Physics 2049 Test 1 Solutions Spring 2003
Dr. Jeff Saul

Problem 1 (Short Answer/Multiple Choice)

Question	Direction of E-field	Magnitude of E-field
(a)	A	d
(b)	A	d
(c)	C	a

The key to this problem is recognizing how much charge is placed where and what electric field each charged object produces at the point in question. The E-field magnitudes are presented in the format (E_x, E_y). So the magnitude list in a can also be represented as follows:

$$\sqrt{8}(1,0) = \sqrt{8}\ (1\hat{i} + 0\hat{j})$$

Remember that during the test you are allowed to ask any question that helps to clarify the problem. This particular problem is almost identical to the last problem in the test notes.

Grading: 3 points for answering each part + 2 points per correct answer

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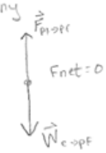
Problem 2 (Estimation) Student Solution

Consider a situation where you have two pennies, the bottom one on a table and the top one held 1 m above the table. Estimate how many electrons would need to be added to both pennies so that you can let go of the top penny and it would stay suspended in midair. Be sure to list all assumptions and explain all approximations.

Assumptions
Mass of penny = 5g = 5.0×10^{-3} kg
The number of electrons added to each is equal.
The mass of the extra electrons is negligable.

FBD on floating penny



e = earth
pf = floating penny
pt = table penny

$$\vec{F}_{electric} = \frac{K|q_1||q_2|}{r^2}$$

We need to set the weight force equal to the electric force so that $F_{net}=0$ so that $\vec{a}=0$. This is in accordance with Newton's 2nd law. $\vec{a} = \frac{\vec{F}_{net}}{m}$

$$|\vec{W}_{e\to pf}| = m_p g$$

$$|\vec{F}_{pt\to pf}| = \frac{K|q_{pf}||q_{pt}|}{r^2} = \frac{K|q|^2}{r^2}$$ since $q_{pf}=q_{pt}$ we can us q for both charges

$$\frac{K|q|^2}{r^2} = m_p g\ ,\quad q^2 = \frac{m_p g r^2}{K} \qquad q = \pm\sqrt{\frac{m_p g r^2}{K}}$$

then to find # of electrons simply use $\frac{1 \text{ electron}}{1.6\times10^{-19}\text{C}}$

$$\#\text{ of electrons} = \frac{\sqrt{\frac{m_p g r^2}{K}}}{1.6\times10^{-19}\text{C}} = \frac{\sqrt{\frac{5.0\times10^{-3}\text{kg}\cdot 10\text{N/kg}\cdot(1.0\text{m})^2}{8.99\times10^{9}\text{Nm}^2/\text{C}^2}}}{1.6\times10^{-19}\text{C}} = 1.474\times10^{13} \text{ electrons}$$

We would need to add 1.474×10^{13} electrons to each penny to make a 5g penny float 1m above the other.

Dr. Saul comments:
This is an excellent example of how an estimation problem should be done. The only improvement I would suggest is explaining the estimate of 5g for the mass of the penny.

Problem 3 (Essay 10 points)
You may use diagrams and equations but no calculations in your response for this problem.

Explain how you could charge two metal spheres to have opposite charges without rubbing them or touching them. Explain how you would test them to make sure they are oppositely charged.

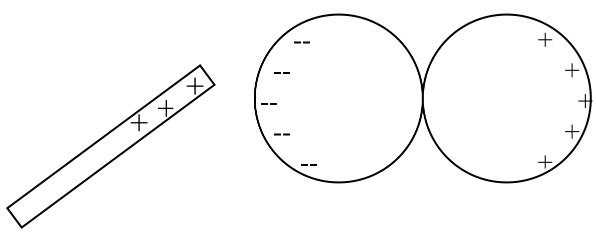
<u>*Key Points*</u>

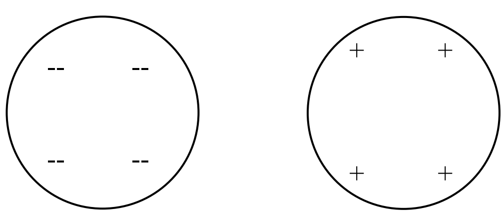
Charging in this way is called induced charge:

