

Appendix A

Optics and Radiance

The power incident on a detector depends on the power emitted by the source and the optical system. Figure 1 defines the essential geometrical parameters. An infrared emitting source has area A_s , and the detector active area is A_d . The radiance L of the source has units of Watts/cm²-sr and is defined as the power emitted per unit source area per unit solid angle. As shown in Figure 1, if the detector is a distance r from the source, then the solid angle Ω the detector's active area subtends is $\frac{A_d}{r^2}$. For large Ω this solid angle is about the same for all area elements of the source, as indicated in the figure by the red and black lines. Integration over the entire source area gives the total power incident on the detector

$$P = L A_s \Omega_d \quad (1)$$

Of course, this assumes that none of the source elements are far off the optical axis. If this is not true, then the detector is tilted with respect to that source element, and the corresponding solid angle is reduced by a factor of $\cos\theta$, where θ is the angle between the ray from the source element and the normal of the detector. However, usually the source is sufficiently distant so that $\cos\theta \sim 1$ for all imaged source elements.

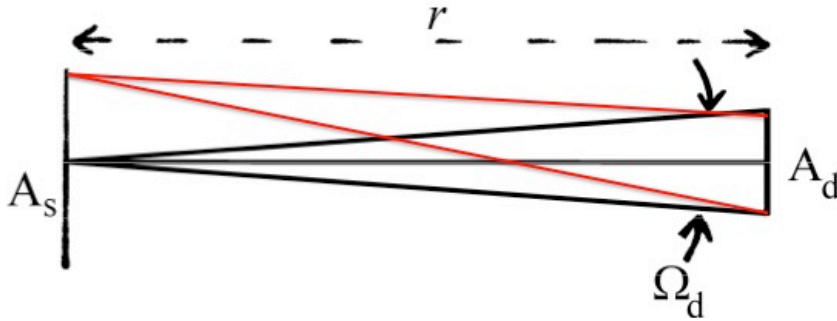


Fig. 1. Schematic of detector and source with geometrical parameters.

Generally, imaging optics are placed between the source and the detector, as shown schematically in Figure 2. In this case, there are 3 important solid angles, namely

$$\Omega_1 = \frac{A_{lens}}{p^2}, \quad \Omega_2 = \frac{A_s}{p^2}, \quad \Omega_3 = \frac{A_d}{q^2} \quad (2)$$

Rays from the extreme edges of the object pass undeflected through the center of the lens to the extremes of the image, which for optimal imaging coincide with the edges of the detector active area. These rays are indicated by red lines Fig. 2, which shows that $\Omega_2 = \Omega_3$.

which can be solved numerically. If we integrate over all wavelengths, we find the total radiance

$$L = \frac{\sigma}{\pi} T^4 \quad (10)$$

which is the power per unit source area per unit solid angle, where σ is the Stefan-Boltzmann constant $\sigma = \frac{2\pi^5 k^4}{15h^3 c^2}$ ($5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$). This assumes a Lambertian emitter where the radiance is proportional to $\cos(\theta)$, where θ is the angle with respect to the surface normal. Integrating over a hemisphere gives

$$L = \sigma T^4 \quad (11)$$

which is the Stefan-Boltzmann Law.

L changes with T according to

$$\frac{\partial L}{\partial T} = + \frac{2k^4 T^3}{h^3 c^2} \int_{x_2}^{x_1} \frac{e^x x^4}{(e^x - 1)^2} dx \quad (12)$$

Note that the integration variable is also a function of T . For small ΔT the dependence is linear, giving

$$\Delta L \approx \left. \frac{\partial L}{\partial T} \right|_{300K} \Delta T \quad (13)$$

for the differential radiance.

These additional factors influence the power incident on the detector: the pixel fill factor (ff), the optical transmission (τ) and the absorptivity of the detector (ϵ). The fill factor is the ratio of active area (intended for high absorption) to total area of the bolometer device. The optical transmission is the ratio of transmitted radiation to total incident radiation of the optical focusing lens. The absorptivity of the detector is the ratio of radiation absorbed by the detector to total incident radiation. Thus, from Eq. 7 and 13, the change in absorbed power resulting from a change in source temperature is

$$\Delta P = \left. \frac{\partial L}{\partial T} \right|_{300K} \Delta T \times \frac{\pi}{4} \times \frac{A_d}{(F/\#)^2} \times \tau \times (ff) \times \epsilon \quad (14)$$

References Cited

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