}{d}\left(\frac{1}{\sqrt{2}} + 1\right) = \frac{kQ^2}{d}\left(2 + \frac{1}{\sqrt{2}}\right)$

(c) How much work is required to move a charge q from the fourth corner to a point where the diagonals of the square intersect?

To move the charge q, you would have to do work

$W_{you} = \Delta PE = q\,\Delta V = q(V_f - V_i)$

$V_i = V_{1i} + V_{2i} + V_{3i} = \frac{kQ}{d} + \frac{kQ}{\sqrt{2}d} + \frac{kQ}{d} = \frac{kQ(4+\sqrt{2})}{2d}$

$V_f = V_{1f} + V_{2f} + V_{3f} = \frac{kQ}{\frac{\sqrt{2}}{2}d} + \frac{kQ}{\frac{\sqrt{2}}{2}d} + \frac{kQ}{\frac{\sqrt{2}}{2}d} = \frac{3\sqrt{2}kQ}{d}$

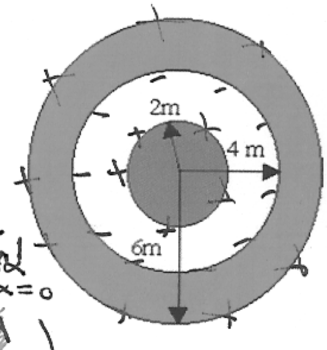
$\frac{1}{\frac{\sqrt{2}}{2}} = \frac{\cancel{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$\Delta PE = q\left(\frac{3\sqrt{2}\,kQ}{d} - \frac{(4+\sqrt{2})\,kQ}{2d}\right)$

$\Delta PE = \frac{5\sqrt{2}-4}{2}\,\frac{kqQ}{d}$

Problem 5 (17 points)

A solid spherical conductor with radius $R_1 = 2$ m is given a net charge of $+Q_0$ and placed inside a spherical conducting shell (with inner radius $R_2 = 4$ m and outer radius $R_3 = 6$ m) with net charge $+q$.



(a) Use Gauss' Law to find a symbolic expression for magnitude of the electric field (if the electric field is zero state that explicitly and show your reasoning) at the following points:

1. $r_1 = 1$ m
2. $r_2 = 3$ m
3. $r_3 = 5$ m
4. $r_4 = 9$ m

+3

$\Phi = \frac{q_{enc}}{\epsilon_0}$ $\quad \Phi = \oint \vec{E} \cdot d\vec{A} = \int |\vec{E}||d\vec{A}| \cdot \cos\alpha$ $\quad \alpha = 0$

$EA = \frac{q_{enc}}{\epsilon_0} \Rightarrow E = \frac{q_{enc}}{\epsilon_0 A} = \frac{q_{enc}}{4\pi r^2 \epsilon_0}$

$\int E \cdot dA = E \int dA$

E is ~~one~~ constant on Gaussian surface

+1 1. $q_{enc} = 0 \Rightarrow E = 0$

+2 2. $E = \frac{Q_0}{36\pi\epsilon_0}$ or $\frac{Q_0 k}{9}$

+1 3. $q_{enc} = 0 \Rightarrow E = 0$

+2 4. $E = \frac{Q_0 + q}{4(81)\pi\epsilon_0}$ or $\frac{(Q_0 + q)k}{81}$

(b) Where and how much charge is on each surface of the two conductors?

+1 Inner sphere — $+Q_0$ on ~~outside~~ edge

+1 Shell
Inner edge $- Q_0$

+1 outer edge ~ $+Q_0 + q$

Picture +1 Bonus

(c) Using your expression for the electric field in part a, find the electric potential difference between the two spheres.

$\Delta V = -\int \vec{E} \cdot d\vec{s} = -\int_2^4 \frac{kQ\,dr}{r^2} = \frac{kQ}{r}\Big|_2^4 = \frac{kQ}{4} - \frac{2kQ}{4} = \frac{-kQ}{4}$

+ +1 +1 +1 +1

$= -\int |\vec{E}||d\vec{s}| \cos\alpha$ $\quad \alpha = 1$

$d\vec{s} = d\vec{r}$

$|\vec{E}| = \frac{kQ_0}{r^2}$ $\quad = -\int_{r=2m}^{r=4m} \frac{kQ}{r^2} dr =$