- *Spheres must be touching at first*
- *A charged object is brought towards one of the spheres on the side opposite where the two spheres are touching*
- *The charges object is brought near but does not touch the closer sphere. This causes the charge in the two neutral spheres to polarize with the charges unlike the charge of the charged object accumulating in the sphere closest to the charged object and the like charges accumulating in the sphere that is farthest away from the charged object.*
- *The spheres are separated while the charge object is near. Thus each sphere now has a net charge. The net charges on each sphere should be equal in magnitude and opposite in sign*
- *The charged object is taken away leaving the two separated spheres on the seen. Both spheres are now charges*

Testing the polarity of the charges:

- *Need at least two objects to test the charge of the spheres.*
    - *A charged object and a neutral object*
    - *Two charged objects with charges of opposing signs*
- *A neutral object should be attracted to both spheres => this would mean they are both charged*
- *A charged object should be attracted to one sphere and repelled by the other*
    - *If we had a positively charged object and a negatively charged object, one sphere would attract one and repel the other. The other sphere would have to repel the object that was attracted to the first sphere and attract the one that was repelled by the first sphere.*



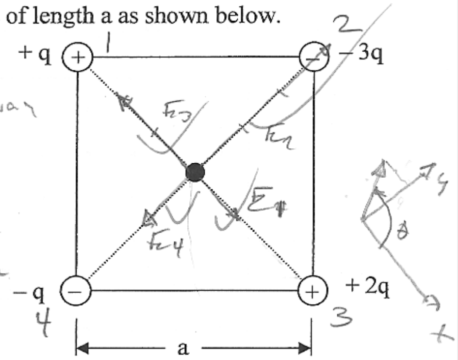


Problem 4 (20 points) *from 2 Student Solutions*

Four point charges are at the corners of a square with sides of length a as shown below.

A. Using graphical vector addition, find the E-field at the center of the square.

They are all the same distance away from the center &, their magnitudes are relative only to the coefficient of the charge. + (+) charges create and E field pointing away, (−) charges create an E field pointing towards. Use superposition to find E_T



subtracting opposite vectors → using parallelogram method

Dr. Saul comments: Excellent

B. Using vector components, algebraically find the E-field at the center of the square.

$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{2a^2}{4}} = \frac{a}{\sqrt{2}}$ same for all $\quad K = 8.99 \times 10^9 \frac{Nm^2}{C^2}$

$\vec{E}_1 = \frac{Kq}{d^2}\hat{i}$

$\vec{E}_2 = \frac{K3q}{d^2}\hat{j}$

$E_3 = -\frac{K2q}{d^2}\hat{i}$

$E_4 = -\frac{Kq}{d^2}\hat{j}$

$\vec{E}_T = \left(\frac{Kq}{d^2} - \frac{2Kq}{d^2}\right)\hat{i} + \left(\frac{3Kq}{d^2} - \frac{Kq}{d^2}\right)\hat{j}$

$\vec{E}_T = \left(-\frac{Kq}{d^2}\right)\hat{i} + \left(\frac{2Kq}{d^2}\right)\hat{j}$

$|E_T| = \sqrt{\left(-\frac{Kq}{d^2}\right)^2 + \left(\frac{2Kq}{d^2}\right)^2} = \sqrt{\frac{K^2q^2 + 4K^2q^2}{d^4}} = \frac{\sqrt{5}Kq}{d^2}$

$\boxed{|\vec{E}_T| = \frac{2\sqrt{5}Kq}{a^2}}$

Dr. Saul comments: This is a very good solution that is missing only 2 steps between calculating the E-field from each charged object and the total E-field $\vec{E}_T$. Note that the direction is indicated by $\vec{E}_T$ in unit vector ($\hat{i}\hat{j}$) form. The missing steps are as follows:

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = \frac{Kq}{d^2}\hat{i} + \frac{K3q}{d^2}\hat{j} + \frac{-K2q}{d^2}\hat{i} + \frac{-Kq}{d^2}\hat{j}$$

C. If q = 0.5 μC and a = 25 cm, what would be the magnitude and direction of the force on an electron placed at the center of the square.

$F = qE$

$= (0.5\,\mu C)\left(-\frac{2(8.99E9\,Nm^2/C^2)(0.5E{-}6C)}{(0.25)^2 m^2}\hat{i} + \frac{4(8.99E9\,Nm^2/C^2)(0.5E{-}6C)}{(0.25\,m)^2}\hat{j}\right)$

$= 0.5E{-}6C\left[143840\,\hat{i}\,N/C + 287680\,\hat{j}\,N/C\right]$

$= 0.07192\,N\hat{i} + 0.14384\,N\hat{j}$ −2

$|F| = \sqrt{(0.07192N)^2 + (0.14384N)^2}$

$= 0.161\,N$ negative charge

$\tan\theta = \frac{0.14384}{0.07192} \rightarrow \theta = \tan^{-1}\left(\frac{0.14384}{0.07192}\right) = 63°$ −1

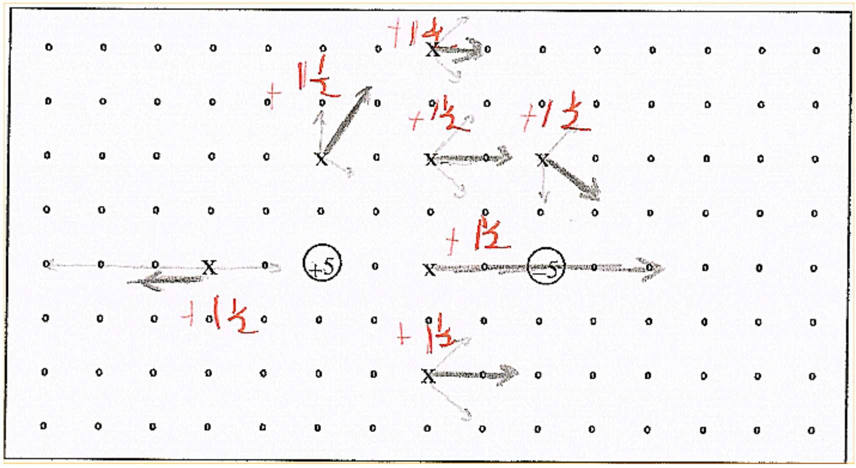
Dr. Saul comments: This is a good solution except for one mistake. Since we want the force on the electron:

$q = 1.6 \times 10^{-19}$ C, not 0.5 μC

Everything else looks good.

Problem 5 (15 points) Student Solutions

A) Sketch electric field vectors at the points marked x for the two equal and opposite point charges shown below.



Dr. Saul comments: A complete solution to this problem would include the E-field vectors form the individual charges as well as the net E-field. This solution has some elements of both E-fields at the 7 points in

(B) An electron (mass = 9.11 x 10^{-31} kg) is released from rest in a uniform electric field of magnitude 5000 N/C. Compute the magnitude of the electron's acceleration.

$\vec{F} = \vec{E}q$ +1   since we assume no other forces $\vec{F}_{net} = \vec{E}q$

$\vec{a} = \frac{\vec{F}_{net}}{m}$ +1 $\vec{a} = \frac{\vec{E}q}{m}$ +1 $\vec{a} = \left| \frac{5000 N/C \cdot -1.6\times10^{-19} C}{9.11\times10^{-31} kg} \right| =$ +1 $\boxed{8.78\times10^{14} m/s^2}$

Dr. Saul comments: Excellent

(C) Compute the distance the electron travels before reaching a speed of 2.0 x 10^6 m/s.

from rest → uniform field means uniform force
mean constant acceleration.

$v^2 = \cancel{v_0^2}^{0} + 2ax$ $v^2 = 2a\Delta x$

$\Delta x = \frac{v^2}{2a}$ = +1 $\frac{(2\times10^6 m/s)^2}{2(8.78\times10^{14} m/s^2)} = \frac{4\times10^{12} m^2/s^2}{1.756\times10^5 m/s^2}$

+1 $\boxed{\Delta x = 2.28\times10^{-3} m}$

Dr. Saul comments: Excellent